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Principles of Linear Equations

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--------------------ABSTRACT--------------------
The research shows a new approach that can be used to solve linear equations .Basically this work shows five different methods of solving linear equations these includes Pick Direct Method, Pick Transpose Method, Pick Back Transpose Method, Pick Down Transpose method, Pick Down Back Transpose Method. Adequate illustrations have been given the paragraph below. The methods given has been tested by the author, it is easy to apply.

Keywords *– Pick Direct Method, Pick Transpose Method, Pick Back Transpose Method, Pick Down Transpose Method and Pick Down Back Transpose Method.*

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I. INTRODUCTION

Linear equation (matrices) is one of the topics in mathematics that cannot be mastered by memorizing the basic principle. It requires, in addition a sound theoretical knowledge, intensive application of basic principles. Most research work, give a fairly detailed explanation of the principles but failed to provide readers with easy understandable approach. Muhammed Adebayo Kassim introduce to you a new method for solving linear equations, these methods are **Pick Direct Method, Pick Transpose Method, Pick Back Transpose Method, Pick Down Transpose Method And Pick Down Back Transpose Method.** These methods have been tested and it had worked for two unknowns, three unknowns, four unknowns etc.

The aim of this work is to demonstrate on how to solve linear equations using Muhammed Adebayo Kassim Picking Methods. MAK'S method may help to meet the need of professional such as Mathematicians, Engineers, Scientists etc. there are larger number of fully worked problem that are followed by different methods for the reader to study. Explanatory note has been given within worked examples for guidance. I would, however, like to stress that MAK'S method are not intended to criticize other method but to serve as substitute to them. I implore you to try the approach given in this book as these will enable you to solve problems on linear equations with ease. Enjoy reading.

1.1 CHAPTER ONE

PICK DIRECT METHOD(TWO EQUATION TWO UNKNOWN)

$a_{11}x + a_{12}\, y = p_1$
$a_{21}x + a_{22}\, y = p_2$

First step, $a_{11}x + a_{12}\, y - p_1 = 0$
$a_{21}x + a_{22}\, y - p_2 = 0$

Next steps pick the first two column coefficient. Leave the first column coefficient. Then, pick the next two column coefficient i.e. 2nd and 3rd column coefficient. Leave the 2nd column coefficient and pick third and first column coefficient.

$$\Delta = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{21}a_{12}$$

$$\Delta_1 = \begin{vmatrix} a_{12} & -P_1 \\ a_{22} & p_2 \end{vmatrix} = a_{12}p_2 - (-a_{22}p_1) = -\, a_{12}p_2 + a_{22}p_1$$

$$\Delta_2 = \begin{vmatrix} -P_1 & -a_{11} \\ -P_2 & a_{21} \end{vmatrix} = -\, P_1a_{21} - (-P_2a_{11}) = -P_1a_{21} + P_2a_{11}$$

$$x = \frac{\Delta_1}{\Delta} = \frac{-\, a_{12}p_2 + a_{22}p_1}{a_{11}p_{22} - a_{21}a_{12}}$$

$y = \frac{\Delta_2}{\Delta} = \frac{- P_1a_{21} + P_2a_{11}}{a_{11}p_{22} - a_{21}p_{12}}$

PICK DIRECT METHOD

$2x + y = 13$
$3x – 2y = -5$
First step, $2x + y – 13 = 0$
$3x – 2y + 5 = 0$

Next steps pick the first two column coefficient. Leave the first column coefficient. Then, pick the next two column coefficient i.e. 2nd and 3rd column coefficient. Leave the 2nd column coefficient and pick third and first column coefficient.

$$\Delta = \begin{vmatrix} 2 & 1 \\ 3 & -2 \end{vmatrix} = -4 – 3 = -7$$

$$\Delta_1 = \begin{vmatrix} 1 & -13 \\ -2 & 5 \end{vmatrix} = 5 – 26 = -21$$

$$\Delta_2 = \begin{vmatrix} -13 & 2 \\ 5 & 3 \end{vmatrix} = - 39 – 10 = -49$$

$$x = \frac{\Delta_1}{\Delta} = \frac{-21}{-7} = 3$$

$$y = \frac{\Delta_2}{\Delta} = \frac{-49}{-7} = 7$$

$x = 3, y = 7$

1.2 PICK DIRECT METHOD (THREE UNKNOWN)

$a_{11}x + a_{12}y + a_{13}z = p_1$
$a_{21}x + a_{22}y + a_{23}z = p_2$
$a_{31}x + a_{32}y + a_{33}z = p_3$

First step, $a_{11}x + a_{12}y + a_{13}z - p_1 = 0$
$a_{21}x + a_{22}y+ a_{23}z – p_2= 0$
$a_{31}x + a_{32}y+ a_{33}z – p_3= 0$

Next step, pick the first three column coefficient i.e. 1st, 2nd and 3rd column coefficient. Leave 1st column coefficient then, pick the next three column coefficients i.e 2nd, 3rd and 4th column coefficients. Leave the 2nd column coefficient then, pick the next three column coefficients i.e 3rd, 4th and 1st column coefficients
Leave the 3rd column coefficient. Then, pick the next three column coefficients i.e. 4th, 1st and 2nd column coefficients

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12} \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13} \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

$$= a_{11} (a_{22} a_{33} - a_{32}a_{23}) - a_{12} (a_{21} a_{33} - a_{31}a_{23}) + a_{13} (a_{21}a_{32} – a_{31}a_{22})$$

$$\Delta_1 = \begin{vmatrix} a_{12} & a_{13} & - p_1 \\ a_{22} & a_{23} & - p_2 \\ a_{32} & a_{33} & - p_3 \end{vmatrix} = a_{12} \begin{vmatrix} a_{23} & - p_2 \\ a_{33} & - p_3 \end{vmatrix} - a_{13} \begin{vmatrix} a_{22} & - p_2 \\ a_{32} & - p_3 \end{vmatrix} - p_1 \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}$$

$$= a_{12} (- a_{23}p_3 + a_{33}p_2) -a_{13} (-a_{22}p_3 + a_{32} p_2) -p_1 (a_{22}a_{33} - a_{32} a_{23})$$

$$\Delta_2 = \begin{vmatrix} a_{13} & – p_1 & a_{11} \\ a_{23} & – p_2 & a_{21} \\ a_{33} & - p_3 & a_{31} \end{vmatrix} = a_{13} \begin{vmatrix} - p_2 & a_{21} \\ - p_3 & a_{31} \end{vmatrix} - (- p_1) \begin{vmatrix} a_{23} & a_{21} \\ a_{33} & a_{31} \end{vmatrix} + a_{11} \begin{vmatrix} a_{23} & - p_2 \\ a_{33} & – p_3 \end{vmatrix}$$

$$= a_{13}(- p_2 a_{31} + p_3a_{21}) + p_1 (a_{23} a_{31} - a_{33} a_{21}) + a_{11} (-a_{23}p_3 + a_{32} p_2)$$

$$\Delta_3 = \begin{vmatrix} -p_1 & a_{11} & a_{12} \\ -p_2 & a_{21} & a_{22} \\ -p_3 & a_{31} & a_{32} \end{vmatrix} = -p_1 \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} - a_{11} \begin{vmatrix} -p_2 & a_{22} \\ -p_3 & a_{32} \end{vmatrix} + a_{12} \begin{vmatrix} -p_2 & a_{21} \\ -p_3 & a_{31} \end{vmatrix}$$

$$= - p_1 (a_{21} a_{32} - a_{31} a_{22}) - a_{11} (-p_2 a_{32} + p_3 a_{22}) + a_{12} (-p_2 a_{31} + p_3 a_{21})$$

$$x = \frac{-\Delta_1}{\Delta} = \frac{-(a_{12}(-a_{23}p_3 + a_{33}p_2) - a_{13}(-a_{22}p_3 + a_{32}p_2) - p_1(a_{22}a_{33} - a_{32}a_{23}))}{a_{11}(a_{22}a_{23} - a_{32}a_{23}) - a_{12}(a_{21}a_{33} - a_{31}a_{23}) + a_{13}(a_{21}a_{32} - a_{31}a_{22})}$$

$$y = \frac{\Delta_2}{\Delta} = \frac{a_{13}(-p_2a_{31} + p_3p_{21}) + p_1(a_{23}a_{31} - a_{33}a_{21}) + a_{11}(-a_{23}p_3 + a_{33}p_2)}{a_{11}(a_{22}a_{33} - a_{32}a_{23}) - a_{12}(a_{21}a_{33} - a_{31}a_{23}) + a_{13}(a_{21}a_{32} - a_{31}a_{22})}$$

$$z = \frac{-\Delta_3}{\Delta} = \frac{-(-p_1(a_{21}a_{32} - a_{31}a_{22}) - a_{11}(-p_2a_{32} + p_3a_{22}) + a_{12}(-p_2a_{31} + p_3a_{21})}{a_{11}(a_{22}a_{33} - a_{32}a_{23}) - a_{12}(a_{21}a_{33} - a_{31}a_{23}) + a_{13}(a_{21}a_{32} - a_{31}a_{22})}$$

THREE EQUATIONS AND THREE UNKNOWN

$2x + y + 3z = 16$
$x + 2y - z = -2$
$3x + y + 22 = 14$

PICK DIRECT METHOD

First step, $2x + y + 3z - 16 = 0$
$x + 2y - z + 2 = 0$
$3x + y + 2z - 14 = 0$

Next step, pick the first three column coefficient i.e. 1st, 2nd and 3rd column coefficient. Leave 1st column coefficient then, pick the next three column coefficients i.e 2nd, 3rd and 4th column coefficients. Leave the 2nd column coefficient then, pick the next three column coefficients i.e 3rd, 4th and 1st column coefficients
Leave the 3rd column coefficient. Then, pick the next three column coefficients i.e. 4th, 1st and 2nd column coefficients

$$\Delta = \begin{vmatrix} 2 & 1 & 3 \\ 1 & 2 & -1 \\ 3 & 1 & 2 \end{vmatrix} = 2\begin{vmatrix} 2 & -1 \\ 1 & 2 \end{vmatrix} - 1\begin{vmatrix} 1 & -1 \\ 3 & 2 \end{vmatrix} + 3\begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix}$$

$= 2(4+1) - 1\ (2+3) + 3\ (1-6)$
$= 2\ (5) - 1\ (5) + 3\ (-5)$
$= 10 - 5 - 15 = -10$

$$\Delta_1 = \begin{vmatrix} 1 & 3 & -16 \\ 2 & -1 & 2 \\ 1 & 2 & -14 \end{vmatrix} = 1\begin{vmatrix} -1 & 2 \\ 2 & -14 \end{vmatrix} - 3\begin{vmatrix} 2 & 2 \\ 1 & -14 \end{vmatrix} - 16\begin{vmatrix} 2 & -1 \\ 1 & 2 \end{vmatrix}$$

$= 1\ (14 - 4) - 3\ (-28-2) - 16\ (4 - (-1)$
$= 1\ (10) - 3\ (-30) - 16\ (5)$
$= 10 + 90 - 80$
$= 20$

$$\Delta_2 = \begin{vmatrix} 3 & -16 & 2 \\ -1 & 2 & 1 \\ 2 & -14 & 3 \end{vmatrix} = 3\begin{vmatrix} 2 & 1 \\ -14 & 3 \end{vmatrix} - (-16)\begin{vmatrix} -1 & 1 \\ 2 & 3 \end{vmatrix} + 2\begin{vmatrix} -1 & 2 \\ 2 & -14 \end{vmatrix}$$

$= 3\ (6 + 14) + 16\ (-3-2) + 2\ (14 - 4)$
$= 3\ (20) + 16\ (-5) + 2\ (10)$
$= 60 - 80 + 20$
$= 0$

$$\Delta_3 = \begin{vmatrix} -16 & 2 & 1 \\ 2 & 1 & 2 \\ -14 & 3 & 1 \end{vmatrix} = -16\begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} - 2\begin{vmatrix} 2 & 2 \\ -14 & 1 \end{vmatrix} + 1\begin{vmatrix} 2 & 1 \\ -14 & 3 \end{vmatrix}$$

$= -16(1-6) - 2\ (2+28) + 1\ (6 + 14)$
$= -16\ (-5) - 2\ (30) + 1\ (20)$
$= +80 - 60 + 20$
$= 40$

$$x = \frac{\Delta_1}{\Delta} = \frac{-20}{-10} = 2$$

$$y = +\frac{\Delta_2}{\Delta} = \frac{0}{-10} = 0$$

$$z = \frac{\Delta_3}{\Delta} = \frac{-40}{-10} = 4$$

$x = 2, y=0, Z =4$

1.3 FOUR EQUATIONS FOUR UNKNOWN

$a_{11}x + a_{12}y + a_{13}z + a_{14}p = l_1$
$a_{21}x + a_{22}y + a_{23}z + a_{24}p = l_2$
$a_{31}x + a_{32}y + a_{33}z + a_{34}p = l_3$
$a_{42}x + a_{42}y + a_{43}z + a_{44}p = l_4$

First step, $a_{11}x + a_{12}y + a_{13}z + a_{14}p - l_1 = 0$
$a_{21}x + a_{22}y + a_{23}z + a_{24}p - l_2 = 0$
$a_{31}x + a_{32}y + a_{33}z + a_{34}p - l_3 = 0$
$a_{41}x + a_{42}y + a_{43}z + a_{44}p - l_4 = 0$

Next steps pick the first four column coefficients i.e 1st, 2nd, 3rd, and 4th column coefficients. Leave the 1st column coefficients and pick the next four column coefficients i.e 2nd, 3rd, 4th and 5th. Leave the 2nd column coefficient and pick the next four column coefficient i.e 3rd, 4th, 5th, & 1st. leave the 3rd column coefficient and pick the next four column coefficients 1.e 4th, 5th, 1st, & 2nd. Leave the fourth column coefficients and pick the next four column coefficients i.e 5th, 1st, 2nd & 3rd.

