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Effect Of MFD Viscosity On Bénard-Marangoni Ferroconvection In A Rotating Ferrofluid Layer

R. Arunkumar, C. E. Nanjundappa
Department of Mathematics, Sai Vidya Institute of Technology, Bangalore- 560 064, India
Department of Mathematics, Dr. Ambedkar Institute of Technology, Bangalore -560 056, India
Corresponding Author: R. Arunkumar

ABSTRACT

The combined effect of Coriolis force due to rotation and magnetic field dependent (MFD) viscosity on the onset of Bénard-Marangoni convection in a horizontal layer of ferrofluid is investigated theoretically. The lower boundary is taken to be rigid with fixed temperature, while the upper free boundary at which temperature-dependent surface tension effect is considered is non-deformable and subject to a general thermal condition. The Rayleigh-Ritz's method is employed to extract the critical stability parameters numerically with thermal Rayleigh number R_t or Marangoni number Ma as the eigenvalue. The results reveal that, the Taylor number Ta, Biot number Bi and MFD viscosity parameter Λ reduces the intensity of Bénard-Marangoni ferroconvection, while an increase in the magnetic Rayleigh number R_m and the non-linearity of fluid magnetization parameter M_3 is to hasten the onset of ferroconvection in a rotating ferrofluid layer. Further the effect of increase in Ta and Bi as well as decrease in M_3 and R_m is to increase the critical wave number a_c and hence their effect is to decrease the dimension of convection cells.

KEYWORDS: Ferrofluid, Bénard-Marangoni convection, MFD viscosity, Coriolis force, Rayleigh-Ritz technique.

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I. INTRODUCTION

Thermogravitational convection in a layer of ferrofluid in the presence of a uniform magnetic field, known as ferroconvection, is analogous to classical Bénard convection and has received due attention in the literature because of promising potential in heat transfer applications. An extensive literature pertaining to this field is given in the books by Rosensweig [1], Berkovsky et al. [2] and Hergt et al. [3]. Ganguly et al. [4] have given an overview of prior research on heat transfer in ferrofluid flows and also discussed the heat transfer augmentation due to the thermomagnetic convection. In his review article, Odenbach [5] has focused on recent developments in the field of ferrofluids and their importance for the general treatment of ferrofluids. Nanjundappa and Shivakumara [6] have considered variety of velocity and temperature boundary conditions on the onset of ferroconvection in an initially quiescent ferrofluid layer. Kaloni and Mahajan [7] have studied the asymptotic stability of both equilibrium and arbitrary flows of ferrofluids. Shivakumara et al. [8] have investigated the onset of thermogravitational convection in a horizontal ferrofluid layer with viscosity depending exponentially on temperature.

A limited number of studies have addressed the effect of surface tension forces on ferroconvection in a horizontal ferrofluid layer. Linear and non-linear stability of combined buoyancy-surface tension effects in a ferrofluid layer heated from below is considered by Qin and Kaloni [9]. The coupling between Marangoni and Rosensweig instabilities by considering two semi-infinite incompressible and immiscible viscous fluids of infinite lateral extent in which one of them is ferromagnetic and the other is a usual Newtonian liquid is studied by Weiplepp et al. [10]. Shivakumara et al. [11] have investigated the effect of different forms of basic temperature gradients on the onset of ferroconvection driven by combined surface tension and buoyancy forces with an idea of understanding control of ferroconvection. The Rayleigh-Bénard-Marangoni instability in a ferrofluid layer in the presence of weak vertical magnetic field normal to the boundaries has been discussed by Hennenberg et al. [12]. The onset of Marangoni ferroconvection with different initial temperature gradients is analyzed by Shivakumara and Nanjundappa [13]. Shivakumara et al. [14] have investigated the onset of Brinkman-Benard-Marangoni convection in an initially quiescent magnetized ferrofluid saturated horizontal layer of a very coarse porous medium in the presence of a uniform vertical magnetic field. Nanjundappa et al. [15] have investigated the onset of Bénard-Marangoni ferroconvection with Internal Heat Generation in the presence of a uniform vertical magnetic field. The same authors[16] have studied effect of the temperature dependent viscosity on the Onset of Marangoni- Bénard ferroconvection in presence of a vertical magnetic field. Recently, Nanjundappa and Arunkumar [17] have studied the effects of cubic temperature profiles on ferroconvection in Brinkman porous medium.

The study of fluids in rotation is in itself an interesting topic for research. Ferrofluids are known to exhibit peculiar characteristics when they are set to rotation. Venkatasubramanian and Kaloni [18] have discussed the effect of rotation on thermo-convective instability of a horizontal layer of ferrofluid confined between stress-free, rigid-paramagnetic and rigid-ferromagnetic boundaries. Thermal convection in a rotating layer of a magnetic fluid is discussed by Auernhammer and Brand [19]. The weakly nonlinear instability of a rotating ferromagnetic fluid layer heated from below is studied by Kaloni and Lou [20]. Shivakumara and Nanjundappa [21] have studied the effects of Coriolis force and different basic temperature gradients on Marangoni ferroconvection. Shivakumara et al. [22] have investigated the onset of coupled Bénard-Marangoni convection in a rotating ferrofluid layer. Mahajan and Arora [23] have investigated the effect of rotation for convective instability in a thin layer of a magnetic nanofluid.

In view of the fact that rotation gives rise to interesting practical situations, the object of this paper is to study the combined effect of rotation and surface tension effects on the linear stability of Bénard-Marangoni ferroconvection. In this study, the lower rigid boundary is considered to be isothermal and the upper non-deformable free boundary is insulating to temperature perturbations. The resulting eigenvalue problem is solved numerically by employing the Galerkin technique with modified Chebyshev polynomials as trail functions. A comparative study is conducted to analyze on the onset of convection and also with the other works under the limiting conditions.

II. MATHEMATICAL FORMULATION

The physical configuration considered is as shown in Fig. 1. We consider an infinite horizontal layer of an electrically non-conducting Boussinesq ferromagnetic fluid of depth d permeated by uniform applied magnetic field H_0 acting in the vertical direction. The layer is rotating uniformly about its vertical axis with angular velocity $\vec{\Omega} = \Omega\,\hat{k},$ which is bounded below by a rigid-isothermal surface and above by a non-deformable free-insulating surface. A temperature drop ΔT is acting across the boundaries and a Cartesian co-ordinate system (x, y, z) is used with the origin at the bottom of the surface and z-axis vertically upwards. The surface tension σ is assumed to vary linearly with temperature as $\sigma = \sigma_0 - \sigma_T\,\Delta T,$ where σ_0 is the unperturbed value and σ_T is the rate of change of surface tension with temperature. The momentum equation is containing viscous force $2\nabla\cdot[\eta\ \underline{\underline{D}}],$ where $\underline{\underline{D}} = [\nabla\vec{q} + (\nabla\vec{q})^T]/2$ is the rate of strain tensor and $\vec{q} = (u, v, w)$ is the velocity. The fluid is assumed to be incompressible having variable viscosity, given by $\eta = \eta_0(1 + \vec{\delta}\cdot\vec{B})$, where $\vec{\delta}$ is the variation coefficient of magnetic field dependent viscosity and is considered to be isotropic (Vaidyanathan et al. [24]), η_0 is taken as viscosity of the fluid when the applied magnetic field is absent and $\vec{B} = (B_x,\ B_y,\ B_z)$ is the magnetic induction. Experimentally, it has been demonstrated that the magnetic viscosity has got exponential variation with respect to magnetic field (Rosenswieg [25]). As a first approximation for small field variation, linear variation of magnetic viscosity has been used.

The basic governing equations for the flow of an incompressible ferrofluid are:

$$\nabla\cdot\vec{q} = 0 \tag{1}$$

$$\rho_0\left[\frac{\partial\vec{q}}{\partial t} + (\vec{q}\cdot\nabla)\,\vec{q}\right] = -\nabla \mathrm{p} + \rho_0\left[1 - \alpha_t\,(T - T_0)\right]\vec{g} + 2\nabla\cdot[\eta\ \underline{\underline{D}}] + \nabla\cdot(\vec{H}\vec{B}) + 2\rho_0\,(\vec{\mathrm{q}}\times\vec{\Omega}) + \frac{\rho_0}{2}\nabla(\left|\vec{\Omega}\times\vec{r}\right|^2) \tag{2}$$

$$\left[\rho_0 C_{V,H} - \mu_0\vec{H}\cdot\left(\frac{\partial\vec{M}}{\partial T}\right)_{V,H}\right]\frac{DT}{Dt} + \mu_0 T\left(\frac{\partial\vec{M}}{\partial T}\right)_{V,H}\cdot\frac{D\vec{H}}{Dt} = k_t\nabla^2 T \tag{3}$$

$$\nabla\cdot\vec{\mathrm{B}} = 0\,,\ \nabla\times\vec{\mathrm{H}} = 0 \text{ or } \vec{H} = \nabla\varphi \tag{4}$$

$$\vec{B} = \mu_0(\vec{M} + \vec{H}) \tag{5}$$

$$\vec{M} = \frac{\vec{H}}{H} M(H,T) \tag{6}$$

$$M = M_0 + \chi(H - H_0) - K(T - \bar{T}) \tag{7}$$

where, p is the pressure, T is the temperature, t is the time, $\vec{B}$ is the magnetic induction, $\vec{H}$ is the intensity of magnetic field. $\vec{M}$ is the magnetization, ρ_0 is the reference density, α_t is the thermal expansion coefficient, μ_0 is the magnetic permeability of vacuum, k_t is the thermal conductivity, $\bar{T} = (T_0 + T_1)/2$ is the average temperature, $\chi = (\partial M / \partial H)_{H_0, T_0}$ is the magnetic susceptibility, $K = -(\partial M / \partial T)_{H_0, T_0}$ is the pyromagnetic co-efficient, $C_{V,H}$ is the specific heat capacity at constant volume and magnetic field per unit mass, $M_0 = M(H_0, \bar{T})$ is the saturation magnetization, $H = |\vec{H}|$ and $M = |\vec{M}|$. The fluid is assumed to be incompressible having variable viscosity.