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{vmatrix}$$

$$\Delta = a_{11}\begin{vmatrix} a_{22} & a_{23} & a_{24} \\ a_{32} & a_{33} & a_{34} \\ a_{42} & a_{43} & a_{44} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} & a_{24} \\ a_{31} & a_{33} & a_{34} \\ a_{41} & a_{43} & a_{44} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} & a_{24} \\ a_{31} & a_{32} & a_{34} \\ a_{41} & a_{42} & 44 \end{vmatrix} - a_{14}\begin{vmatrix} a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \\ a_{41} & a_{42} & a_{43} \end{vmatrix}$$

$= a_{11} (a_{22}(a_{33} a_{44} – a_{43} a_{34}) - a_{23} (a_{32} a_{44} – a_{42} a_{34}) + a_{24} (a_{32} a_{43}– a_{42} a_{33})$
$- a_{12} (a_{21}(a_{33} a_{44} – a_{43} a_{34}) - a_{23} (a_{31} a_{44} – a_{41} a_{34}) +a_{24} (a_{31} a_{43} – a_{41} a_{33})$
$+ a_{13} (a_{21}(a_{32} a_{44} – a_{42} a_{34}) -a_{22} (a_{31} a_{44} – a_{41} a_{34}) +a_{24} (a_{31} a_{43} – a_{41} a_{32})$
$- a_{14} (a_{21}(a_{32} a_{43} – a_{42} a_{33}) -a_{22} (a_{31} a_{43} – a_{41} a_{33}) +a_{23} (a_{31} a_{42} – a_{41} a_{32})$

$$\Delta_1 = \begin{vmatrix} a_{12} & a_{13} & a_{14} & -l_1 \\ a_{22} & a_{23} & a_{24} & -l_2 \\ a_{32} & a_{33} & a_{34} & -l_3 \\ a_{42} & a_{43} & a_{44} & -l_4 \end{vmatrix}$$

$$\Delta_1 = a_{12}\begin{vmatrix} a_{23} & a_{24} – l_2 \\ a_{33} & a_{34} - l_3 \\ a_{43} & a_{44} - l_4 \end{vmatrix} - a_{13}\begin{vmatrix} a_{22} & a_{24} - l_2 \\ a_{32} & a_{34} - l_3 \\ a_{42} & a_{44} - l_4 \end{vmatrix} + a_{14}\begin{vmatrix} a_{22} & a_{23} - l_2 \\ a_{32} & a_{33} -l_3 \\ a_{42} & a_{43} - l_4 \end{vmatrix} -(-l_1)\begin{vmatrix} a_{22} & a_{23} & a_{24} \\ a_{32} & a_{33} & a_{34} \\ a_{42} & a_{43} & a_{44} \end{vmatrix}$$

$a_{12}(a_{23}(-a_{34}l_4+a_{44}l_3) –a_{24} (-a_{33}ly +a_{43}l_3) –l_2 (a_{33}a_{44} – a_{43}a_{34})) –a_{13} (a_{22}(-a_{34}ly + a_{44}l_3) -a_{24} (-a_{24}(a_{32}ly + a_{42}l_3) – l_2 (a_{32}a_{44}-a_{42}a_{34})) –a_{14} (a_{22}(-a_{33}ly + a_{43}l_3) – a_{23} (a_{32}ly + a_{42}l_3) –l_2 (a_{32}a_{43} – a_{42}a_{33})) + l_1 (a_{22}(a_3a_{44} – a_{43}a_{34} – a_{23} (a_{32}a_{44} – a_{42}a_{34}) + a_{24} (a_{32}a_{43} – a_{42} a_{33}))$

$$\Delta_2 = \begin{vmatrix} a_{13} & a_{14} & -l_1 & a_{11} \\ a_{23} & a_{24} & -l_2 & a_{21} \\ a_{33} & a_{34} & -l_3 & a_{31} \\ a_{43} & a_{44} & -l_4 & a_{41} \end{vmatrix}$$

$$\Delta_2 = a_{13}\begin{vmatrix} a_{24} & -l_2 & a_{21} \\ a_{34} & -l_3 & a_{31} \\ a_{34} & -l_4 & a_{41} \end{vmatrix} - a_{14}\begin{vmatrix} a_{23} & -l_2 & a_{21} \\ a_{33} & -l_3 & a_{31} \\ a_{43} & -l_4 & a_{41} \end{vmatrix} - l_1\begin{vmatrix} a_{23} & a_{24} & a_{21} \\ a_{33} & a_{34} & a_{31} \\ a_{43} & a_{44} & a_{41} \end{vmatrix} - a_{11}\begin{vmatrix} a_{23} & a_{24} & -l_2 \\ a_{33} & a_{34} & -l_3 \\ a_{43} & a_{44} & -l_4 \end{vmatrix}$$

$= a_{13}\ (a_{24}(-a_{41}l_3 + a_{31}l_4) + l_2\,(a_{33}a_{41} - a_{44}a_{31}) + a_{21}\,(-a_{34}l_4 + a_{44}l_3))\ -$
$a_{14}\,(a_{23}(-a_{41}l_3 + a_{31}l_4) + l_2\,(a_{33}a_{41} - a_{43}a_{31}) + a_{21}\,(a_{33}l_4 + a_{43}l_3)\ -$
$l_1\,(a_{23}(a_{34}a_{41} - a_{44}a_{31}) - a_{24}\,(a_{33}\,a_{41} - a_{43}a_{31}) + a_{21}\,(a_{33}a_{44} - a_{43}a_{34})$
$+\ a_{11}(a_{23}(-a_{34}l_4 + a_{44}l_3)) - a_{24}\,(-a_{33}l_4 + a_{43}l_3) - l_2\,(a_{33}a_{44} - a_{44}a_{34}))$

$$\Delta_3 = \begin{vmatrix} a_{14} & -l_1 & a_{11} & a_{12} \\ a_{24} & -l_2 & a_{21} & a_{22} \\ a_{34} & -l_3 & a_{31} & a_{32} \\ a_{44} & -l_4 & a_{41} & a_{42} \end{vmatrix}$$

$$= a_{14}\begin{vmatrix} -l_2 & a_{21} & a_{22} \\ -l_3 & a_{31} & a_{32} \\ -l_4 & a_{41} & a_{42} \end{vmatrix} - (-l_1)\begin{vmatrix} a_{24} & a_{21} & a_{22} \\ a_{34} & a_{31} & a_{32} \\ a_{44} & a_{41} & a_{42} \end{vmatrix} + a_{11}\begin{vmatrix} a_{24} & -l_2 & a_{22} \\ a_{34} & -l_3 & a_{32} \\ a_{44} & -l_4 & a_{42} \end{vmatrix} - a_{22}\begin{vmatrix} a_{24} & -l_2 & a_{23} \\ a_{34} & -l_3 & a_{33} \\ a_{44} & -l_4 & a_{41} \end{vmatrix}$$

$= a_{14}\,(-l_1(a_{31}a_{42} - a_{41}a_{32}) - a_{21}\,(-l_3a_{42} + l_4a_{32}) + a_{22}\,(-l_3a_{41} + l_4a_{31}))$
$+ l_1\,(a_{24}(a_{31}\,a_{42} - a_{41}a_{32}) - a_{21}\,(a_{34}\,a_{42} - a_{44}\,a_{32}) + a_{22}\,(-a_{32}a_{41} - a_{44}a_{31})$
$+ a_{11}\,(a_{24}(-l_3\,a_{42} + l_4a_{32}) - l_2(a_{34}\,a_{42} - a_{44}\,a_{32}) + a_{22}\,(-a_{34}l_4 + a_{44}l_3)$
$-\ a_{12}\,(-l_3\,a_{41} + a_{31}l_4) + l_2(a_{34}\,a_{41} - a_{44}\,a_{31}) + a_{21}\,(-a_{34}l_4 + a_{44}l_3)$

$$\Delta_4 = \begin{vmatrix} -l_1 & - a_{11} & a_{12} & a_{13} \\ -l_2 & - a_{21} & a_{22} & a_{23} \\ -l_3 & - a_{31} & a_{32} & a_{33} \\ -l_4 & - a_{11} & a_{42} & a_{44} \end{vmatrix}$$

$$= l_1\begin{vmatrix} a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \\ a_{41} & a_{42} & a_{43} \end{vmatrix} - a_{11}\begin{vmatrix} -l_2 & a_{22} & a_{23} \\ -l_3 & a_{32} & a_{33} \\ -l_4 & a_{42} & a_{43} \end{vmatrix} + a_{12}\begin{vmatrix} -l_2 & a_{21} & a_{23} \\ -l_3 & a_{31} & a_{33} \\ -l_4 & a_{41} & a_{43} \end{vmatrix} - a_{13}\begin{vmatrix} -l_2 & a_{21} & a_{22} \\ -l_3 & a_{31} & a_{32} \\ -l_4 & a_{41} & a_{42} \end{vmatrix}$$

$= -l_1(a_{21}(a_{32}a_{43}-a_{42}a_{33})-a_{22}(a_{31}a_{43}-a_{41}a_{33})+a_{23}(a_{31}a_{42}-a_{41}a_{32}))-a_{11}(-a_{11}(-l_2(a_{32}a_{43}-a_{42}a_{33})$
$-a_{22}(-l_3a_{43}+l_4a_{33})+a_{23}(-l_3a_{41}+l_4a_{31})-a_{13}(-l_1(a_{31}a_{42}a_{32})-a_{21}(-l_3a_{42}+l_4a_{32})+a_{22}(-l_3(a_{41}+l_4\,a_{31}))$

$x = \dfrac{\Delta_1}{\Delta}$, $y = \dfrac{\Delta_2}{\Delta}$, $z = \dfrac{\Delta_3}{\Delta}$, $1 = \dfrac{\Delta_4}{\Delta}$

PICK DIRECT METHOD (FOUR UNKNOWNS)

$2x + y + z + p = 5$
$x + 2y + z - 2p = -6$
$x + 3y + 3z + 4p = 13$
$4x + 2y - 5z + 2p = 3$

First step, $2x + y + z + p - 5 = 0$
$x + 2y + z - 2p + 6 = 0$
$x + 3y + 3z + 4p - 13 = 0$
$4x + 2y - 5z + 2p - 3 = 0$

Next step, pick the first four column coefficient i.e 1st, 2nd 3rd and 4th column coefficients. Leave the 1st column coefficients and pick the next four column coefficients i.e 4th 5th & 2nd. Leave the fourth column coefficient and pick the four column coefficient i.e 5th, 1st, 2nd & 3rd.

$$\Delta = \begin{vmatrix} 2 & 1 & 1 & 1 \\ 1 & 2 & 1 & -2 \\ 13 & & 3 & 4 \\ 42 & & -5 & 2 \end{vmatrix}$$

$$= 2\begin{vmatrix} 2 & 1 & -2 \\ 3 & 3 & 4 \\ 2 & -5 & 2 \end{vmatrix} - 1\begin{vmatrix} 1 & 1 & -2 \\ 1 & 1 & 4 \\ 4 & -5 & 2 \end{vmatrix} + 1\begin{vmatrix} 1 & 2 & 1 \\ 1 & 3 & 4 \\ 4 & 2 & 2 \end{vmatrix} - 1\begin{vmatrix} 1 & 2 & 1 \\ 1 & 3 & 3 \\ 4 & 2 & -5 \end{vmatrix}$$