The undisturbed basic quiescent state is given as follows:

$$\vec{q} = 0, \;\; p = p_b(z), \;\; \eta = \eta_b(z), \;\; T_b = T_0 - \beta z \left(\beta = \frac{\Delta T}{d} \right), \tag{8}$$

$$\vec{H}_b = \left[H_0 - \frac{K \beta z}{1+\chi} \right] \hat{k}, \quad \vec{M}_b = \left[M_0 + \frac{K \beta z}{1+\chi} \right] \hat{k}. \tag{9}$$

To study the stability of the system, we perturb all the variables in the form

$$[q, p, \eta, T, \vec{H}, \vec{M}] = [\vec{q}', p_b(z) + p', \eta = \eta_b(z) + \eta', T_b(z) + T', \vec{H}_b(z) + \vec{H}', \vec{M}_b(z) + \vec{M}'] \tag{10}$$

where, $\vec{q}'$, p', η', T', $\vec{H}'$ and $\vec{M}'$ are perturbed variables and are assumed to be small.

Taking curl of Eq. (2), using Eq. (10) and linearizing, the z-component of resulting equation is (after neglecting primes)

$$\rho_0 \frac{\partial \xi}{\partial t} = \eta \nabla^2 \xi + 2 \rho_0 \Omega \frac{\partial w}{\partial z} \tag{11}$$

which is the vorticity transport equation, where $\xi = \partial v / \partial x - \partial u / \partial y$ is the z-component of vorticity.

Substituting Eq. (10) in Eq. (2), on taking curl twice, linearizing and together with $\vec{H} = \nabla \varphi,$ the z-component of the resulting equation can be written as (after neglecting the primes)

$$\left[\rho_0 \frac{\partial}{\partial t} - \eta \nabla^2 \right] \nabla^2 w = \rho_0 \alpha_t g \nabla_1^2 T - 2 \rho_0 \Omega \frac{\partial \xi}{\partial z} - \mu_0 K \beta \frac{\partial}{\partial z} (\nabla_1^2 \varphi) + \frac{\mu_0 K^2 \beta}{1+\chi} (\nabla_1^2 T) \tag{12}$$

where, $\nabla_1^2 = \partial^2 / \partial x^2 + \partial^2 / \partial y^2$ is the horizontal Laplacian operator.

As before, Substituting Eq. (10) in Eq. (3) and linearizing, we obtain(neglecting primes)

$$\rho_0 C_0 \frac{\partial T}{\partial t} - \mu_0 K T_0 \frac{\partial}{\partial t} \left(\frac{\partial \varphi}{\partial z} \right) = \left(\rho_0 C_0 - \frac{\mu_0 K^2 T_0}{(1+\chi)} \right) w \beta + k_t \nabla^2 T \tag{13}$$

Equations (4), after substituting Eq. (10), may be written as (after dropping the primes)

$$\left(1 + \frac{M_0}{H_0} \right) \nabla_1^2 \varphi + (1+\chi) \frac{\partial^2 \varphi}{\partial z^2} - K \frac{\partial T}{\partial z} = 0 . \tag{14}$$

As is customary in convective instability analysis we assume the normal mode hypothesis or separation of variables. Each variable is expanded in the form

$$f(x,y,z,t)=f(z,t)e^{i(lx+my)} \quad (15)$$

where l and m are wave numbers in the x and y directions respectively.
Substituting Eq. (15) into Eqs. (11)-(14), we get

$$\left[\rho_0\frac{\partial}{\partial t}-\eta\left(\frac{\partial^2}{\partial z^2}-a^2\right)\right]\left(\frac{\partial^2}{\partial z^2}-a^2\right)w=-a^2\alpha_t g\,\theta+a^2\mu_0 K\beta\frac{\partial\varphi}{\partial z}-\frac{a^2\mu_0 K^2\beta}{(1+\chi)}\theta-2\rho_0\Omega\frac{\partial\xi}{\partial z} \quad (16)$$

$$\rho_0\frac{\partial\xi}{\partial t}=\eta\left(\frac{\partial^2}{\partial z^2}-a^2\right)\xi+2\rho_0\Omega\frac{\partial w}{\partial z} \quad (17)$$

$$\frac{\partial\theta}{\partial t}-\kappa\left(\frac{\partial^2}{\partial z^2}-a^2\right)\theta-\frac{\mu_0 K T_0}{\rho_0 C_0}\frac{\partial}{\partial t}\left(\frac{\partial\varphi}{\partial z}\right)=\left(1-\frac{\mu_0 K^2 T_0}{(1+\chi)\rho_0 C_0}\right)w\beta \quad (18)$$

$$(1+\chi)\frac{\partial^2\varphi}{\partial z^2}-\left(1+\frac{M_0}{H_0}\right)a^2\varphi-K\frac{\partial\theta}{\partial z}=0. \quad (19)$$

Thus, Eqs. (16)-(19) are governing linearized perturbation equations. The form of above equations are simplified by introducing the following non-dimensionalized quantities

$$(x^*,y^*,z^*)=\left(\frac{x}{d},\frac{y}{d},\frac{z}{d}\right),\quad \Lambda^*=\mu_0 H_0(1+\chi)\delta,\ w^*=\frac{d}{\nu}w,\quad a^*=a\,\mathrm{d},\ t^*=\frac{\nu}{d^2}t,$$

$$\xi^*=\frac{d^2}{\nu}\xi,\quad \theta^*=\frac{\kappa}{\beta\,\nu\,d}\theta,\quad \varphi^*=\frac{(1+\chi)\,\kappa}{K\,\beta\,\nu\,d^2}\varphi. \quad (20)$$

Thus Eqs. (16)-(19) become (after neglecting the asterisks)

$$\left[(1+\Lambda)(D^2-a^2)-\frac{\partial}{\partial t}\right](D^2-a^2)w=Ta^{1/2}D\xi+R_t\,a^2\theta-R_m\,a^2(D\varphi-\theta) \quad (21)$$

$$\left[(1+\Lambda)(D^2-a^2)-\frac{\partial}{\partial t}\right]\xi=-Ta^{1/2}Dw \quad (22)$$

$$\left(D^2-a^2-Pr\frac{\partial}{\partial t}\right)\theta+Pr\,M_2\frac{\partial}{\partial t}D\varphi=-(1-M_2)w \quad (23)$$

$$(D^2-a^2M_3)\varphi-D\theta=0. \quad (24)$$

Here, Ta is the Taylor number, R_t the thermal Rayleigh number, R_m the magnetic Rayleigh number, M_3 the measure of nonlinearity of magnetization, Pr the Prandtl number, Λ the non dimensional magnetic field dependent viscosity parameter, M_2 the non-dimensional parameter and is neglected in the subsequent analysis since its value is negligible.

All the above parameters affect the stability of the system in one way or the other, as the subsequent analysis only deals with the dimensionless variables. We set

$$\{w,\theta,\varphi,\xi\}(z,t)=\{W(z),\Theta(z),\Phi(z),\xi(z)\}e^{\omega t} \quad (25)$$

where, ω is the real or complex.

Using Eq. (25), Eqs. (21)-(24) can be written as

$$\left[(1+\Lambda)(D^2-a^2)-\omega\right](D^2-a^2)W=Ta^{1/2}D\xi+R_t\,a^2\Theta+R_m\,a^2(\Theta-D\Phi) \quad (26)$$

$$(D^2-a^2-Pr\omega)\Theta=-W \quad (27)$$

$$(D^2 - a^2 M_3)\,\Phi - D\Theta = 0 \quad (28)$$

$$\left[(1+\Lambda)(D^2 - a^2) - \omega\right]\xi = -Ta^{1/2}\,DW\,. \quad (29)$$

The corresponding boundary conditions for the perturbed non-dimensional variables take the form

W = DW = $\Theta = \Phi = \xi = 0$ at z = 0 (30)

$W = (1+\Lambda)D^2W + Ma\;a^2\,\Theta = D\Phi = D\Theta + Bi\,\Theta = D\xi = 0$ at z = 1, (31)

where, $Ma = \sigma_T\,\Delta T\,d\,/\,\mu\kappa$ is the Marangoni number and $Bi = h\,d\,/\,k_t$ is the Biot number.