$= 2\,(96) - 1\,(74) + 1(46) - 1(3)$
$= 161$

$$\Delta_1 = \begin{vmatrix} 1 & 1 & 1 & -5 \\ 2 & 1 & -2 & 6 \\ 3 & 3 & 4 & -13 \\ 2 & -5 & 2 & -3 \end{vmatrix}$$

$$= 1\begin{vmatrix} 1 & -2 & 6 \\ 3 & 4 & -13 \\ 5 & 2 & -3 \end{vmatrix} -1\begin{vmatrix} 2 & -2 & 6 \\ 3 & 4 & -13 \\ 2 & 2 & -3 \end{vmatrix} +1\begin{vmatrix} 2 & 1 & 6 \\ 3 & 3 & -13 \\ 2 & -5 & -3 \end{vmatrix} -5\begin{vmatrix} 2 & 1 & -2 \\ 3 & 3 & 4 \\ 2 & -5 & 2 \end{vmatrix}$$

$= 1 (22) - (150) +1 (-291) +1 (96)$
$= 161$

$$\Delta_2 = \begin{vmatrix} 1 & 1 & -5 & 2 \\ 1 & -2 & 6 & 1 \\ 3 & 4 & -13 & 1 \\ 5 & 2 & -3 & 4 \end{vmatrix}$$

$$= 1\begin{vmatrix} -2 & 6 & 1 \\ 4 & -13 & 1 \\ 2 & -3 & 4 \end{vmatrix} -1\begin{vmatrix} 1 & 6 & 1 \\ 3 & -13 & 1 \\ -5 & -3 & 4 \end{vmatrix} -5\begin{vmatrix} 1 & -2 & 1 \\ 3 & 4 & 1 \\ -5 & 2 & 4 \end{vmatrix} +2\begin{vmatrix} 1 & -2 & 6 \\ 3 & 4 & -13 \\ -5 & 2 & -3 \end{vmatrix}$$

$= 1 (28) -1 (225) -5 (74) -2 (22)$
$= \Delta_2 = -161$

$$\Delta_3 = \begin{vmatrix} 1 & -5 & 2 & 1 \\ -2 & 6 & 1 & 2 \\ 4 & -13 & 1 & 3 \\ 2 & -3 & 4 & 2 \end{vmatrix}$$

$$= 1\begin{vmatrix} 6 & 1 & 2 \\ -13 & 1 & 3 \\ -3 & 4 & 2 \end{vmatrix} -(-5)\begin{vmatrix} -2 & 1 & 2 \\ 4 & 1 & 3 \\ 2 & 4 & 2 \end{vmatrix} +2\begin{vmatrix} -2 & 6 & 2 \\ 4 & -13 & 3 \\ 2 & -3 & 2 \end{vmatrix} -1\begin{vmatrix} -2 & 6 & 1 \\ 4 & -13 & 1 \\ 2 & -3 & 4 \end{vmatrix}$$

$= 1 (-141) + 5 (46) +2 (50) -1 (28)$

$= 161$

$$\Delta_4 = \begin{vmatrix} -5 & 2 & 1 & 1 \\ 6 & 1 & 2 & 1 \\ -13 & 1 & 3 & 3 \\ -3 & 4 & 2 & -5 \end{vmatrix}$$

$$= -5\begin{vmatrix} 1 & 2 & 1 \\ 1 & 3 & 3 \\ 4 & 2 & -5 \end{vmatrix} -2\begin{vmatrix} 6 & 2 & 1 \\ -13 & 3 & 3 \\ -3 & 2 & -5 \end{vmatrix} +1\begin{vmatrix} 6 & 1 & 1 \\ -13 & 1 & 3 \\ -3 & 4 & -5 \end{vmatrix} -1\begin{vmatrix} 6 & 1 & 1 \\ -13 & 1 & 3 \\ -3 & 4 & 2 \end{vmatrix}$$

$= -5(3) -2 (-291) +1 (-225) -1 (-141)$
$= 483$

$$x = \frac{\Delta_1}{\Delta} = \frac{161}{161} = 1$$

$$y = \frac{\Delta_2}{\Delta} = \frac{-161}{161} = -1$$

$$z = \frac{\Delta_3}{\Delta} = \frac{161}{161} = 1$$

$$p = \frac{\Delta_4}{\Delta} = \frac{483}{161} = 3$$

2.0 PICK TRANSPOSE METHOD

$a_{11}x + a_{12}y = p_1$
$a_{21} + a_{22}y = p_2$
First step, $a_{11}x + a_{12}y - p_1 = 0$
$a_{21}x + a_{22}y - p_2 = 0$

Next steps, pick the first two column coefficient i.e 1st and 2nd column coefficients. Then transpose. Leave the first column coefficient. Then, pick the next two column coefficient i.e 2nd & 3rd column coefficients then, transpose. Leave the 2nd column coefficient, pick the 3rd and 1st column coefficient then, transpose.

$$\Delta = \begin{vmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$$

$$\Delta_1 = \begin{vmatrix} a_{12} & a_{22} \\ -p_1 & -p_2 \end{vmatrix} \begin{matrix} = - a_{12}p_2 - (-p_1 a_{22}) \\ = - a_{12}p_2 + p_1a_{22} \end{matrix}$$

$$\Delta_2 = \begin{vmatrix} -p_1 & -p_2 \\ a_{11} & a_{21} \end{vmatrix} \begin{matrix} = - p_1 a_{21} - (-a_{11}P_2) \\ = - p_1a_{21} + a_{11}p_2 \end{matrix}$$

$$x = \frac{\Delta_1}{\Delta} = \frac{-a_{12}p_2 + p_1a_{22}}{a_{11}a_{22} - a_{12}a_{21}}$$

$$y = \frac{\Delta_2}{\Delta} = \frac{-p_1a_{21} + a_{11}p_2}{a_{11}a_{22} - a_{12}a_{21}}$$

PICK TRANSPOSE METHOD

$2x + y = 13$
$3x - 2y = -5$
First step, $2x + y - 13 = 0$
$3x - 2y + 5 = 0$

Next steps, pick the first two column coefficient i.e 1st and 2nd column coefficients. Then transpose. Leave the first column coefficient. Then, pick the next two column coefficient i.e 2nd & 3rd column coefficients then, transpose. Leave the 2nd column coefficient, pick the 3rd and 1st column coefficient then, transpose.

$$\Delta = \begin{vmatrix} 2 & 3 \\ 1 & -2 \end{vmatrix} \begin{matrix} = (2x -2) - (1 \times 3) \\ = - 4 -3 = -7 \end{matrix}$$

$$\Delta_1 = \begin{vmatrix} 1 & -2 \\ -13 & 5 \end{vmatrix} \begin{matrix} = (1 \times 5) - (-2 \times -13) \\ = 5 - 26 = -21 \end{matrix}$$

$$\Delta_2 \begin{vmatrix} -13 & 5 \\ 2 & 3 \end{vmatrix} \begin{matrix} = (-13 \times 3) - 2 \times 5) \\ = -39 - 10 = - 49 \end{matrix}$$

$$x = \frac{\Delta_1}{\Delta} = \frac{-21}{-7} = 3$$

$$y = \frac{\Delta_2}{\Delta} = \frac{-49}{-7} = 7$$

2.1 THREE EQUATION THREE UNKNOWN
PICK TRANSPOSE METHOD

$a_{11} + a_{12}y + a_{13}z = p_1$
$a_{21}x + a_{22}y + a_{23}z = p_2$
$a_{31}x + a_{32}y + a_{33}z = p_3$
First step, $a_{11}x + a_{12}y + a_{13}z - p_1 = 0$
$a_{21}x + a_{22}y + a_{23}z - p_1 = 0$
$a_{31}x + a_{32}y + a_{33}z - p_3 = 0$

Next steps pick the first three column coefficient i.e 1st, 2nd & 3rd then, transpose. Leave the 1st column coefficient then pick the next three column coefficient i.e 2nd, 3rd, and 4th column coefficient then, transpose. Leave the 2nd column coefficient then, pick the next three column coefficients i.e 3rd, 4th & 1st coefficients then, transpose. Leave the third column coefficients. Then, pick the next three column coefficient i.e 4th, 1st, & 2nd column coefficient then, transpose.

$$\Delta=\begin{vmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{12} & a_{22} & a_{32} \end{vmatrix} = a_{11}\begin{vmatrix} a_{22} & a_{32} \\ a_{23} & a_{32} \end{vmatrix} - a_{21}\begin{vmatrix} a_{12} & a_{32} \\ a_{13} & a_{33} \end{vmatrix} + a_{31}\begin{vmatrix} a_{12} & a_{22} \\ a_{13} & a_{23} \end{vmatrix}$$

$$=a_{11}(a_{22}\,a_{23} - a_{23}a_{32}) - a_{21}(a_{12}a_{33} - a_{13}a_{32}) + a_{31}(a_{12}a_{23} - a_{13}\,a_{22})$$

$$\Delta_1 = \begin{vmatrix} a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \\ -p_1 & -p_2 & -p_3 \end{vmatrix} = a_{12}\begin{vmatrix} a_{23} & a_{33} \\ -p_2 & -p_3 \end{vmatrix} + a_{22}\begin{vmatrix} a_{13} & a_{33} \\ -p_1 & -p_3 \end{vmatrix} + a_{32}\begin{vmatrix} a_{13} & a_{23} \\ -p_1 & -p_2 \end{vmatrix}$$

$$= a_{12}(-_{23}p_3 + p_2\,a_{33}) +_{22}(-a_{13}p_2 + p_1a_{23}) + a_{32}(-a_{13}p_2 + p_1a_{23})$$

$$\Delta_2 = \begin{vmatrix} a_{13} & a_{23} & a_{33} \\ -p_1 & -p_2 & -p_3 \\ a_{11} & a_{21} & a_{31} \end{vmatrix} = a_{13}\begin{vmatrix} -p1 & -p_3 \\ a_{21} & a_{31} \end{vmatrix} - a_{23}\begin{vmatrix} -p_1 & -p_3 \\ a_{11} & a_{31} \end{vmatrix} + a_{33}\begin{vmatrix} -p_1 & -p_2 \\ a_{11} & a_{21} \end{vmatrix}$$

$$= a_{13}\,(-P_2\,a_{31}+a_{21}p_3)-a_{23}(-p_1a_{31}+a_{11}p_3)+a_{33}(-p_1a_{21}+a_{11}p_2)$$

$$\Delta_3 = \begin{vmatrix} -p_1 & -p_2 & -p_3 \\ a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \end{vmatrix} = -p_1\begin{vmatrix} a_{21} & a_{31} \\ a_{22} & a_{32} \end{vmatrix} -(-p_2)\begin{vmatrix} a_{11} & a_{31} \\ a_{12} & a_{32} \end{vmatrix} -p_3\begin{vmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \end{vmatrix}$$

$$=-p_1\,(a_{21}a_{32}-a_{22}a_{31}) + p_2\,(a_{11}a_{32} - a_{12}a_{31}) - p_3\,(a_{11}a_{22} - a_{12}\,a_{21})$$

$$x = -\frac{\Delta_1}{\Delta} = \frac{-\,(a_{12}\,(-a_{23}p_3 + p_2a_{33}) + a_{22}\,(-a_{13}p_2+p_1a_{23}) + a_{32}(-a_{32}p_2 + p1a23}{a_{11}(a_{22}a_{33}-a23a_{32}) - a_{21}(a_{12}a_{33}-a_{13}a_{32})+a_{31}(a_{12}a_{23}-a_{13}a_{22})}$$

$$y = +\frac{\Delta_2}{\Delta} = \frac{a_{13}(-p_2a_{31} + a_{21}p_3) - a_{23}\,(-p_1a_{31} + a_{11}p_3) + a_{33}(-p_1a_{21}+a_{11}p_2)}{a_{11}(a_{22}a_{33} - a_{2332}) - a_{21}(a_{12}\,a_{33} - a_{13}a_{32}) + a_{31}(a_{12}a_{23} - a_{13}a_{22})}$$

$$z = -\frac{\Delta_3}{\Delta} = \frac{-(-p_1(a_{2132}-a_{22}a_{31}) + p_1\,(a_{11}a_{32} - a_{12}a_{31}) - p_3\,(a_{11}a_{22} - a_{12}a_{21})}{a_{11}(a_{22}a_{33} - a_{23}) - a_{21}\,(a_{12}a_{33} - a_{13}a_{32}) + a3_1(a_{12}a_{23} - a_{13}a_{22})}$$

PICK TRANSPOSE METHOD

$2x + y + 3z = 16$

$x + 2y - z = 2$

$3x + y + 2z = 14$

First steps, $2x + y + 3z - 16 = 0$

$x + 2y - z + 2 = 0$

$3x + y + 2z - 14 = 0$

Next steps pick the first three column coefficient i.e 1st, 2nd & 3rd then, transpose. Leave the 1st column coefficient then pick the next three column coefficient i.e 2nd, 3rd, and 4th column coefficient then, transpose. Leave the 2nd column coefficient then, pick the next three column coefficients i.e 3rd, 4th & 1st coefficients then, transpose. Leave the third column coefficients. Then, pick the next three column coefficient i.e 4th, 1st, & 2nd column coefficient then, transpose.

$$\Delta = \begin{vmatrix} 2 & 1 & 3 \\ 1 & 2 & 1 \\ 3 & -1 & 2 \end{vmatrix} = 2\begin{vmatrix} 2 & 1 \\ -1 & 2 \end{vmatrix} - 1\begin{vmatrix} 1 & 1 \\ 3 & 2 \end{vmatrix} + 3\begin{vmatrix} 1 & 2 \\ 3 & -1 \end{vmatrix}$$

$=2(4+1)-1(2-3)+3(-1-6)$

$=2(5)-1(-1)+3(-7)$

$= 10+1-21$

$= -10$

$$\Delta_1 = \begin{vmatrix} 1 & 2 & 1 \\ 3 & -1 & 2 \\ -16 & 2 & -14 \end{vmatrix} = 1\begin{vmatrix} -1 & 2 \\ 2 & -14 \end{vmatrix} -2\begin{vmatrix} 3 & 2 \\ -16 & -14 \end{vmatrix} +1\begin{vmatrix} 3 & -1 \\ -16 & 2 \end{vmatrix}$$

$= 1(14 – 4) -2 (-42 + 32) +1 (6 – 16)$
$= 1(10) -2 (-10) + 1 (-10)$
$= 10 + 20 – 10$
$= 10 + 20 – 10$
$\Delta_1 = 20$

$$\Delta_2 = \begin{vmatrix} 3 & -1 & 2 \\ -16 & 2 & -14 \\ 2 & 1 & 3 \end{vmatrix} = 3\begin{vmatrix} 2 & -14 \\ 1 & 3 \end{vmatrix} +1\begin{vmatrix} -16 & -14 \\ 2 & 3 \end{vmatrix} +2\begin{vmatrix} 16 & 2 \\ 2 & 1 \end{vmatrix}$$

$=3(6+14)+1(-48+28)+2(-16-4)$
$=3(20)+1(-20)+2(-20)$
$=60 \ -20 \ -40$
$= 0$

$$\Delta_3 = \begin{vmatrix} -16 & 2 & -14 \\ 2 & 1 & 3 \\ 1 & 2 & 1 \end{vmatrix} = -16\begin{vmatrix} 1 & 3 \\ 2 & 1 \end{vmatrix} -2\begin{vmatrix} 2 & 3 \\ 1 & 1 \end{vmatrix} -14\begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix}$$

$= - 16 (1-6) -2 (2-3) -14 (4-1)$
$= - 16(-5) -2 (-1) – 14(3)$
$= 40$

$x = -\frac{\Delta_1}{\Delta} = \frac{-20}{-10} = 2$

$y = +\frac{\Delta_2}{\Delta} = \frac{0}{-10} = 0$

$z = -\frac{\Delta_3}{\Delta} = \frac{-40}{-10} = 4$

PICK TRANSPOSE METHOD

$2x + y + z + p = 5$
$x + 2y + z -2p = -6$
$x + 3y + 3z + 4p = 13$
$4x + 2y – 5z + 2p = 3$

First Step,
$2x + y + z + p – 5 = 0$
$x + 2y + z -2p + 6 = 0$
$x + 3y +3z + 4p – 13 = 0$
$4x + 2y – 5z + 2p – 3= 0$

Next steps, pick the first four column coefficient i.e 1st, 2nd, 3rd, & 4th then, transpose. Leave the 1st coefficient i.e 2nd, 3rd, 4th & 5th, then, transpose. Leave the 2nd column coefficient and pick the next four column coefficient i.e 3rd, 4th, 5th & 1st, then, transpose. Leave the 3rd column coefficient and pick next four column coefficient i.e 4th, 5th, 1st & 2nd then, transpose. Leave the 4th column coefficients and pick the next four column coefficients i.e 5th, 1st, 2nd & 3rd then transpose.

$$\Delta = \begin{vmatrix} 2 & 1 & 1 & 4 \\ 1 & 2 & 3 & 2 \\ 1 & 1 & 3 & -5 \\ 2 & -2 & 4 & 2 \end{vmatrix}$$

$$= 2\begin{vmatrix} 2 & 3 & 2 \\ 1 & 3 & -5 \\ -2 & 4 & 2 \end{vmatrix} -1\begin{vmatrix} 1 & 3 & 2 \\ 1 & 3 & -5 \\ 1 & 4 & 2 \end{vmatrix} +1\begin{vmatrix} 1 & 2 & 2 \\ 1 & 1 & -5 \\ 1 & -2 & 2 \end{vmatrix} -4\begin{vmatrix} 1 & 2 & 3 \\ 1 & 1 & 3 \\ 1 & -2 & 4 \end{vmatrix}$$

$= 2(96) – (7) + (-28) -4(1)$
$\Delta = 161$

$$\Delta_1 = \begin{vmatrix} 1 & 2 & 3 & 2 \\ 1 & 1 & 3 & -5 \\ 1 & -2 & 4 & 2 \\ -5 & 6 & -13 & -3 \end{vmatrix}$$

$$= 1\begin{vmatrix} 1 & 3 & -5 \\ -2 & 4 & 2 \\ 6 & -13 & -3 \end{vmatrix} - 2\begin{vmatrix} 1 & 3 & -5 \\ 1 & 4 & 2 \\ -5 & -13 & -3 \end{vmatrix} + 3\begin{vmatrix} 1 & 1 & -5 \\ 1 & -2 & 2 \\ -5 & 6 & -3 \end{vmatrix} - 2\begin{vmatrix} 1 & 1 & 3 \\ 1 & -2 & 4 \\ -5 & 6 & -13 \end{vmatrix}$$

$= 1\ (22) -2\ (-42) +3(7) -2\ (-17)$

$\Delta_1 = 161$

$$\Delta_2 = \begin{vmatrix} 1 & 1 & 3 & -5 \\ 1 & -2 & 4 & 2 \\ -5 & 6 & -13 & -3 \\ 2 & 1 & 1 & 4 \end{vmatrix}$$

$$= 1\begin{vmatrix} -2 & 4 & 2 \\ 6 & -13 & -3 \\ 1 & 1 & 4 \end{vmatrix} - 1\begin{vmatrix} 1 & 4 & 2 \\ -5 & -13 & -3 \\ 2 & 1 & 4 \end{vmatrix} + 3\begin{vmatrix} 1 & -2 & 2 \\ -5 & 6 & -3 \\ 2 & 1 & 4 \end{vmatrix} + 5\begin{vmatrix} 1 & -2 & 4 \\ -5 & 6 & -13 \\ 2 & 1 & 1 \end{vmatrix}$$

$= 1\ (28) – 1(49) + 3\ (-35) + 5\ (-7)$

$\Delta_2 = -161$

$$\Delta_3 = \begin{vmatrix} 1 & -2 & 4 & 2 \\ -5 & 6 & -13 & -3 \\ 2 & 1 & 1 & 4 \\ 1 & 2 & 3 & 2 \end{vmatrix}$$

$$= 1\begin{vmatrix} 6 & -13 & -3 \\ 1 & 1 & 4 \\ 2 & 3 & 2 \end{vmatrix} - (-2)\begin{vmatrix} -5 & -13 & -3 \\ 2 & 1 & 4 \\ 1 & 3 & 2 \end{vmatrix} + 4\begin{vmatrix} -5 & 6 & -3 \\ 2 & 1 & 4 \\ 1 & 2 & 2 \end{vmatrix} - 2\begin{vmatrix} -5 & 6 & -13 \\ 2 & 1 & 1 \\ 1 & 2 & 3 \end{vmatrix}$$

$= 1(-141) +2\ (35)\ +4\ (21)\ -2\ (-74)$

$\Delta_3 = 161$

$$\Delta_4 = \begin{vmatrix} -5 & 6 & -13 & -3 \\ 2 & 1 & 1 & 4 \\ 1 & 2 & 3 & 2 \\ 1 & 1 & 3 & -5 \end{vmatrix}$$

$$= -5\begin{vmatrix} 1 & 1 & 4 \\ 2 & 3 & 2 \\ 1 & 3 & -5 \end{vmatrix} - 6\begin{vmatrix} 2 & 1 & 4 \\ 1 & 3 & 2 \\ 1 & 3 & -5 \end{vmatrix} - 13\begin{vmatrix} 2 & 1 & 4 \\ 1 & 2 & 2 \\ 1 & 1 & -5 \end{vmatrix} + 3\begin{vmatrix} 2 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 1 & 2 \end{vmatrix}$$

$= -5\ (3) -6(-35) -13\ (-21) +3\ (5)$

$\Delta_4 = 483$

$$x = \frac{\Delta_1}{\Delta} = \frac{161}{161} = 1$$

$$y = \frac{\Delta_2}{\Delta} = \frac{-161}{161} = -1$$

$$z = \frac{\Delta_3}{\Delta} = \frac{161}{161} = 1$$

$$p = \frac{\Delta_4}{\Delta} = \frac{483}{161} = 3$$

PICK BACK TRANPOSE METHOD

$a_{11}x + a_{12}y = P_1$
$a_{21}x + a_{22}y = p_2$
First step, $a_{11}x + a_{12}y - p_1 = 0$
$a_{21}x + a_{22}y - p_2 = 0$

Next steps, pick the first two column coefficients i.e 1st and 2nd column coefficients then, back transpose. Leave the first column coefficient then, pick the next two column coefficient i.e 2nd & 3rd column coefficients then, back transpose. Leave the 2nd column coefficient and pick the 3rd & 1st column coefficients then, back transpose.

$$\Delta = \begin{vmatrix} a_{12} & a_{22} \\ a_{11} & a_{21} \end{vmatrix} = a_{12}a_{21} - a_{11}a_{22}$$

$$\Delta_1 = \begin{vmatrix} -p_1 & -p_2 \\ a_{12} & a_{22} \end{vmatrix} = -p_1 a_{22} - (-a_{12}p_2) = -p_1a_{22} + a_{12}p_2$$

$$\Delta_2 = \begin{vmatrix} a_{11} & a_{21} \\ -p_1 & -p_2 \end{vmatrix} = -a_{11}p_2 - (-p_1a_{21}) = -a_{11}p_2 + p_1a_{21}$$

$$x = \frac{\Delta_1}{\Delta} = \frac{-p_1a_{22} + a_{12}p_2}{a_{12}a_{21} - a_{11}a_{22}}$$

$$y = \frac{\Delta_2}{\Delta} = \frac{-a_{11}p_2 + p_1a_{21}}{a_{12}a_{21} - a_{11}a_{22}}$$

PICK BACK TRANSPOSE METHOD
$2x + y, = 13$
$3x - 2y = -5$
First step, $2x + y - 13 = 0$
$3x - 2y + 5 = 0$

Next steps, pick the first two column coefficients i.e 1st and 2nd column coefficients then, back transpose. Leave the first column coefficient then, pick the next two column coefficient i.e 2nd & 3rd column coefficients then, back transpose. Leave the 2nd column coefficient and pick the 3rd & 1st column coefficients then, back transpose.