III. METHOD OF SOLUTION

The Eqs.(26)-(29) together with the boundary conditions (30) and (31) constitute an eigenvalue problem with R_t or Ma as an eigenvalue. To solve the resulting eigenvalue problem, Rayleigh-Ritz method is used. Accordingly, the variables are written in a series of basis functions as

$$W = \sum_{i=1}^{n} A_i\,W_i(z)\,,\;\; \Theta(z) = \sum_{i=1}^{n} C_i\,\Theta_i(z)\,,\;\; \Phi(z) = \sum_{i=1}^{n} D_i\,\Phi_i(z)\,,\;\; \xi = \sum_{i=1}^{n} E_i\,\xi_i(z) \quad (32)$$

where, the trial functions $W_i(z)$, $\xi_i(z)$, $\theta_i(z)$ and $\Phi_i(z)$ will be generally chosen in such a way that they satisfy the respective boundary conditions and $A_i\,, C_i\,,\, D_i$ and E_i are constants.

Substituting Eq. (32) into Eqs. (26)-(29), multiplying the resulting momentum Eq. (26) by $W_j(z)$ energy Eq. (27) by $\Theta_j(z)$, magnetic potential Eq. (28) by $\Phi_j(z)$ and vorticity Eq. (29) by $\xi_j(z)$, performing the integration by parts with respect to z between z = 0 and z = 1 and using the boundary conditions, we obtain the following system of linear homogeneous algebraic equations:

$$C_{ji}A_i + D_{ji}C_i + E_{ji}D_i + F_{ji}E_i = 0$$

(33)

$$G_{ji}A_i + H_{ji}C_i = 0$$

(34)

$$I_{ji}C_i + J_{ji}D_i = 0$$

(35)

$$K_{ji}A_i + L_{ji}E_i = 0\,.$$

(36)

The coefficients $C_{ji} - L_{ji}$ involve the inner products of the basis functions and are given by

$$C_{ji} = (1+\Lambda)\left[< D^2W_j D^2W_i > + 2a^2 < DW_j DW_i > + a^4 < W_j W_i >\right] + \omega\left[< DW_j DW_i > + a^2 < W_j W_i >\right]$$

$$D_{ji} = -a^2(R_t + R_m) < \Theta_j W_i > + a^2 Ma\,DW_j(1)\,\Theta_i(1)$$

$$E_{ji} = a^2 R_m < W_j D\Phi_i >$$

$$F_{ji} = -Ta^{1/2} < W_j D\zeta_i >$$

$$G_{ji} = - < \Theta_j W_i >$$

$$H_{ji} = < D\Theta_j D\Theta_i > + (a^2 + \omega Pr) < \Theta_j \Theta_i > + Bi\,\Theta_j(1)\Theta_i(1)$$

$$I_{ji} = < \Phi_j D\Theta_i >$$

$$J_{ji} = < D\Phi_j D\Phi_i > + a^2 M_3 < \Phi_j \Phi_i >$$

$$K_{ji} = -Ta^{1/2} < \zeta_j DW_i >$$

$$L_{ji} = (1+\Lambda)\left[< D\zeta_j D\zeta_i > + a^2 < \zeta_j \zeta_i >\right] + \omega < \zeta_j \zeta_i >$$

where, the inner product is defined as $<\cdots\cdots> = \int_0^1 (\cdots)\, dz.$

The above set of homogeneous algebraic equations can have a non-trivial solution if and only if

$$\begin{vmatrix} C_{ji} & D_{ji} & E_{ji} & F_{ji} \\ G_{ji} & H_{ji} & 0 & 0 \\ 0 & I_{ji} & J_{ji} & 0 \\ K_{ji} & 0 & 0 & L_{ji} \end{vmatrix} = 0. \qquad (37)$$

The eigenvalues have to be extracted from the above characteristic equation. For this, we select the trial functions as

$$\mathrm{W_i} = z^2(1-\mathrm{z})\,\mathrm{T}^*_{\mathrm{i-1}}, \quad \Theta_i = z(1-\mathrm{z}/2)\,\mathrm{T}^*_{\mathrm{i-1}}, \quad \Phi_i = \xi_i = \mathrm{z}^2(1-2\,z/3)\,\mathrm{T}^*_{\mathrm{i-1}} \qquad (38)$$

where, $\mathrm{T}^*_{\mathrm{i}}$ s are the modified Chebyshev polynomials such that they satisfy all the corresponding boundary conditions except the one, namely $D^2W + Ma\, a^2\Theta = 0 = D\Theta + Bi\,\Theta$ at z = 1 but the residual from this equation is included as a residual from the differential equation.

At this juncture, it would be instructive to look at the results for $i = j = 1$ and for this order Eq. (37) gives the following characteristic equation

$$Ma = -\frac{(\eta_1 + 2\,\omega Pr)}{1575\,a^2 <W\,\Theta>}\left[\frac{147\,Ta}{2(\eta_2 + 13\,\omega)} + 2(\eta_3 + \eta_4\omega)\right] - \frac{63\,R_m <WD\Phi>}{2\eta_5} - 2(R_m + R_t) <W\Theta> \qquad (39)$$

where, $\eta_1 = 2\,a^2 + 5 + 15Bi/4$, $\eta_2 = (1+\Lambda)(42 + 13\,a^2)$, $\eta_3 = (1+\Lambda)(a^4 + 28\,a^2 + 420)$, $\eta_4 = 14 + a^2$ and $\eta_5 = 42 + 13\,M_3\,a^2$.

To examine the stability of the system, we take $\omega = i\,\omega$ in Eq. (39) and clear the complex quantities, we obtain,

$$Ma = -\frac{1}{1575\,a^2 < W\,\Theta >}\left[\frac{147\,Ta\,(\eta_1\eta_2 + 26\,\omega^2\;Pr)}{2(\eta_2^2 + 169\,\omega^2)} + 2\,(\eta_1\eta_3 - 2\,\omega^2\,\eta_4 Pr)\right] - 2\,(R_m + R_t) <W\,\Theta> - \frac{63\,R_m < W\,D\Phi >}{2\,\eta_5} + i\,\omega\,\Delta \qquad (40)$$

where, $\Delta = -\dfrac{1}{1575a^2 < W\,\Theta>}\left[\dfrac{147\,Ta\,(2\,\eta_2 Pr - 13\eta_1)}{2(\eta_2^2 + 169\omega^2)} + 2\,(2\eta_3 Pr + \eta_1\eta_4)\right].$

Since Ma *is* a physical quantity it must be real, so that it implies either $\omega = 0$ or $\Delta = 0$ (i.e. $\omega \neq 0$) and accordingly the condition for steady and oscillatory onset is obtained.

The steady onset is governed by $\omega = 0$ and it occurs at $Ma = Ma^s$, where

$$Ma^s = -\frac{\eta_1}{1575\,a^2 < W\,\Theta >}\left(\frac{147\,Ta}{2\eta_2} + 2\,\eta_3\right) - 2\,(R_m + R_t) <W\,\Theta> - \frac{63\,R_m < W\,D\Phi >}{2\eta_5}. \qquad (41)$$

The oscillatory convection occurs at $Ma = Ma^0$, where

$$Ma^0 = -\frac{2(a_1 a_4^2 + a_2 a_4 + a_3/2)}{1575 a_4 a^2 <W\Theta>} - 2(R_m + R_t)<W\Theta> - \frac{63 R_m <W D\Phi>}{2\eta_5}. \quad (42)$$

Here, $a_1 = \eta_1 \eta_2 - \frac{26}{169} Pr\, \eta_2^2$, $\quad a_2 = \eta_1 \eta_3 + \frac{2}{169} Pr\, \eta_4 \eta_2^2 + \frac{147}{26} Ta\, Pr$,

$$a_3 = -\frac{147}{169} Ta\, Pr\, \eta_4 \quad \text{and} \quad a_4 = \frac{\eta_1 \eta_4 + 2Pr\, \eta_3}{13\eta_1 - 2Pr\, \eta_2}.$$

The corresponding frequency of oscillations is given by

$$\omega^2 = -\frac{\eta_2^2}{169} + \frac{147 Ta}{52\eta_4}\left[\frac{1 - 2\beta_1 Pr}{1 + 2\beta_2 Pr}\right] \quad (43)$$

where, $\beta_1 = \frac{(1+\Lambda)(42 + 13a^2)}{65 + 26a^2 + 195Bi/4}$ and $\beta_2 = \frac{(1+\Lambda)(a^4 + 28a^2 + 420)}{2a^4 + 33a^2 + 70 + (14 + a^2)Bi/4}$.

For the occurrence of oscillatory onset ω^2 should be positive and the necessary conditions for the same are

$$Pr < \frac{(a^2 + 2.5 + 1.875Bi)}{(1+\Lambda)(a^2 + 3.23)} \quad \text{and} \quad Ta > \frac{52}{24843}\eta_2^2 \eta_4 \left[\frac{1 + 2\beta_2 Pr}{1 - 2\beta_1 Pr}\right].$$

(44)

It is thus evident that for the oscillatory onset to exist the Prandtl number Pr should be less than unity as observed in the classical viscous liquids. But for most of the ferrofluids, whether it is water based or any other organic liquid based, Prandtl number is greater than unity and hence the overstability is not a preferred mode of instability. In what follows we restrict ourselves to the case of steady onset and put $\omega = 0$ in Eq. (37). A nontrivial solution to the system requires the characteristic determinant of the coefficient matrix must vanish and this leads to a relation involving the parameters R_t, Ma, R_m, M_1, M_3, Λ, Bi, Ta and a in the form

$$f(R_t, Ma, R_m, M_1, M_3, \Lambda, Bi, Ta, a) = 0. \quad (45)$$

The critical values of R_{tc} or Ma_c are found as a function of wave number a for various values of physical parameters. The results presented here are for $i = j = 8$ the order at which the convergence is achieved, in general.

IV. RESULTS AND DISCUSSIONS

The linear stability theory is used to investigate the effects of Coriolis force and MFD viscosity on coupled Bénard-Marangoni ferroconvection in a ferrofluid layer rotating about the vertical axis. The fluid layer is heated from below and its upper surface is subjected to a surface tension decreasing with temperature. The resulting eigenvalue problem is solved by employing Rayleigh-Ritz's method with either thermal Rayleigh number (R_t) or Marangoni number (Ma) as the eigenvalue. Computations reveal that the convergence in finding Ma_c crucially depends on the value of Ta, and for higher value of Ta more number of terms in the expansion of dependent variables were found to be required. The results presented here are for $i = j = 8$ the order at which the convergence is achieved, in general. The critical Marangoni number is determined as a function of wave number by taking all the other parameters as given. The results thus obtained for different values of physical parameters are presented in Tables 1- 3 and graphically in Figs.2-8.

In order to validate the numerical solution procedure used, first the critical values (Ma_c, a_c) obtained from the present study under the limiting conditions are compared with the previously published results of Vidal and Acrivos [26] in Table 1. The results tabulated in Table 1 for different values of *Ta* are for $Bi = R_t = R_m = \Lambda = 0$ (i.e., classical Marangoni convection for non-ferrofluids). In order to compare the results of the present analysis with those of Qin and Kaloni [9] obtained numerically a new magnetic parameter S was introduced in the analysis. The critical values obtained for different values of Ma_c with values of $S\,(=10^{-4})$ and $Bi\,(=0,\ 10)$ are exhibited in Table 2. From the values presented in Tables 1 and 2, it is evident that there is an excellent agreement between the results of the present study and the previously published ones. This verifies the applicability and accuracy of the method used in solving the convective instability problem considered.

The tight coupling between buoyancy, surface tension, magnetic and Coriolis forces is exhibited quantitatively by tabulating the values of triplets (R_{tc} , Ma_c , R_{mc}) for different values of Ta with $\Lambda = 0.2$ and $Bi = 2$ in Table 3. From the table, it can be seen that an increase in M_3 is to decrease R_{mc} but only marginally and thus it has a destabilizing effect on the stability of the system. This may be due to the fact that the application of magnetic field makes the ferrofluid to acquire larger magnetization which in turn interacts with the imposed magnetic field and releases more energy to drive the flow faster. Hence, the system becomes unstable with a smaller temperature gradient as the value of M_3 increases. From the Table 3, we note that an increase in M_3 is to increase a_c and hence its effect is to decrease the dimension of convection cells. Besides, as M_3 increases, R_{mc} decreases and the results reduce to that of classical Bénard-Marangoni problem for ordinary viscous fluids as $M_3 \to \infty$. That is, $R_{mc} = R_{tc}$ as $M_3 \to \infty$.
The salient characteristics of these physical parameters are exhibited graphically in Figs.2-8 for various values of Taylor number Ta. Figs. 2(*a*)-4(*a*) show the locus of the critical Marangoni number Ma_c and thermal Rayleigh number R_{tc} for different Λ, Bi and M_1 respectively. From these figures, it is obvious that the curves are slightly convex and there is a strong coupling between the critical thermal Rayleigh and the Marangoni numbers, and an increase in the thermal Rayleigh number has a destabilizing effect on the system. Thus, when the buoyancy force is predominant, the surface tension force becomes negligible and vice-versa.

The effects of both buoyancy and surface tension forces are considered together on the onset of ferroconvection in a rotating ferrofluid layer. Fig. 2(*a*) shows the locus of Ma_c and R_{tc} for different values of Ta with two values of Λ (= 0 and 0.5) when $M_3 = 1$, $M_1 = 2$ and $Bi = 2$. From the figure, the extent to which the surface tension effect is diminished due to R_{tc} however, depends on the strength of rotation and also the viscosity variation with respect to magnetic field dependent viscosity parameter Λ. The critical thermal Rayleigh number R_{tc} and Marangoni number Ma_c increase with an increase in the Taylor number and this indicates the presence of Coriolis force due to rotation is to suppress the Bénard-Marangoni ferroconvection. For Taylor number $Ta \le 10^3$, the effect of Coriolis force is not so significant, while for $Ta > 10^3$ a rapid increase in the critical thermal Rayleigh number and Marangoni number could be seen. As $Ta \to \infty$, the Bénard-Marangoni ferroconvection ceases to exist and the corresponding R_{tc} and Ma_c become infinite. Besides, from Fig. 2(*a*), it is seen that R_{tc} and Ma_c increase with an increase in the MFD viscosity parameter Λ and thus it has a stabilizing effect on the system. That is, the effect of increasing Λ is to delay the onset of Bénard-Marangoni ferroconvection. From Fig. 2(b), we note that increase in the value of Λ is to decrease the critical wave number a_c and thus to widen the size of convection cells and opposite is the case with an increasing in the value of Taylor number Ta.

The plots in Fig. 3(*a*) represents the locus of Ma_c and R_{tc} for different values of Ta with two values of Biot number Bi (= 1 and 2) when $\Lambda = 0.2$, $M_3 = 1$, $M_1 = 2$. From the figure it is evident that an increase in the value of heat transfer coefficient Bi (i.e., Biot number) is to increase R_{tc} as well as Ma_c and thus its effect is to delay the onset of Bénard-Marangoni ferroconvection. This may be attributed to the fact that with increasing Bi, the thermal disturbances can easily dissipate into the ambient surrounding due to a better convective heat transfer coefficient at the top surface and hence higher heating is required to make the system unstable. Fig. 3(b) represents the corresponding critical wave number a_c and it indicates that increase in the value of Bi and R_t is to increase a_c and thus their effect is to reduce the size of convection cells.

The locus of R_{tc} and Ma_c is shown in Fig. 4(*a*) for different values of Ta with two values of M_1 (= 0 and 1) when $M_3 = 1$, $Bi = 2$, $\Lambda = 0.2$. It is observed that an increase in the value of M_1 (i.e., increase in the destabilizing magnetic force) is to decrease the values of R_{tc}, Ma_c and makes the system more unstable due to an increase in the destabilizing magnetic force. That is to say that the buoyancy and magnetic forces are complementary to each other. Fig. 4(b) illustrates that increase in the value of M_1 is to decrease the critical wave number a_c slightly and thus to increase the size of convection cells.

Figures 5-8 show the critical values Ma_c and R_{tc} as well as corresponding a_c for different values of Ta, Bi, R_m and M_3 respectively as a function of MFD viscosity parameter Λ. From the figures, it is seen that $Ma_c < R_{tc}$ and the effect of increasing Λ is to delay the onset of Bénard/Marangoni ferroconvection. Further, increase in Ta (Fig. 5*a*) and Bi (Fig. 6*a*), and decrease in R_m (Fig. 7*a*) and M_3 (Fig. 8*a*) is to increase the critical thermal Rayleigh/Marangoni number and hence has a stabilizing effect on the system. Moreover, increase in Ta (Fig. 5b), Bi (Fig. 6b), R_m (Fig. 7b) and M_3 (Fig. 8b) is to decrease the width of convection cells. The critical wave numbers a_c for Bénard ferroconvection are always found to be higher than those of pure Marangoni ferroconvection (see Figs. 5(b)-8(b)). Further inspection of these figures reveals that an increasing the values of Λ is to decrease the critical wave number a_c and thus to increase the size of convection cells.

V. CONCLUSIONS

The combined effect of Coriolis force due to rotation and magnetic field dependent (MFD) viscosity on the onset of Bénard-Marangoni convection in a horizontal layer of ferrofluid is investigated theoretically. The lower boundary is taken to be rigid with fixed temperature, while the upper free boundary at which temperature-dependent surface tension effect is considered is non-deformable and subject to a general thermal condition. The Rayleigh-Ritz's method is employed to extract the critical stability parameters numerically with thermal Rayleigh number R_t or Marangoni number Ma as the eigenvalue. Comparisons with previously published works are performed and excellent agreement between the results is obtained. From the foregoing study, the following conclusions may be drawn:

1. The critical thermal Rayleigh number R_{tc} and Marangoni number Ma_c increases with an increase in the Taylor number Ta and this indicates the presence of Coriolis force due to rotation is to reduce the intensity of Bénard-Marangoni ferroconvection.
2. The critical thermal Rayleigh number R_{tc} increases with an increase in the value of Biot number Bi and MFD viscosity parameter Λ and thus their effect is to delay the onset of Bénard-Marangoni ferroconvection.
3. The effect of increasing the value of magnetic Rayleigh number R_m and the non-linearity of fluid magnetization parameter M_3 is to hasten the onset of ferroconvection in a rotating ferrofluid layer.
4. The buoyancy force and surface tension force complement with each other and it is always found that $Ma_c < R_{tc}$; a result in accordance with ordinary viscous fluids.
5. As $M_3 \to \infty$, the results reduce to that of the Bénard-Marangoni convection problem for ordinary viscous fluids.
6. The effect of increase in Ta and Bi as well as decrease in M_3 and R_m is to increase the critical wave number a_c and hence their effect is to decrease the dimension of convection cells.