$$\Delta = \begin{vmatrix} 1 & -2 \\ 2 & 3 \end{vmatrix} = (1 \times 3) - (2 \times -2) = 3 - (-4) = 3 + 4 = 7$$

$$\Delta_1 = \begin{vmatrix} -13 & 5 \\ 1 & -2 \end{vmatrix} = (-13 \times -2) - (1 \times 5) = 26 - 5 = 21$$

$$\Delta_2 = \begin{vmatrix} 2 & 3 \\ -13 & 5 \end{vmatrix} = (2 \times 5) - (3x - 13) = 10 - (-39) = 49$$

$$x = \frac{\Delta_1}{\Delta} = \frac{21}{7} = 3$$

$$y = \frac{\Delta 2}{\Delta} = \frac{49}{7} = 7$$

$x = 3, \quad y = 7$

THREE LINEAR EQUATION THREE UNKNOWN

PICK BACK TRANSPOSE MEHTOD

$a_{11}x + a_{12}y + a_{13}z = p_1$
$a_{21}x + a_{22}y + a_{23}z = p_2$
$a_{31}x + a_{32}y + a_{33}z = p_3$
First step, $a_{11}x + a_{12}y + a_{13}z - p_1 = 0$
$a_{21}x + a_{22}y + a_{23}z - p_2 = 0$
$a_{31}x + a_{32}y + a_{33}z - p_3 = 0$
Next steps, pick the first three column coefficient i.e 1st, 2nd and 3rd column coefficient. Then, back transpose. Leave the 1st column coefficient. Then, pick the next three column coefficients 2nd, 3rd and 4th column coefficients then, back transpose. Leave the 2nd column coefficient. then, pick the next three column i.e 3rd, 4th and 1st column coefficient then, back transpose. Leave the 3rd column coefficient i.e. a_{13} then, pick the next three column coefficients i.e 4th, 1st and 2nd column coefficients then, back transpose.

$$\Delta = \begin{vmatrix} a_{13} & a_{23} & a_{33} \\ a_{12} & a_{22} & a_{32} \\ a_{11} & a_{21} & a_{31} \end{vmatrix} = a_{13}\begin{vmatrix} a_{22} & a_{32} \\ a_{21} & a_{31} \end{vmatrix} - a_{23}\begin{vmatrix} a_{12} & a_{32} \\ a_{11} & a_{31} \end{vmatrix} + a_{33}\begin{vmatrix} a_{12} & a_{22} \\ a_{11} & a_{21} \end{vmatrix}$$
$$= a_{13}(a_{22}a_{31} - a_{21}a_{32}) - a_{23}(a_{12}a_{31} - a_{11}a_{32}) + a_{33}(a_{12}a_{21} - a_{11}a_{22})$$

$$\Delta_1 = \begin{vmatrix} -p_1 & -p_2 & -p_3 \\ a_{13} & a_{23} & a_{33} \\ a_{12} & a_{22} & a_{32} \end{vmatrix} = -p_1\begin{vmatrix} a_{23} & a_{33} \\ a_{22} & a_{32} \end{vmatrix} - (-p_2)\begin{vmatrix} a_{13} & a_{33} \\ a_{12} & a_{32} \end{vmatrix} - p_3\begin{vmatrix} a_{13} & a_{23} \\ a_{12} & a_{22} \end{vmatrix}$$
$$= -p_1(a_{23}a_{32} - a_{22}a_{33}) + p_2(a_{13}a_{32} - a_{12}a_{33}) - p_3(a_{13}a_{22} - a_{12}a_{23})$$

$$\Delta_2 = \begin{vmatrix} a_{11} & a_{21} & a_{31} \\ -p_1 & -p_2 & -p_3 \\ & & \end{vmatrix} = a_{11}\begin{vmatrix} -p_2 & -p_3 \\ a_{12}\ a_{23} & a_{33} \end{vmatrix} - a_{21}\begin{vmatrix} -p_1 & -p_2 \\ a_{23} & a_{33} \end{vmatrix} + a_{31}\begin{vmatrix} -p_1 & -p_2 \\ a_{13} & a_{33} \end{vmatrix} \quad a_{13} \quad a_{23}$$
$$= a_{11}(-p_2a_{33} + a_{23}p_3) - a_{21}(-p_1a_{33} + a_{13}p_3) + a_{11}(-p_1a_{23} + a_{13}p_2)$$

$$\Delta_3 = \begin{vmatrix} a_{12} & a_{22} & a_{32} \\ & a_{11} & a_{21} \\ -p_1 & -p_2 & -p_3 \end{vmatrix} a_{31} = \begin{vmatrix} a_{12} & a_{21} \\ -p_2 & -p_3 \end{vmatrix} a_{31} \quad \begin{vmatrix} a_{22} & a_{11} \\ -p_1 & -p_3 \end{vmatrix} a_{31} + \begin{vmatrix} a_{32} & a_{11} \\ -p_1 & -p_2 \end{vmatrix} a_{21}$$
$$= a_{12}(-a_{21}p_3 + p_2a_{31}) - a_{22}(-a_{11}p_3 + p_1a_{31}) + a_{32}(-a_{11}p_2 + p_1a_{21})$$

$$x = \frac{-\Delta_1}{\Delta} = \frac{-(-p_1(a_{23}a_{32} - a_{22}a_{33}) + p_2(a_{13}a_{32} - a_{12}a_{33}) - p_1(a_{13}a_{22} - a_{12}a_{23})}{a_{13}(a_{22}a_{31} - a_{21}a_{32}) - a_{23}(a_{12}a_{31} - a_{11}a_{32}) + a_{33}(a_{12}a_{21} - a_{11}a_{22})}$$

$$y = \frac{+\Delta_2}{\Delta} = \frac{a_{11}(-p_2a_{33} + a_{23}p_3) - a_{21}(-p_1a_{33} + a_{13}p_3) + a_{31}(-p_1a_{33} + a_{23}p_2)}{a_{13}(a_{22}a_{31} - a_{21}a_{32}) - a_{23}(a_{12}a_{31} - a_{11}a_{32}) + a_{33}(a_{12}a_{21} - a_{11}a_{22})}$$

$$z = \frac{-\Delta_3}{\Delta} = \frac{-(a_{12}(-a_{21}p_3 + p_2a_{31}) - a_{22}(-a_{11}p_3 + p_2a_{31}) - a_{22}(-a_{11}p_2 + p_1a_{21})}{a_{13}(a_{22}a_{31} - a_{21}a_{32}) - a_{23}(a_{12}a_{31} - a_{11}a_{32}) + a_{33}(a_{12}a_{21} - a_{11}a_{22})}$$

PICK BACK TRANSPOSE METHOD
$2x + y + 3z = 16$
$x + 2y - z = -2$
$3x + y + 2z = 4$

First step, $2x + y + 3z - 16 = 0$
$x + 2y - z + 2 = 0$
$3x + y + 2z - 4 = 0$
Next steps, pick the first three column coefficient i.e 1st, 2nd and 3rd column coefficient. Then, back transpose leave the 1st column coefficient then pick the next three column coefficient 2nd, 3rd and 4th column coefficients then, back transpose. Leave the 2nd column coefficient then, pick the next three column i.e 3rd, 4th and 1st

column coefficient back transpose. Leave the 3rd column coefficient then, pick the next three column coefficients i.e 4th, 1st and 2nd column coefficients then, back transpose.

$$\Delta = \begin{vmatrix} 3 & -1 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & 3 \end{vmatrix} = 3\begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix} +1\begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} +2\begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix}$$

$=3(6 – 1) +1 (3 -2) +2 (1- 4)$
$= 3 (5) +1 (1) +2 (-3)$
$= 15 + 1 – 6$
$= 10$

$$\Delta_1 = \begin{vmatrix} 16 & 2 & -14 \\ 3 & -1 & 2 \\ 1 & 2 & 1 \end{vmatrix} = -16\begin{vmatrix} -1 & 2 \\ 2 & 1 \end{vmatrix} -2\begin{vmatrix} 3 & 2 \\ 1 & 1 \end{vmatrix} -14\begin{vmatrix} 3 & -1 \\ 1 & 2 \end{vmatrix}$$

$= -16(-1-4) -2 (3-2)-14 (6 + 1)$
$= -16 (-5) -2 (1) – 14(7)$
$= 80 – 2 – 98$
$= -20$

$$\Delta_2 = \begin{vmatrix} 2 & 1 & 3 \\ -16 & 2 & -14 \\ 3 & -1 & 2 \end{vmatrix} = 2\begin{vmatrix} 2 & -14 \\ -1 & 2 \end{vmatrix} -1\begin{vmatrix} -16 & -14 \\ 3 & 2 \end{vmatrix} +3\begin{vmatrix} -16 & 2 \\ 3 & -1 \end{vmatrix}$$

$= 2(4 – 14) -1 (10) +3 (10)$
$=2(-10) -1 (10) +3 (10)$
$= - 20 – 10 +30$
$= 0$

$$\Delta_3 = \begin{vmatrix} 1 & 2 & 1 \\ 2 & 1 & 3 \\ -16 & 2 & -14 \end{vmatrix} = 1\begin{vmatrix} 1 & 3 \\ 2 & -14 \end{vmatrix} -2\begin{vmatrix} 2 & 3 \\ -16 & -14 \end{vmatrix} +1\begin{vmatrix} 2 & 1 \\ -16 & 2 \end{vmatrix}$$

$= 1(-14 – 6) -2 (-28 + 48) +1 (4 + 16)$
$= 1(-20) -2(20) + 1 (20)$
$= -20 + 40 +20$
$\Delta_3 = - 40$

$$x = -\frac{\Delta_1}{\Delta} = -\frac{(-20)}{10} = \frac{20}{10} = 2$$

$$y = +\frac{\Delta_2}{\Delta} = \frac{0}{10} = 0$$

$$z = -\frac{\Delta_3}{\Delta} = -\frac{(-40)}{10} = \frac{40}{10} = 4$$

$x = 2, y = 0, z = 4$

PICK BACK TRANSPOSE METHOD (4 by 4 MATRICES)

$2x + y + z + p = 5$
$x + 2y + z -2p = -6$
$x + 3y + 3z + 4p = 13$
$4x + 2y – 5z + 2p = 3$

First step, $2x + y + z + p -5 = 0$
$x +2y + z -2p + 6 = 0$
$x + 3y + 3z + 4p -13 = 0$
$4x + 2y - 5z +2p -3 = 0$

Next steps pick the first four column coefficient i.e 1st, 2nd, 3rd, & 4th. Then, back transpose. Leave the 1st, 2nd, 3rd & 4th. Then, back transpose. Leave the 1st column coefficients and pick the next four column coefficient i.e 2nd, 3rd, 4th, & 5th. Then, back transpose. Leave the 2nd column coefficient and pick the next four column coefficient i.e 3rd, 4th, 5th & 1st. then, back transpose. Leave the 4th column coefficients and pick the next four column coefficients i.e 5th, 1st, 2nd, & 4th then, back transpose.

$$\Delta = \begin{vmatrix} 1 & -2 & 4 & 2 \\ 1 & 1 & 3 & -5 \\ 1 & 2 & 3 & 2 \\ 2 & 1 & 1 & 4 \end{vmatrix}$$

$$= 1\begin{vmatrix} 1 & 3 & -5 \\ 2 & 3 & 2 \\ 1 & 1 & 4 \end{vmatrix} - (-2)\begin{vmatrix} 1 & 3 & -5 \\ 1 & 3 & 2 \\ 2 & 1 & 4 \end{vmatrix} +4\begin{vmatrix} 1 & 1 & -5 \\ 1 & 2 & 2 \\ 2 & 1 & 4 \end{vmatrix} -2\begin{vmatrix} 1 & 1 & 3 \\ 1 & 2 & 3 \\ 2 & 1 & 1 \end{vmatrix}$$

$= 1(-3) +2 (35) +4 (21) -2 (-5)$

$\Delta = 161$

$$\Delta_1 = \begin{vmatrix} -5 & 6 & -13 & 3 \\ 1 & -2 & 4 & 2 \\ 1 & 1 & 3 & -5 \\ 1 & 2 & 3 & 2 \end{vmatrix}$$

$$= -5\begin{vmatrix} -2 & 4 & 2 \\ 1 & 3 & -5 \\ 2 & 3 & 2 \end{vmatrix} -6\begin{vmatrix} 1 & 4 & 2 \\ 1 & 3 & -5 \\ 1 & 3 & 2 \end{vmatrix} -13\begin{vmatrix} 1 & -2 & 2 \\ 1 & 1 & -5 \\ 1 & 2 & 2 \end{vmatrix} +3\begin{vmatrix} 1 & -2 & 4 \\ 1 & 1 & 3 \\ 1 & 2 & 3 \end{vmatrix}$$

$= -5(-96) - 6(-7) -13(28) +3 (1)$

$\Delta_1 = 161$

$$\Delta_2 = \begin{vmatrix} 2 & 1 & 1 & 4 \\ & & & -5 \\ 1 & -2 & 4 & 2 \\ 1 & 1 & 3 & -5 \end{vmatrix} \begin{matrix} \\ 6 \;\; -13 \;\; -3 \\ \\ \\ \end{matrix}$$

$$= 2\begin{vmatrix} 6 & -13 & -3 \\ -2 & 4 & 2 \\ 1 & 3 & -5 \end{vmatrix} -1\begin{vmatrix} -5 & -13 & -3 \\ 1 & 4 & 2 \\ 1 & 3 & -5 \end{vmatrix} +1\begin{vmatrix} -5 & 6 & -3 \\ 1 & -2 & 2 \\ 1 & 1 & -5 \end{vmatrix} -4\begin{vmatrix} -5 & 6 & -13 \\ 1 & -2 & 4 \\ 1 & 1 & 3 \end{vmatrix}$$

$= 2(-22) -1 (42) +1(-7)-4(17)$

$\Delta_2 = -161$

$$\Delta_3 = \begin{vmatrix} 1 & 2 & 3 & 2 \\ 2 & 1 & 1 & 4 \\ -5 & 6 & -13 & -3 \\ 1 & -2 & 4 & 2 \end{vmatrix}$$

$$=1\begin{vmatrix} 1 & 1 & 4 \\ 6 & -13 & -3 \\ -2 & 4 & 2 \end{vmatrix} -2\begin{vmatrix} 2 & 1 & 4 \\ -5 & -13 & -3 \\ 1 & 4 & 2 \end{vmatrix} +3\begin{vmatrix} 2 & 1 & 4 \\ -5 & 6 & -3 \\ 1 & -2 & 2 \end{vmatrix} -2\begin{vmatrix} 2 & 1 & 1 \\ -5 & 6 & -13 \\ 1 & -2 & 4 \end{vmatrix}$$

$= 1(-28) -2 (-49) +3 (35) -2 (7)$

$= 161$

$$\Delta_4 = \begin{vmatrix} 1 & 1 & 3 & -5 \\ 1 & 2 & 3 & 2 \\ 2 & 1 & 1 & 4 \\ -5 & 6 & -13 & -3 \end{vmatrix}$$

$$=1\begin{vmatrix} 2 & 3 & 2 \\ 1 & 1 & 4 \\ 6 & -13 & -3 \end{vmatrix} -1\begin{vmatrix} 1 & 3 & 2 \\ 2 & 1 & 4 \\ -5 & -13 & -3 \end{vmatrix} +3\begin{vmatrix} 1 & 2 & 2 \\ 2 & 1 & 4 \\ -5 & 6 & -3 \end{vmatrix} +5\begin{vmatrix} 1 & 2 & 3 \\ 2 & 1 & 1 \\ -5 & 6 & -13 \end{vmatrix}$$

$= 1(141) -1 (-35) +3 (-21) +5 (74)$

$= 483$

$$x = \frac{\Delta_1}{\Delta} = \frac{161}{161} = 1$$

$y = \Delta_2 = -161 = -1$

$\overline{\Delta} \quad \overline{161}$

$z = \frac{\Delta_3}{\Delta} = \frac{161}{161} = 1$

$p = \frac{\Delta_4}{\Delta} = \frac{483}{161} = 3$

TWO LINEAR EQUATIONS TWO UNKNOWN

PICK DOWN TRANSPOSE METHOD

$a_{11}x + a_{12}y = p_1$
$a_{21}x + a_{22}y = p_2$
First step, $a_{11}x + a_{12}y - p_1 = 0$
$a_{21}x \quad a_{22}y - p_2 = 0$

next steps, pick the first two column coefficient then, down transpose. Leave the first column coefficient and pick the next two column coefficient i.e. 2nd & 3rd column coefficient then, down transpose. Leave the 2nd column coefficient. Pick the third and first column coefficient then, down transpose

$$\Delta = \begin{vmatrix} a_{21} & a_{11} \\ a_{22} & a_{12} \end{vmatrix} = a_{21}a_{12} - a_{22}a_{11}$$

$$\Delta_1 = \begin{vmatrix} a_{22} & a_{21} \\ -p_2 & -p_1 \end{vmatrix} = -a_{22}p_1 - (-p_2a_{12}) = -a_{22}p_1 + p_2a_{12}$$

$$\Delta_2 = \begin{vmatrix} -p_2 & -p_1 \\ a_{21} & a_{11} \end{vmatrix} = -p_2a_{11} - (-a_{21}p_1) = -p_2a_{11} + a_{21}p_1$$

$$x = \frac{\Delta_1}{\Delta} = \frac{-a_{22}p_1 + p_2a_{12}}{a_{21}a_{12} - a_{22}a_{11}}$$

$$y = \frac{\Delta_2}{\Delta} = \frac{-p_2a_{11} + a_{21}p_1}{a_{21}a_{12} - a_{22}a_{11}}$$

PICK DOWN TRANPOSE METHOD

$2x + y = 13$
$3x - 2y = -5$

First step, $2x + y - 13 = 0$
$3x - 2y + 5 = 0$

Next steps, pick the first two column coefficient then, down transpose. Leave the first column coefficient then, pick the next two column coefficient i.e. 2nd & 3rd column coefficient then, down transpose. Leave the 2nd column coefficients. Pick the third & first column coefficient then, down transpose.

$$\begin{vmatrix} 3 & 2 \\ -2 & 1 \end{vmatrix} = 3 + 4 = 7$$

$$\begin{vmatrix} -2 & 1 \\ 5 & -13 \end{vmatrix} = 26 - 5 = 21$$

$$\begin{vmatrix} 5 & -13 \\ 3 & 2 \end{vmatrix} = 10 + 39 = 49$$

$$x = \frac{\Delta_1}{\Delta} = \frac{21}{7} = 3$$

$$y = \frac{\Delta_1}{\Delta} = \frac{49}{7} = 7$$

$x = 3,\ y = 7$

THREE EQUATION IN THREE UNKNOWN

PICK DOWN TRANPOSE METHOD

$a_{11}x + a_{12}y + a_{13}z = p_1$
$a_{12}x + a_{22}y + a_{23}z = p_2$
$a_{31}x + a_{32}y + a_{33}z = p_3$

First step, $a_{11}x + a_{12}y + a_{13}z – p_1 = 0$
$a_{12}x + a_{22}y + a_{23}z – p_2 = 0$
$a_{31}x + a_{32}y + a_{33}z – p_3 = 0$

Next steps, pick the first three column coefficient then down transpose. Leave the first column efficient then, pick the next three column coefficient i.e. 2nd, 3rd, &4th column coefficient then, down transpose. Leave the 2nd column coefficient then, pick the next three column coefficient 3rd, 4th & 1st column coefficient then, down transpose. Leave the 3[rd] column coefficient then, pick the next three column coefficient i.e. 4th, 1st & 2nd column coefficients then, down transpose.

$$\Delta = \begin{vmatrix} a_{31} & a_{21} & a_{11} \\ a_{32} & a_{22} & a_{12} \\ a_{33} & a_{23} & a_{13} \end{vmatrix}$$

$$= a_{31}\begin{vmatrix} a_{22} & a_{12} \\ a_{23} & a_{13} \end{vmatrix} - a_{21}\begin{vmatrix} a_{32} & a_{12} \\ a_{33} & a_{13} \end{vmatrix} + a_{11}\begin{vmatrix} a_{32} & a_{22} \\ a_{33} & a_{23} \end{vmatrix}$$

$$= a_{31}(a_{22}a_{13} - a_{23}a_{12}) - a_{21}(a_{32}a_{13} - a_{33}a_{12}) + a_{11}(a_{32}a_{23} - a_{33}a_{22})$$

$$\Delta_1 = \begin{vmatrix} a_{32} & a_{22} & a_{12} \\ a_{33} & a_{23} & a_{13} \\ -p_3 & -p_2 & -p_1 \end{vmatrix} = a_{32}\begin{vmatrix} a_{23} & a_{13} \\ -p_2 & -p_1 \end{vmatrix} - a_{22}\begin{vmatrix} a_{33} & a_{13} \\ -p_3 & -p_1 \end{vmatrix} + a_{12}\begin{vmatrix} a_{33} & a_{23} \\ -p_3 & -p_2 \end{vmatrix}$$

$$= a_{32}(-a_{23}p_1 + p_2a_{13}) - a_{22}(-a_{33}p_1 + p_3a_{13}) + a_{12}(-a_{33}p_2 + p_3a_{23})$$

$$\Delta_2 = \begin{vmatrix} a_{33} & a_{23} & a_{13} \\ -p_3 & -p_2 & -p_1 \\ a_{31} & a_{21} & a_{11} \end{vmatrix} = a_{33}\begin{vmatrix} -p_2 & -p_1 \\ a_{21} & a_{11} \end{vmatrix} - a_{23}\begin{vmatrix} -p_3 & -p_1 \\ a_{31} & a_{11} \end{vmatrix} + a_{13}\begin{vmatrix} -p_3 & -p_2 \\ a_{31} & a_{21} \end{vmatrix}$$

$$= a_{33}(-p_2a_{11} + a_{21}p_1) - a_{23}(-p_3a_{11} + a_{31}p_1) + a_{13}(-p_3a_{21} + a_{31}p_2)$$

$$\Delta_3 = \begin{vmatrix} -p_3 & -p_2 & -p_1 \\ a_{31} & a_{21} & a_{11} \\ a_{32} & a_{22} & a_{12} \end{vmatrix} = -p_3\begin{vmatrix} a_{21} & a_{11} \\ a_{22} & a_{12} \end{vmatrix} - (-p_2)\begin{vmatrix} a_{31} & a_{11} \\ a_{32} & a_{12} \end{vmatrix} - p_1\begin{vmatrix} a_{31} & a_{21} \\ a_{32} & a_{22} \end{vmatrix}$$

$$= -p_3(a_{21}a_{12} - a_{22}a_{11}) + p_2(a_{31}a_{12} - a_{32}a_{11}) - p_1(a_{31}a_{22} - a_{32}a_{21})$$

$$x = \frac{-\Delta_1}{\Delta} = \frac{-(a_{32}(-a_{23}p_1 + p_2a_{13}) - a_{12}(-a_{33}p_1 + p_3a_{13}) + a_{12}(-a_{33}p_2 + p_3a_{23}))}{a_{31}(a_{22}a_{13} - a_{23}a_{12}) - a_{21}(a_{32}a_{13} - a_{33}a_{12}) + a_{11}(a_{32}a_{23} - a_{33}a_{22})}$$

$$y = \frac{+\Delta_2}{\Delta} = \frac{a_{33}(-p_2a_{11} + a_{21}p_1) - a_{23}(-p_3a_{11} + a_{31}p_1) + a_{13}(-p_3a_{21} + a_{31}p_2)}{a_{31}(a_{22}a_{13} - a_{22}a_{11}) - a_{21}(a_{32}a_{13} - a_{33}a_{12}) + a_{11}(a_{32}a_{23} - a_{33}a_{22})}$$

$$z = \frac{-\Delta 3}{\Delta} = \frac{-(-p_3a_{21}a_{12} - a_{22}a_{11}) + p_2(a_{31}a_{12} - a_{32}a_{11}) - p_1(a_{31}a_{22} - a_{32}a_{21}))}{a_{31}(a_{22}a_{13} - a_{22}a_{12}) - a_{21}(a_{32}a_{13} - a_{33}a_{12}) + a_{11}(a_{32}a_{23} - a_{33}a_{22})}$$

PICK DOWN TRANSPOSE METHOD

$2x + y + 3z = 16$
$x + 2y – z = -2$
$3x + y + 22 = 14$
First step, $2x + y + 3z – 16 = 0$

x + 2y – z +2 = 0
3x + y + 2z – 14 = 0

$$\Delta = \begin{vmatrix} 3 & 1 & 2 \\ 1 & 2 & 1 \\ 2 & -1 & 3 \end{vmatrix} = 3\begin{vmatrix} 2 & 1 \\ -1 & 3 \end{vmatrix} -1 \begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} +2 \begin{vmatrix} 1 & 2 \\ 2 & -1 \end{vmatrix}$$

= 3(6 + 1) – 1 (3 -2) +2 (-1-4)
= 3(7) -1(1) +2 (-5)
= 21 – 1 – 10
= 10

$$\Delta_1 = \begin{vmatrix} 1 & 2 & 1 \\ 2 & -1 & 3 \\ -14 & 2 & -16 \end{vmatrix} = 1\begin{vmatrix} -1 & 3 \\ 2 & -16 \end{vmatrix} -2 \begin{vmatrix} 2 & 3 \\ -14 & -16 \end{vmatrix} +1 \begin{vmatrix} 2 & -1 \\ -14 & 2 \end{vmatrix}$$

= 1(16 – 6) -2 (-32+42) +1 (4 – 14)
= 1(10) -2(10) +1 (-10)
= 10 – 20 – 10
= -20

$$\Delta_2 = \begin{vmatrix} 2 & -1 & 3 \\ -14 & 2 & -16 \\ 3 & 1 & 2 \end{vmatrix} = 2\begin{vmatrix} 2 & -16 \\ 1 & 2 \end{vmatrix} -(-1) \begin{vmatrix} -14 & -16 \\ 3 & 2 \end{vmatrix} +3 \begin{vmatrix} -14 & 2 \\ 3 & 1 \end{vmatrix}$$