Table 1 Comparison of Ma_c and a_c for different values of Ta when $R_m = 0$, $\Lambda = 0$ and $R_t = 0$.

Ta	Vidal and Acrivos [26] Ma_c	Vidal and Acrivos [26] a_c	Present study Ma_c	Present study a_c
0	80	2.0	79.61	1.99
10^2	92	2.2	91.31	2.17
10^3	164	3.0	163.11	2.97
10^4	457	5.0	456.21	4.99
10^5	1400	8.6	1400.45	8.82

Table 2 Comparison of critical values of R_{tc} and R_{mc} for different values of Ma and Bi when $\Lambda = 0$, $M_3 = 1$, $Ta = 0$ and $S = 10^{-4}$.

Bi	Ma	Present Analysis R_{tc}	Present Analysis R_{mc}	Qin and Kaloni [9] R_{tc}	Qin and Kaloni [9] R_{mc}
0	**0**	637.875	40.688	652.87	42.624
	10	566.418	32.083	572.11	32.731
	20	492.593	24.265	493.33	24.426
	30	416.358	17.335	414.72	17.199
	40	337.656	11.401	335.98	11.255
	50	256.414	6.575	254.06	6.455
	60	172.539	2.977	171.44	2.939
	70	85.9213	0.738	85.67	0.734
	79.61	0.000	0.000	0.000	0.000
10	**0**	934.009	87.237	892.06	79.577
	50	843.914	71.219	809.25	65.489
	100	748.641	56.046	721.01	51.981
	150	647.822	41.967	628.88	39.298
	200	540.996	29.268	526.21	27.690
	250	427.582	18.283	418.23	17.492
	300	306.831	9.414	301.89	9.114
	350	177.771	3.160	176.10	3.101
	413.44	0.000	0.000	0.000	0.000

Table 3 Critical instability parameters R_{tc} and R_{mc} for different values of Ma and Ta when Λ =0.2 and $Bi = 2$.

Ta	Ma	$R_m = 0$		$R_t = 0$, $M_3 = 1$		$R_t = 0$, $M_3 = 15$		$R_t = 0$, $M_3 = 25$		$R_t = 0$, $M_3 \to \infty$	
		R_{tc}	a_c	R_{mc}	a_c	R_{mc}	a_c	R_{mc}	a_c	R_{mc}	a_c
0	0	997.524	2.392	1256.036	2.453	1052.822	2.465	1033.453	2.444	997.524	2.392
	50	747.670	2.358	934.177	2.398	789.697	2.413	774.991	2.397	747.670	2.358
	100	478.679	2.348	592.955	2.371	505.603	2.382	496.178	2.373	478.679	2.348
	150	189.353	2.363	232.373	2.371	199.833	2.376	196.168	2.372	189.353	2.363
	180.815	0.0	2.386	0.0	2.386	0.0	2.386.	0.0	2.386	0.0	2.386
10^2	0	1098.155	2.544	1376.386	2.612	1153.609	2.611	1133.925	2.591	1098.155	2.544
	50	851.795	2.509	1059.861	2.556	895.292	2.561	879.862	2.546	851.795	2.509
	100	585.018	2.496	721.991	2.524	614.901	2.531	604.290	2.520	585.018	2.496
	150	296.462	2.463	362.621	2.519	311.418	2.523	306.094	2.518	296.462	2.463
	197.515	0.0	2.540	0.0	2.540	0.0	2.540	0.0	2.540	0.0	2.540
10^3	0	1763.335	3.272	2153.833	3.373	1822.971	3.320	1800.921	3.304	1763.345	3.272
	50	1538.723	3.239	1869.601	3.320	1590.632	3.282	1571.433	3.268	1538.732	3.239
	100	1290.302	3.218	1558.476	3.280	1333.555	3.254	1317.549	3.242	1290.310	3.218
	150	1015.721	3.212	1218.796	3.255	1049.386	3.239	1036.916	3.230	1015.727	3.212
	300.275	0.0	3.307	0.0	3.307	0.0	3.307	0.0	3.307	0.0	3.307
5×10^3	0	3547.423	4.383	4616.724	3.566	3620.804	4.417	3592.942	4.405	3547.435	4.383
	50	3363.088	4.363	4355.888	3.566	3431.565	4.395	3405.549	4.384	3363.099	4.363
	100	3158.108	4.346	4075.692	3.566	3221.322	4.376	3197.289	4.365	3158.118	4.346
	150	2929.717	4.333	3774.269	3.566	2987.289	4.360	2965.387	4.351	2929.727	4.333
	529.256	0.0	4.559	0.0	4.559	0.0	4.559	0.0	4.559	0.0	4.559

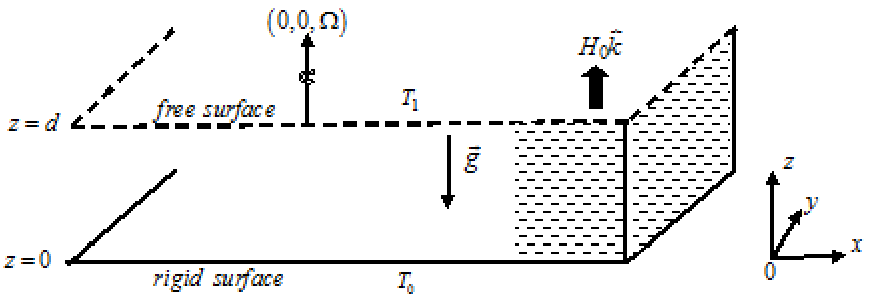
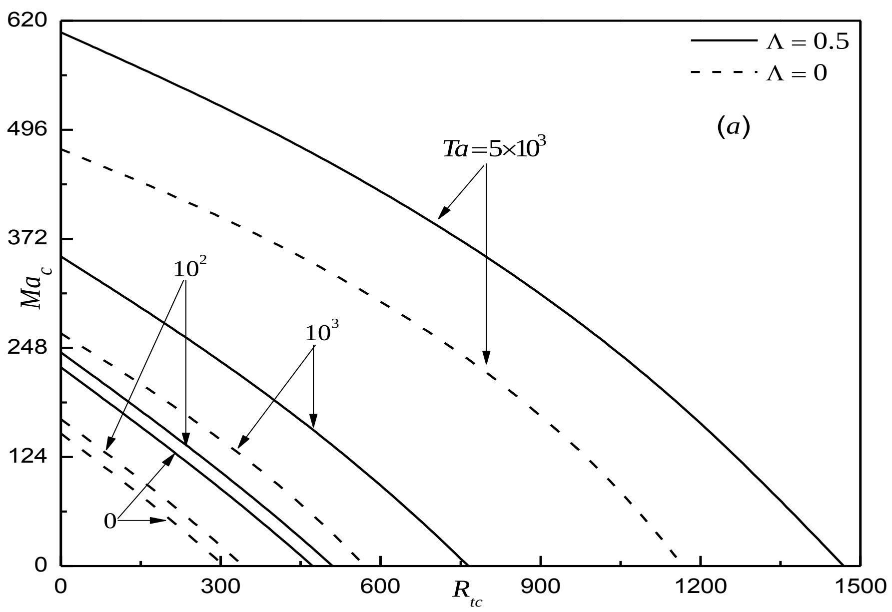


Fig. 1. Physical configuration



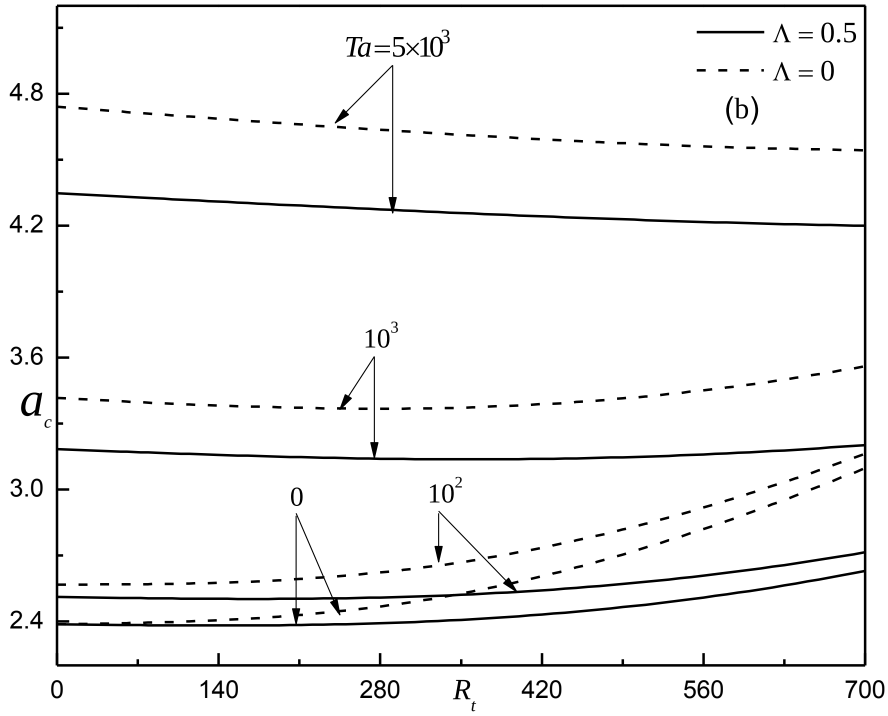
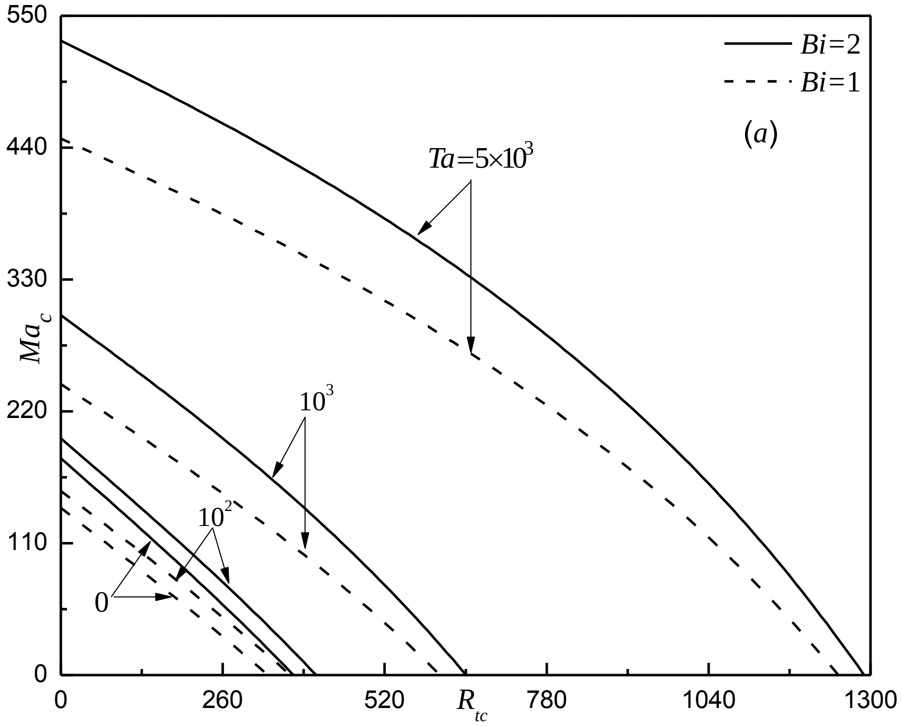


Fig. 2 Plots of (*a*) Ma_c verses R_{tc} and (b) a_c verses R_t for different values of Ta with two values of Λ when $M_3 = 1,\ M_1 = 2$ and $Bi = 2$.



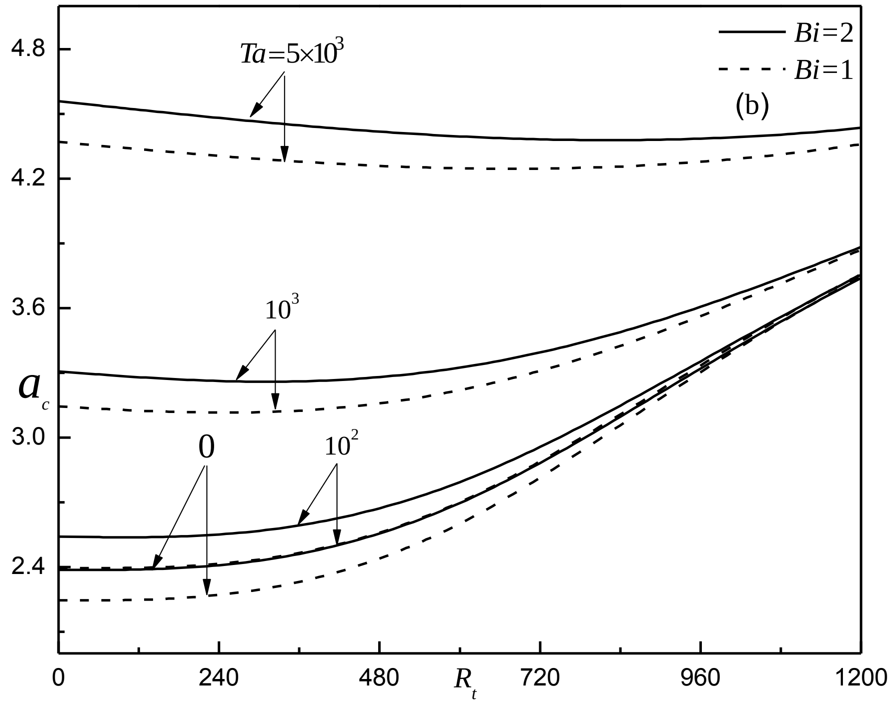
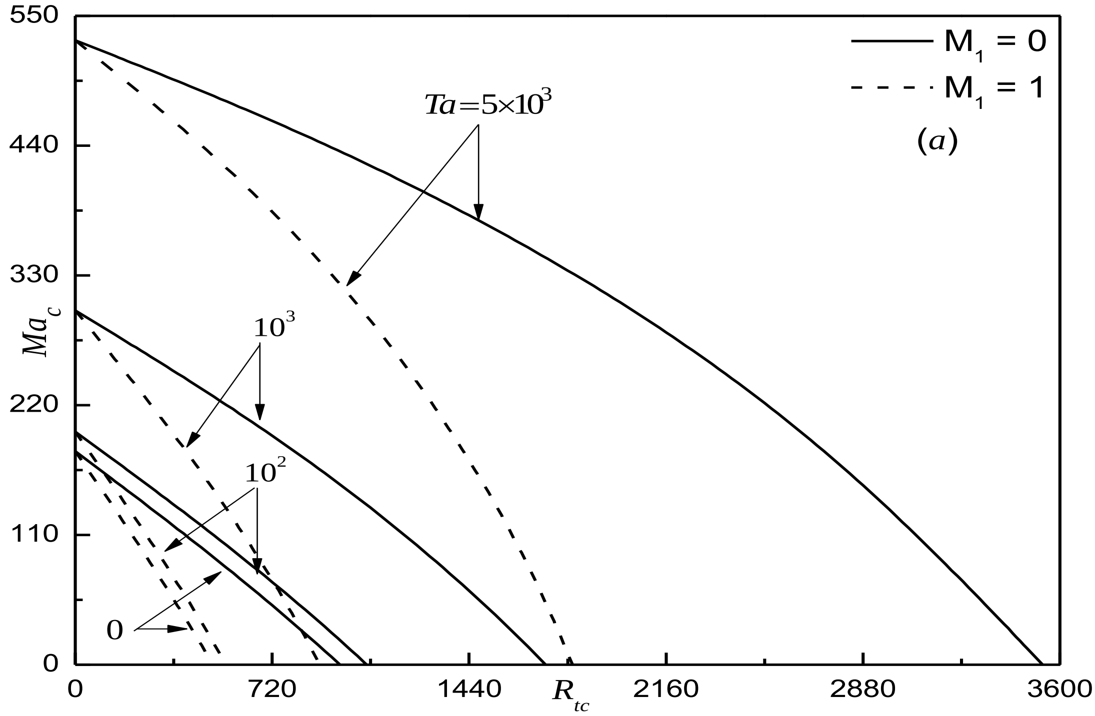


Fig. 3 Plots of (*a*) Ma_c verses R_{tc} and (b) a_c verses R_t for different values of Ta with two values of Bi when $M_3 = 1$, $M_1 = 2$ and $\Lambda = 0.2$.