= 2(4+16) +1 (-28+48) +3(-14-6)
= 2(20) + 1 (20) +3 (-20)
= 40 + 20 – 60
= 0

$$\Delta_3 = \begin{vmatrix} -14 & 2 & -16 \\ 3 & 1 & 2 \\ 1 & 2 & 1 \end{vmatrix} = -14\begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} -2 \begin{vmatrix} 3 & 2 \\ 1 & 1 \end{vmatrix} -16 \begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix}$$

= -14 (1-4) -2 (3-2) -16(6-1)
-14(-3) -2(1) -16(5)
= 42 – 2 – 80
= - 40

$$x = -\frac{\Delta_1}{\Delta} = \frac{-(-20)}{10} = \frac{20}{10} = 2$$

$$y = \frac{+\Delta_2}{\Delta} = \frac{0}{10} = 0$$

$$z = \frac{-\Delta_3}{\Delta} = \frac{-(-40)}{10} = \frac{40}{10} = 4$$

x = 2, y = 0, z = 4

FOUR EQUATION FOUR UNKNOWN

PICK DOWN TRANSPOSE METHOD

2x +y + z + p = 5
x +2y + z – 2p = -6
x + 3y + 3z + 4p = 13
4x + 2y – 5z + 2p = 3

First steps, 2x + y + z + p – 5 = 0
x + 2y + z – 2p + 6 = 0
x + 3y + 3z + 4p -13 = 0
4x + 2y – 5z + 2p – 3 = 0

Next steps, pick the first four column coefficients i.e 1st, 2nd, 3rd, & 4th. Then, down transpose. Leave the 1st column coefficient and pick the next four column coefficient i.e 2nd, 3rd, 4th & 5th. Then, down transpose. leave the 2nd column coefficient and pick the next four column coefficients i.e 3rd, 4th 5^{th} & 1st then, down transpose. leave the 3^{rd} column coefficient ad pick the next four column coefficients i.e 4th, 5th, 1st & 2nd and 3rd. then, down transpose.

$$\Delta_1 = \begin{vmatrix} 4 & 1 & 1 & 2 \\ 2 & 3 & 2 & 1 \\ -5 & 3 & 1 & 1 \\ 2 & 4 & -2 & 1 \end{vmatrix}$$

$$=4 \begin{vmatrix} 3 & 2 & 1 \\ 3 & 1 & 1 \\ -4 & -2 & 1 \end{vmatrix} -1 \begin{vmatrix} 2 & 2 & 1 \\ -5 & 1 & 1 \\ 2 & -2 & 1 \end{vmatrix} +1 \begin{vmatrix} 2 & 3 & 1 \\ -5 & 3 & 1 \\ 2 & 4 & 1 \end{vmatrix} -2 \begin{vmatrix} 2 & 3 & 2 \\ -5 & 3 & 1 \\ 2 & 4 & -2 \end{vmatrix}$$

$= 4(1)-1(28) + (-7) -2 (-96)$

$= 161$

$$\Delta_2 = \begin{vmatrix} -5 & 3 & 1 & 1 \\ 2 & 4 & -2 & 1 \\ -3 & -13 & 6 & -5 \\ 4 & 1 & 1 & 2 \end{vmatrix}$$

$$= 5 \begin{vmatrix} 4 & -2 & 1 \\ -13 & 6 & -5 \\ 1 & 1 & 2 \end{vmatrix} -3 \begin{vmatrix} 2 & -2 & 1 \\ -3 & 6 & -5 \\ 4 & 1 & 2 \end{vmatrix} +1 \begin{vmatrix} 2 & 4 & 1 \\ -3 & -13 & -5 \\ 4 & 1 & 2 \end{vmatrix} -1 \begin{vmatrix} 2 & 4 & -2 \\ -3 & -13 & 6 \\ 4 & 1 & 1 \end{vmatrix}$$

$= -5(7) – 3(35) +1 (-49) -1 (-28)$

$= - 161$

$$\Delta_3 = \begin{vmatrix} 2 & 4 & -2 & 1 \\ -3 & -13 & 6 & -5 \\ 4 & 1 & 1 & 2 \\ 2 & 3 & 2 & 1 \end{vmatrix}$$

$$=2 \begin{vmatrix} -13 & 6 & -5 \\ 1 & 1 & 2 \\ 3 & 2 & 1 \end{vmatrix} -4 \begin{vmatrix} -3 & 6 & -5 \\ 4 & 1 & 2 \\ 2 & 2 & 1 \end{vmatrix} -2 \begin{vmatrix} -3 & -13 & -5 \\ 4 & 1 & 2 \\ 2 & 3 & 1 \end{vmatrix} -1 \begin{vmatrix} -3 & -13 & 6 \\ 4 & 1 & 1 \\ 2 & 3 & 2 \end{vmatrix}$$

$= 2(74) – 4 (-21) -2 (-35) -1 (141)$

$= 161$

$$\Delta_4 = \begin{vmatrix} -3 & -13 & 6 & -5 \\ 4 & 1 & 1 & 2 \\ 2 & 3 & 2 & 1 \\ -5 & 3 & 1 & 1 \end{vmatrix}$$

$$= -3 \begin{vmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 3 & 1 & 1 \end{vmatrix} -(-13) \begin{vmatrix} 4 & 1 & 2 \\ 2 & 2 & 1 \\ -5 & 1 & 1 \end{vmatrix} +6 \begin{vmatrix} 4 & 1 & 2 \\ 2 & 3 & 1 \\ -5 & 3 & 1 \end{vmatrix} +5 \begin{vmatrix} 4 & 1 & 1 \\ 2 & 3 & 2 \\ -5 & 3 & 1 \end{vmatrix}$$

$= -3(-5) + 13 (21) +6(35) + 5 (-3)$

$= 483$

$$x = \frac{\Delta_1}{\Delta} = \frac{161}{161} = 1$$

$$y = \frac{\Delta_2}{\Delta} = \frac{-161}{161} = -1$$

$z = \Delta_3 = \underline{161} = 1$

$\overline{\Delta}$ 161

$p = \frac{\Delta_4}{\Delta} = \frac{483}{161} = 3$

CHAPTER FIVE
TWO LINEAR EQUATION IN TWO UNKNOWN

PICK DOWN BACK TRANSPOSE METHOD

$a_{11}x + a_{12}y = p_1$

$a_{21}x + a_{22}y = p_2$

First Step, $a_{11}x + a_{12}y - p_1 = 0$

$a_{21}x + a_{22}y - p_2 = 0$

Next steps, the first two column coefficients then, down back transpose. Leave the 1st column coefficient. then, pick the next two column coefficient i.e. 2nd & 3rd column coefficient.Then, down back transpose. Leave the 2nd column coefficient then, down back transpose

$$\Delta = \begin{vmatrix} a_{22} & a_{12} \\ a_{21} & a_{11} \end{vmatrix} = a_{22}\, a_{11} \;\; -a_{21}\, a_{12}$$

$$\Delta_1 = \begin{vmatrix} -p_2 & -p_1 \\ a_{22} & a_{12} \end{vmatrix} = -p_2\, a_{12} - (-a_{22}p_1) = -p_2\, a_{12} + a_{22}\, p_1$$

$$\Delta_2 = \begin{vmatrix} a_{21} & a_{11} \\ -p_2 & -p1 \end{vmatrix} = -a_{21}\, p_1 - (-p_2\, a_{11}) = -a_{21}\, p_1 + p_2\, a_{11}$$

$$x = \frac{\Delta_1}{\Delta} = \frac{-p_2a_{12} + a_{22}p_1}{a_{22}a_{11} - a_{21}a_{12}}$$

$$y = \frac{\Delta_2}{\Delta} = \frac{-a_{21}p_1 + p_2a_{11}}{a_{22}a_{11} - a_{21}a_{12}}$$

PICK DOWN BACK TRANSPOSE METHOD

$2x + y = 13$

$3x - 2y = -5$

First Step, $2x + y - 13 = 0$

$3x - 2y + 5 = 0$

Next steps, the first two column coefficients then, down back transpose. Leave the 1st column coefficient. then, pick the next two column coefficient i.e. 2nd & 3rd column coefficient. Then, down back transpose. Leave the 2nd column coefficient then, down back transpose

$$\Delta = \begin{vmatrix} -2 & 1 \\ 3 & 2 \end{vmatrix} = (-2 \text{ x } 2) - (3 \text{ x } 1) = -4 -3 = -7$$

$$\Delta_1 = \begin{vmatrix} 5 & -13 \\ -2 & 1 \end{vmatrix} = (5 \text{ x } 1) - (-2\text{x } -13) = 5 - 26 = -21$$

$$\Delta_2 = \begin{vmatrix} 3 & 2 \\ 5 & -13 \end{vmatrix} = (3\text{x}-13) - (5 \text{ x } 2) = -39 - 10 = -49$$

$$x = \frac{\Delta_1}{\Delta} = \frac{-21}{-7} = 3$$

$$y = \frac{\Delta_2}{\Delta} = \frac{-49}{-7} = 7$$

$x = 3, y = 7$

THREE LINEAR EQUATION IN THREE UNKNOWN

PICK DOWN BACK TRANSPOSE METHOD

$a_{11}x + a_{12}y + a_{13}z = p_1$
$a_{21}x + a_{22}y + a_{23}z = p_1$
$a_{31}x + a_{32}y + a_{33}z = p_3$

First step, $a_{11}x + a_{12}y + a_{13}z - p_1 = 0$
$a_{21}x + a_{22}y + a_{23}z - p_2 = 0$
$a_{31}x + a_{32}y + a_{33}z - p_3 = 0$

Next steps, pick the first three column coefficient i.e 1st, 2nd, & 3rd column coefficient then, down back transpose. Leave the first column coefficient then, pick the next three column coefficients i.e. 2nd, 3rd & 4th column coefficient then, down back transpose. Leave the 2nd column coefficient then, pick the next three column coefficient i.e 3rd, 4th & 1st column coefficients then, down back transpose. Leave the 3rd column coefficient then, pick the next three column coefficient i.e. 4th, 1st, & 2nd then, down back transpose.

$$\Delta = \begin{vmatrix} a_{33} & a_{23} & a_{13} \\ a_{32} & a_{22} & a_{12} \\ a_{31} & a_{21} & a_{11} \end{vmatrix} = a_{23} \begin{vmatrix} a_{22} & a_{12} \\ a_{21} & a_{11} \end{vmatrix} - a_{23} \begin{vmatrix} a_{32} & a_{12} \\ a_{31} & a_{11} \end{vmatrix} + a_{13} \begin{vmatrix} a_{32} & a_{22} \\ a_{31} & a_{21} \end{vmatrix}$$

$$= a_{33}(a_{22}a_{11} - a_{21}a_{12}) - a_{23}(a_{32}a_{11} - a_{31}a_{12}) + a_{13}(a_{32}a_{21} - a_{31}a_{22})$$

$$\Delta_1 = \begin{vmatrix} -p_3 & -p_2 & -p_1 \\ a_{33} & a_{23} & a_{13} \\ a_{32} & a_{22} & a_{12} \end{vmatrix} = -p_3 \begin{vmatrix} a_{23} & a_{13} \\ a_{22} & a_{12} \end{vmatrix} - (-p_3) \begin{vmatrix} a_{33} & a_{13} \\ a_{32} & a_{12} \end{vmatrix} - p_1 \begin{vmatrix} a_{33} & a_{23} \\ a_{32} & a_{22} \end{vmatrix}$$

$$= -p_3(a_{23}a_{12} - a_{22}a_{13}) + p_2(a_{33}a_{12} - a_{32}a_{13}) - p_1(a_{33}a_{22} - a_{32}a_{23})$$

$$\Delta_2 = \begin{vmatrix} a_{31} & a_{21} & a_{11} \\ -p_3 & -p_2 & -p_1 \\ a_{33} & a_{23} & a_{13} \end{vmatrix} = a_{31} \begin{vmatrix} -p_2 & -p_1 \\ a_{23} & a_{13} \end{vmatrix} - a_{21} \begin{vmatrix} -p_3 & -p_1 \\ a_{33} & a_{13} \end{vmatrix} + a_{11} \begin{vmatrix} -p_3 & -p_2 \\ a_{33} & a_{23} \end{vmatrix}$$