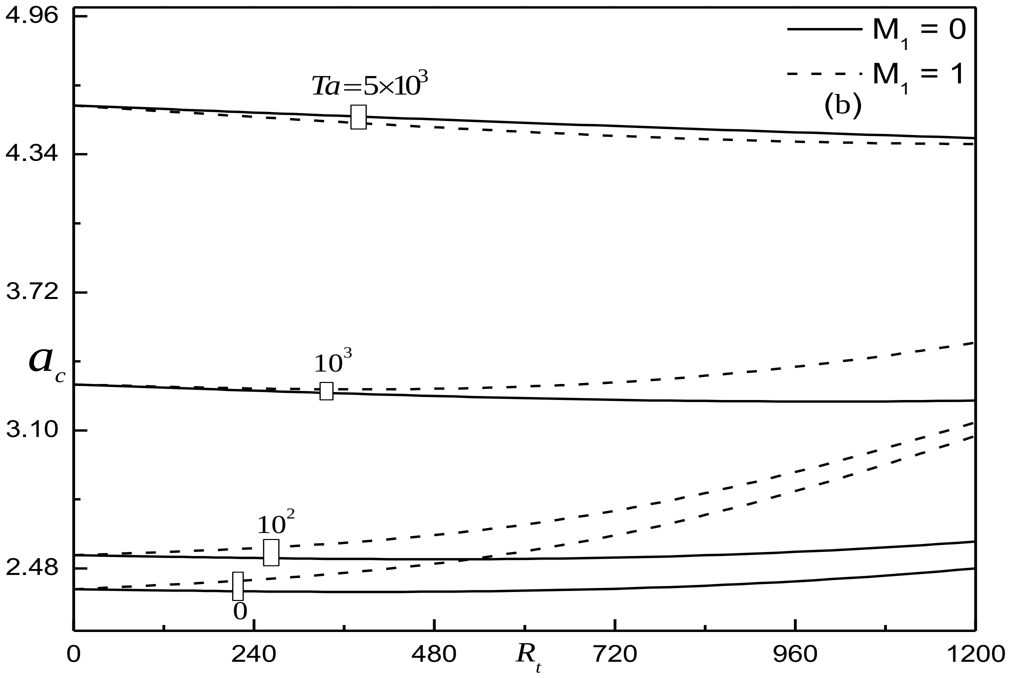
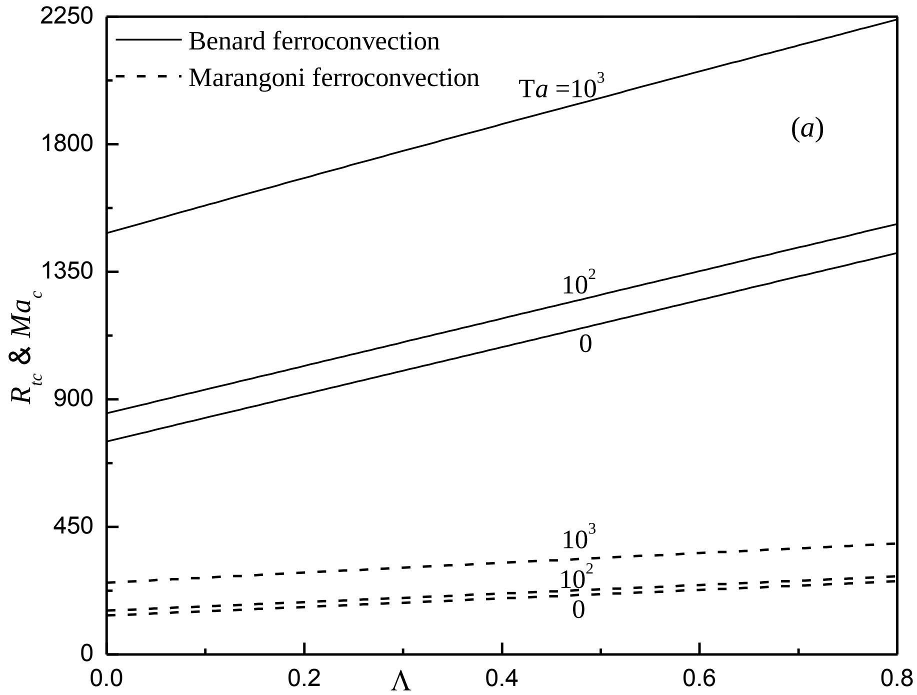


Fig. 4 Plots of (*a*) Ma_c verses R_{tc} and (b) a_c verses R_t for different values of Ta with two values of M_1 when $M_3 = 1,\ Bi = 2$ and $\Lambda = 0.2$.



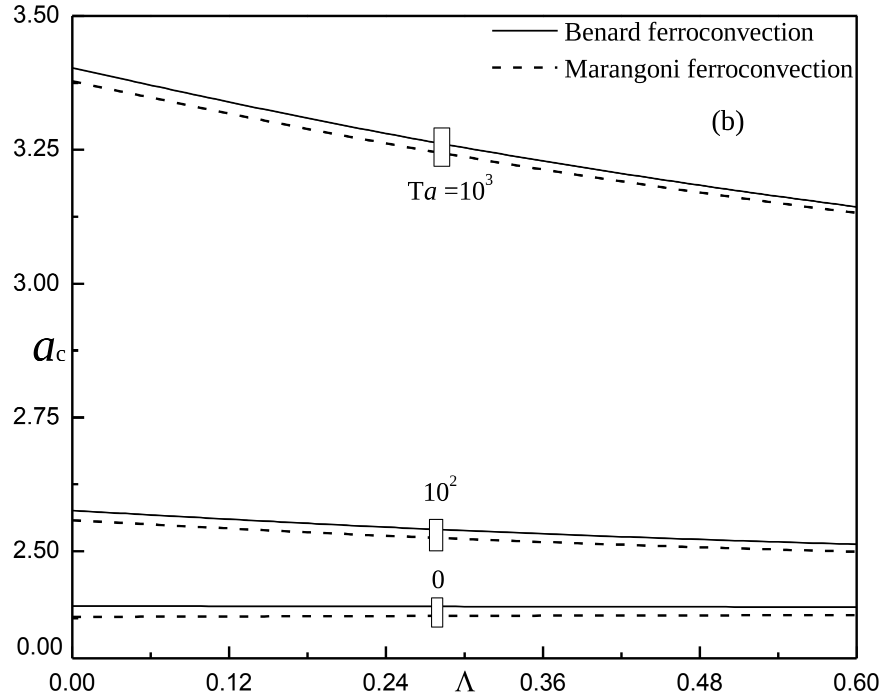
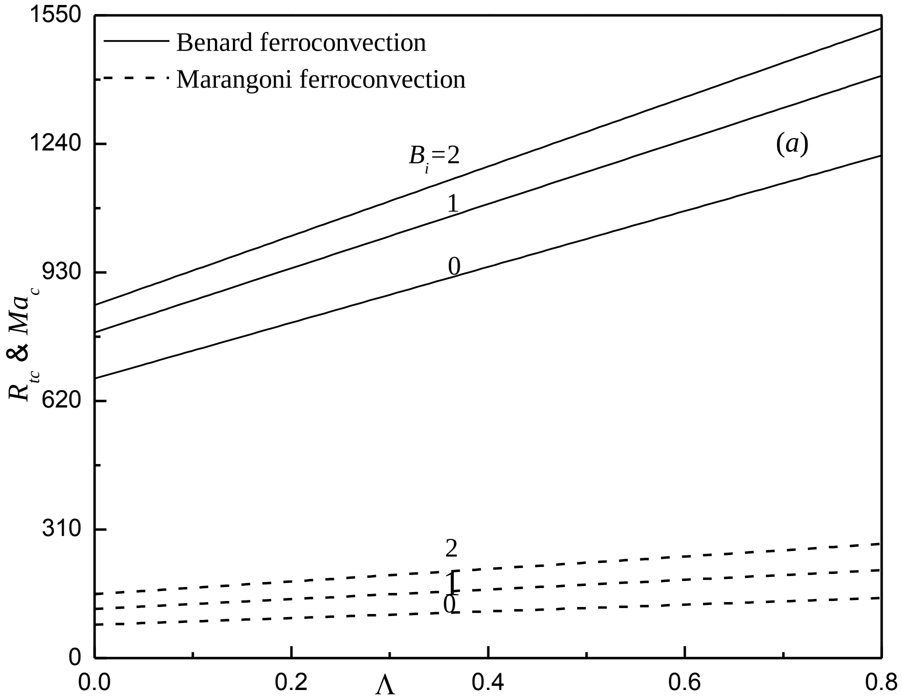


Fig. 5 Variations of (*a*) Ma_c and R_{tc} and (b) a_c as a function of Λ for different values of Ta when $M_3 = 1$, $R_m = 100$ and $Bi = 2$.



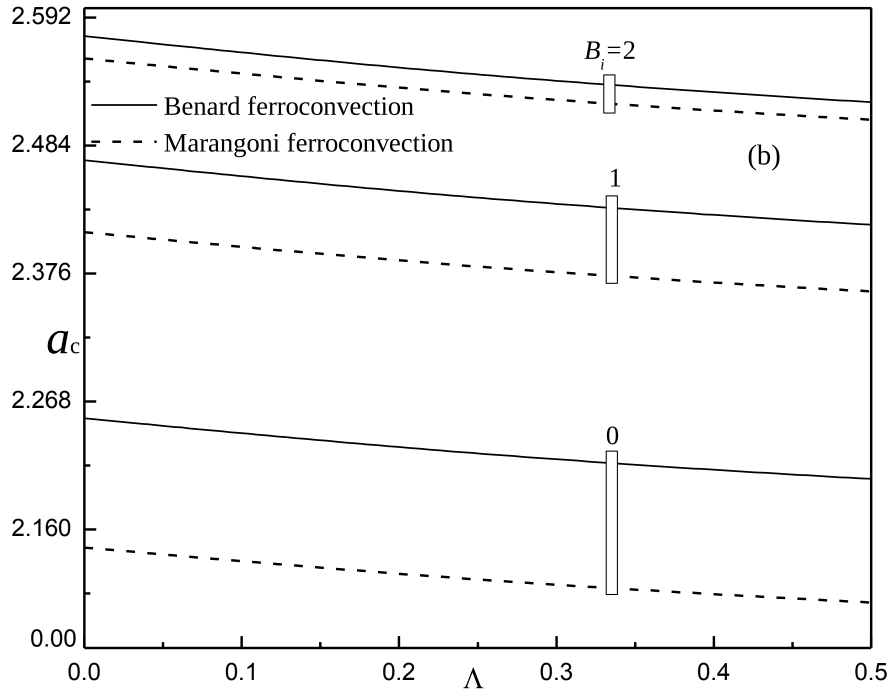
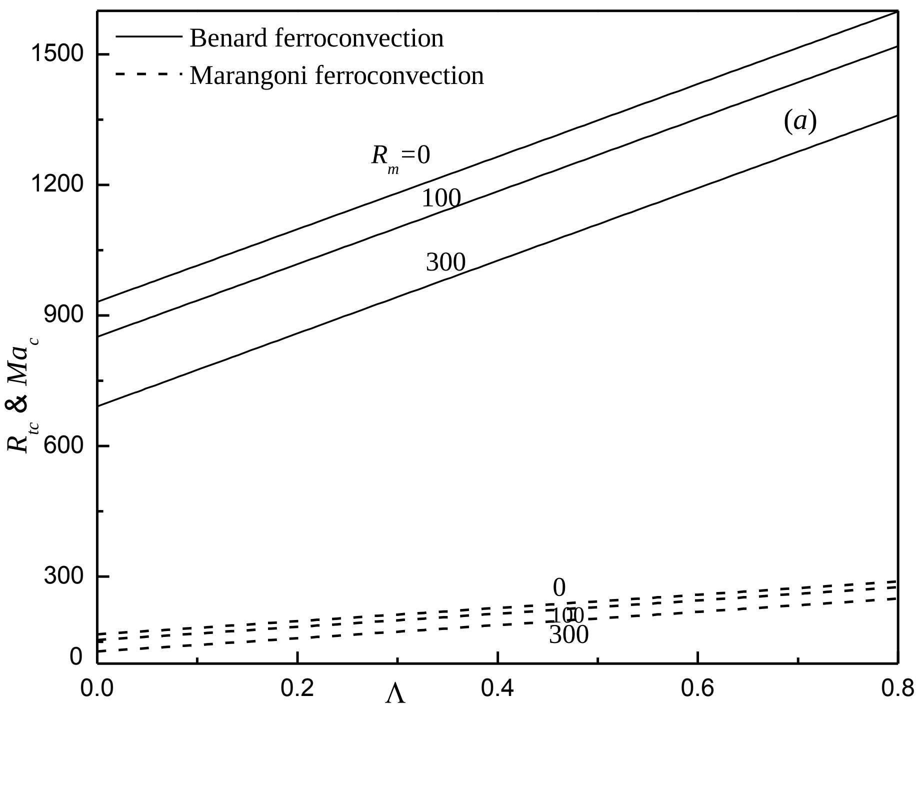


Fig. 6 Variations of (*a*) Ma_c and R_{tc} and (b) a_c as a function of Λ for different values of Bi when $M_3 = 1$, $R_m = 100$ and $Ta = 10^2$.



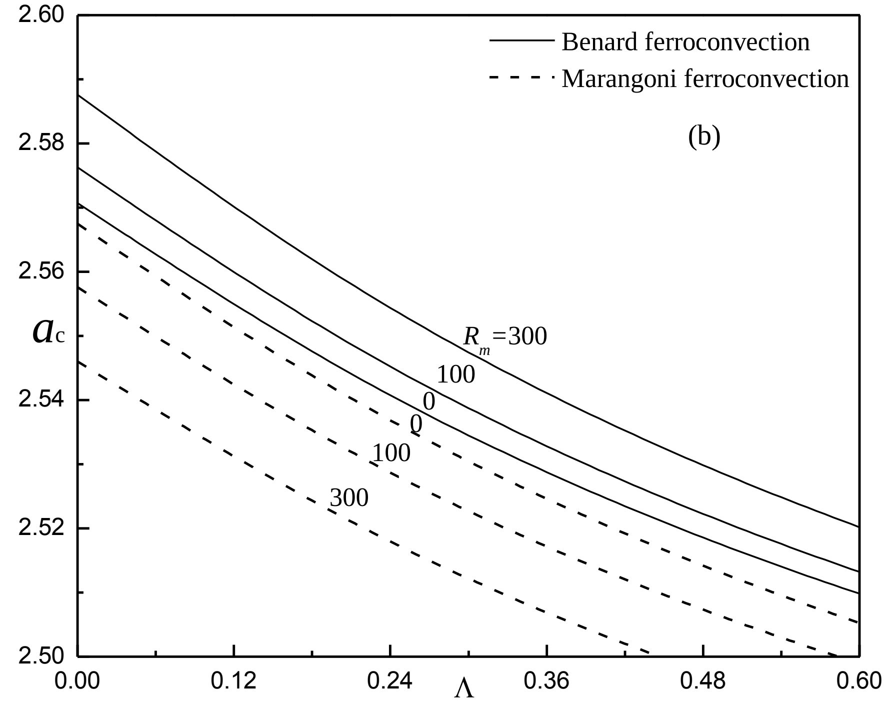
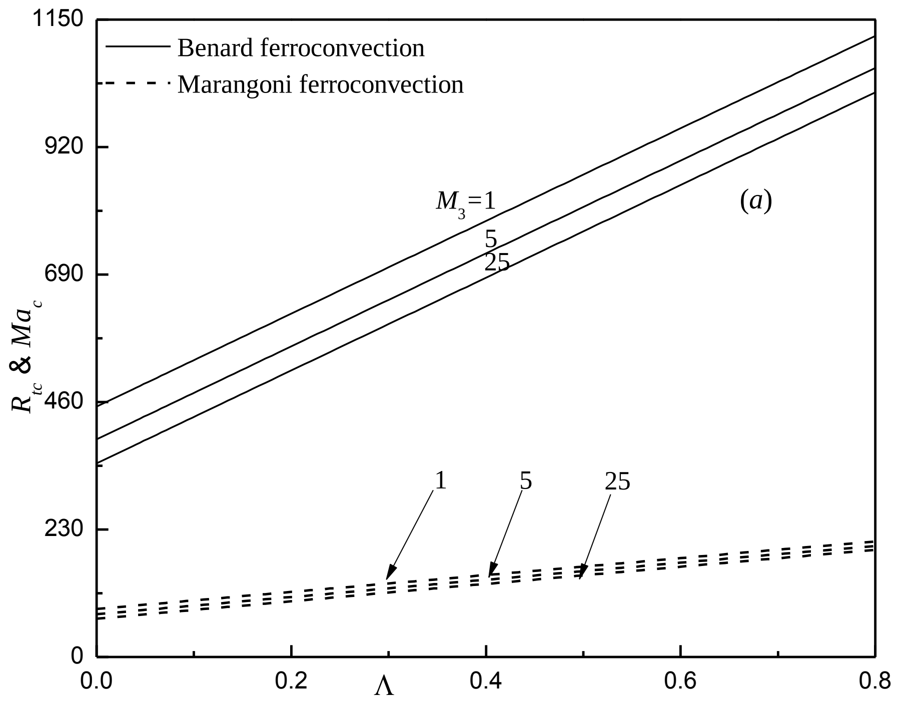


Fig. 7 Variations of (*a*) Ma_c and R_{tc} and (b) a_c as a function of Λ for different values of R_m when $M_3 = 1,\ Bi = 2$ and $Ta = 10^2$.



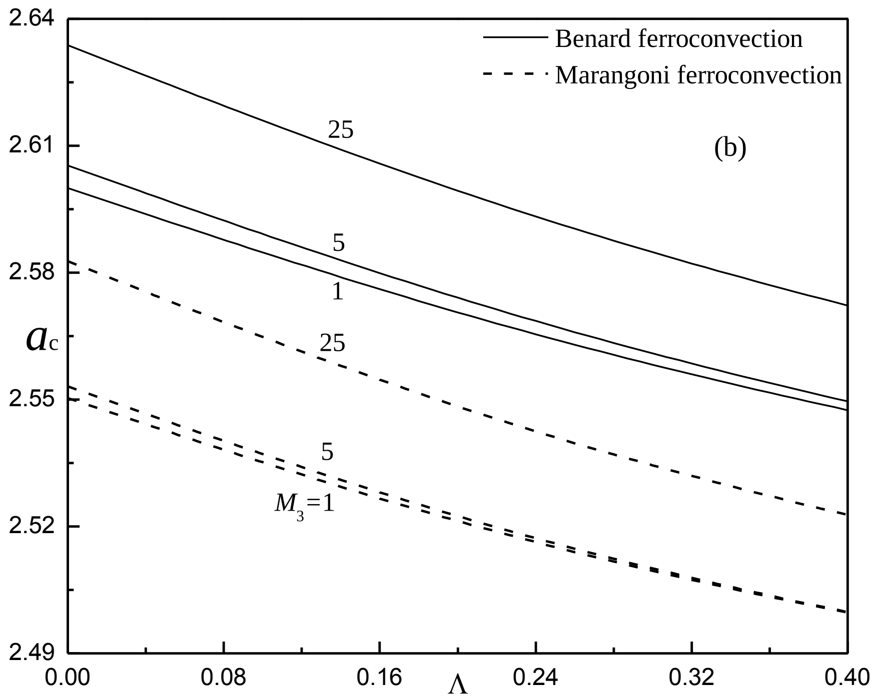


Fig. 8 Variations of (a) Ma_c and R_{tc} and (b) a_c as a function of Λ for different values of M_3 when $R_m = 600$, $Bi = 2$ and $Ta = 10^2$.

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Dr. Arunkumar R, obtained his M. Sc from Bangalore University, Bangalore and Ph. D from Visvesvaraya Technological University Belgaum. He has been teaching Engineering Mathematics from past 16 years and guiding students to pursue their Ph. D. Presently he is working as Associate Professor in the department of Mathematics in Sai Vidya Institute of Technology, Bangalore.

Dr. C. E. Nanjundappa, obtained his M. Sc, and Ph. D from Bangalore University, Bangalore. He has been teaching for undergraduates and post graduates from past several years and guiding students to pursue their Ph. D. Presently he is working as Professor and Hear in the department of Mathematics in Dr. Ambedkar Institute of Technology, Bangalore.

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