$$= a_{31}(-p_2a_{13} + a_{23}p_1) - a_{21}(-p_3a_{13} + a_{33}p_1) + a_{11}(-p_3a_{23} + a_{33}p_2)$$

$$\Delta_3 = \begin{vmatrix} a_{32} & a_{22} & a_{12} \\ a_{31} & a_{21} & a_{11} \\ -p_3 & -p_2 & -p_1 \end{vmatrix} = a_{32} \begin{vmatrix} a_{21} & a_{11} \\ -p_2 & -p_1 \end{vmatrix} - a_{22} \begin{vmatrix} a_{31} & a_{11} \\ -p_3 & -p_1 \end{vmatrix} + a_{12} \begin{vmatrix} a_{31} & a_{21} \\ -p_3 & -p_2 \end{vmatrix}$$

$$= a_{32}(-a_{21}p_1 - (-p_2a_{11})) - a_{22}(-p_1a_{31} + p_3a_{11}) + a_{12}(-a_{31}p_2 + p_3a_{21})$$
$$= a_{32}(-a_{21}p_1 + p_2a_{11}) - a_{22}(-p_1a_{31} + p_3a_{11}) + a_{12}(-a_{31}p_2 + p_3a_{21})$$

$$x = \frac{\Delta_1}{\Delta} = \frac{-(-p_3(a_{23}a_{12} - a_{22}a_{33}) + p_2(a_{33}a_{12} - a_{32}a_{13}) - p_1(a_{33}a_{22} - a_{32}a_{23}))}{a_{33}(a_{22}a_{11} - a_{21}a_{12}) - a_{23}(a_{32}a_{11} - a_{31}a_{12}) + a_{13}(a_{33}a_{21} - a_{31}a_{22})}$$

$$y = +\frac{\Delta_2}{\Delta} = \frac{a_{31}(-p_2a_{13} + a_{23}p_1) - a_{21}(-p_3a_{13} + a_{33}p_1) + a_{11}(-p_3a_{23} + a_{33}p_2)}{a_{33}(a_{22}a_{11} - a_{21}a_{12}) - a_{23}(a_{32}a_{11} - a_{31}a_{12}) + a_{13}(a_{33}a_{21} - a_{31}a_{22})}$$

$$z = \frac{-\Delta_3}{\Delta} = \frac{-(a_{32}(-a_{21}p_1 + p_2a_{11}) - a_{22}(-a_{31}p_1 + p_3a_{11}) + a_{12}(-a_{31}p_2 + p_3a_{21}))}{a_{33}(a_{22}a_{11} - a_{21}a_{22}) - a_{23}(a_{32}a_{11} - a_{31}a_{12}) + a_{13}(a_{33}a_{21} - a_{31}a_{22})}$$

PICK DOWN BACK TRANSPOSE METHOD

$2x + y + 3z = 16$
$x + 2y - z = -2$
$3x + y + 2z = 14$
$2x + y + 3z - 16 = 0$
$x + 2y - z + 2 = 0$
$3x + y + 2z - 14 = 0$

Next steps, pick the first three column coefficient i.e 1st, 2nd, & 3rd column coefficient then, down back transpose. Leave the first column coefficient then, pick the next three column coefficients i.e. 2nd, 3rd & 4th column coefficient then, down back transpose. Leave the 2nd column coefficient then, pick the next three column coefficient i.e 3rd, 4th & 1st column coefficients then, down back transpose. Leave the 3rd column coefficient then, pick the next three column coefficient i.e. 4th, 1st, & 2nd then, down back transpose.

$$\Delta = \begin{vmatrix} 2 & -1 & 3 \\ 1 & 2 & 1 \\ 3 & 1 & 2 \end{vmatrix} = 2 \begin{matrix} 2 & 1 \\ 1 & 2 \end{matrix} -(-1) \begin{matrix} 1 & 1 \\ 3 & 2 \end{matrix} +3 \begin{matrix} 1 & 2 \\ 3 & 1 \end{matrix}$$

$=2(4-1)+1(2-3)+3(1-6)$

$= 2(3)+1(-1)+ 3(-5)$

$= 6 – 1 – 15$

$= -10$

$$\Delta_1 = \begin{vmatrix} -14 & 2 & -16 \\ 2 & -1 & 3 \\ 1 & 2 & 1 \end{vmatrix} = -14 \begin{vmatrix} -1 & 3 \\ 2 & 1 \end{vmatrix} -2 \begin{vmatrix} 2 & 3 \\ 1 & 1 \end{vmatrix} -16 \begin{vmatrix} 2 & -1 \\ 1 & 2 \end{vmatrix}$$

$= -14(-1-6)-2(2-3)-16(4+1)$

$= - 14(-7) -2(-1)-16(5)$

$= 98+2-80$

$=20$

$$\Delta_1 = \begin{vmatrix} 3 & 1 & 2 \\ -14 & 2 & -16 \\ 2 & -1 & 3 \end{vmatrix} = 3 \begin{vmatrix} 2 & -16 \\ -1 & 3 \end{vmatrix} -1 \begin{vmatrix} -14 & -16 \\ 2 & 3 \end{vmatrix} +2 \begin{vmatrix} -14 & 2 \\ 2 & -1 \end{vmatrix}$$

$=3(6 - 16)-1(-42 + 32)+2(14 - 4)$

$=3(-10) -1(-10) +2 (10)$

$= - 30 +10+20$

$= 0$

$$\Delta_3 = \begin{vmatrix} 1 & 2 & 1 \\ 3 & 1 & 2 \\ -14 & 2 & -16 \end{vmatrix} = 1 \begin{vmatrix} 1 & 2 \\ 2 & -16 \end{vmatrix} -2 \begin{vmatrix} 3 & 2 \\ -14 & -16 \end{vmatrix} +1 \begin{vmatrix} 3 & 1 \\ -14 & 2 \end{vmatrix}$$

$=1(-16 – 4) -2 (-48 + 28) +1 (6 +14)$

$=1(-20) -2 (-20) +1 (20)$

$= -20 + 40 +20$

$\Delta_3 = 40$

$x = \frac{-\Delta_1}{\Delta}$, $y = \frac{+\Delta_2}{\Delta}$, $z = \frac{-\Delta_3}{\Delta}$

$x = \frac{-20}{-10}$, $y = \frac{0}{-10}$, $z = \frac{-40}{-10}$

$x = 2, y = 0, z = 4$

FOUR LINEAR EQUATION IN FOUR UNKNOWN

PICK DOWN BACK TRANSPOSE METHOD

$2x + y + z + p = 5$
$x + 2y + z -2p = - 6$
$x + 3y + 3z + 4p = 13$
$4x + 2y – 5z + 2p = 3$

First step, $2x + y + z + p – 5 = 0$
$x + 2y + z – 2p + 6 =0$
$x + 3y + 3z + 4p – 13 = 0$
$4x + 2y – 5z + 2p – 3 = 0$

Next steps, pick the first four column coefficients i.e 1st, 2nd, 3rd, & 4th column coefficient then, down back transpose. Leave the 1st column coefficient and pick the next four column coefficient i.e 2nd, 3rd, & 4th column & 1st column coefficient then, down back transpose. Leave the 2nd column coefficient and pick the next four column coefficient i.e 3rd, 4th, & 2nd column coefficient then, down back transpose. Leave the 3rd column coefficient and pick the next four column coefficient i.e 4th, 5th, 1st & 2nd. Then, down back transpose. Leave the 4th column coefficient and pick the next four column coefficient i.e 5th, 1st, & 2nd. Then, down back transpose.

$$\Delta = \begin{vmatrix} 2 & 4 & -2 & 1 \\ -5 & 3 & 1 & 1 \\ 2 & 3 & 2 & 1 \\ 4 & 1 & 1 & 2 \end{vmatrix}$$

$$= 2\begin{vmatrix} 3 & 1 & 1 \\ 3 & 2 & 1 \\ 1 & 1 & 2 \end{vmatrix} -4\begin{vmatrix} -5 & 1 & 1 \\ 2 & 2 & 1 \\ 4 & 1 & 2 \end{vmatrix} -2\begin{vmatrix} -5 & 3 & 1 \\ 2 & 3 & 1 \\ 4 & 1 & 2 \end{vmatrix} -1\begin{vmatrix} -5 & 3 & 1 \\ 2 & 3 & 2 \\ 4 & 1 & 1 \end{vmatrix}$$

$= 2 (5) – 4 (-21) -2 (-35) – 1 (3)$

$= 161$

$$\Delta_1 = \begin{vmatrix} -3 & -13 & 6 & -5 \\ 2 & 4 & -2 & 1 \\ -5 & 3 & 1 & 1 \\ 2 & 3 & 2 & 1 \end{vmatrix}$$

$$= -3\begin{vmatrix} 4 & -2 & 1 \\ 3 & 1 & 1 \\ 3 & 2 & 1 \end{vmatrix} -(-13)\begin{vmatrix} 2 & -2 & 1 \\ -5 & 1 & 1 \\ 2 & 2 & 1 \end{vmatrix} +6\begin{vmatrix} 2 & 4 & 1 \\ -5 & 3 & 1 \\ 2 & 3 & 1 \end{vmatrix} -(-5)\begin{vmatrix} 2 & 4 & -2 \\ -5 & 3 & 1 \\ 2 & 3 & 2 \end{vmatrix}$$

$= -3 (-1) +13 (-28) + 6(7) + 5(96)$

$= 161$

$$\Delta_2 = \begin{vmatrix} 4 & 1 & 1 & 2 \\ -3 & -13 & 6 & -5 \\ 2 & 4 & -2 & 1 \\ -5 & 3 & 1 & 1 \end{vmatrix}$$

$$= 4\begin{vmatrix} -13 & 6 & -5 \\ 4 & -2 & 1 \\ 3 & 1 & 1 \end{vmatrix} -1\begin{vmatrix} -3 & 6 & -5 \\ 2 & -2 & 1 \\ -5 & 1 & 1 \end{vmatrix} +1\begin{vmatrix} -3 & -13 & -5 \\ 2 & 4 & 1 \\ -5 & 3 & 1 \end{vmatrix} -2\begin{vmatrix} -3 & -13 & 6 \\ 2 & 4 & -2 \\ -5 & 3 & 1 \end{vmatrix}$$

$=4(-17) – (7) +1 (-42) -2 (22)$

$= -161$

$$\Delta_3 = \begin{vmatrix} 2 & 3 & 2 & 1 \\ 4 & 1 & 1 & 2 \\ -3 & -13 & 6 & -5 \\ 2 & 4 & -2 & 1 \end{vmatrix}$$

$$= 2\begin{vmatrix} 1 & 1 & 2 \\ -13 & 6 & -5 \\ 4 & -2 & 1 \end{vmatrix} -3\begin{vmatrix} 4 & 1 & 2 \\ -3 & 6 & -5 \\ 2 & -2 & 1 \end{vmatrix} +2\begin{vmatrix} 4 & 1 & 2 \\ -3 & -13 & -5 \\ 2 & 4 & 1 \end{vmatrix} -1\begin{vmatrix} 4 & 1 & 1 \\ -3 & -13 & 6 \\ 2 & 4 & -2 \end{vmatrix}$$

$= 2(-7) -3 (-35) +2 (49) – 1 (28)$

$= 161$

$$\Delta_4 = \begin{vmatrix} -5 & 3 & 1 & 1 \\ 2 & 3 & 2 & 1 \\ 4 & 1 & 1 & 2 \\ -3 & -13 & 6 & -5 \end{vmatrix}$$

$$= -5\begin{vmatrix} 3 & 2 & 1 \\ 1 & 1 & 2 \\ -13 & 6 & -5 \end{vmatrix} -3\begin{vmatrix} 2 & 2 & 1 \\ 4 & 1 & 2 \\ -3 & 6 & -5 \end{vmatrix} +1\begin{vmatrix} 2 & 3 & 1 \\ 4 & -1 & 2 \\ -3 & -13 & 5 \end{vmatrix} -1\begin{vmatrix} 2 & 3 & 2 \\ 4 & +16 & 1 \\ -3 & -13 & 6 \end{vmatrix}$$

$= -5(-74) -3 (21) +1 (35) -1 (-141)$

$= 483$

$$x = \frac{\Delta_1}{\Delta} = \frac{161}{161} = 1$$

$y = \frac{\Delta_2}{\Delta} = \frac{-161}{161} = -1$

$z = \frac{\Delta_3}{\Delta} = \frac{161}{161} = 1$

$p = \frac{\Delta_4}{\Delta} = \frac{483}{161} = 3$

$x = 1, y = -1, z = 1, p = 3$

CHAPTER SIX

PROOVING FOR TWO EQUATION TWO UNKNOWN

$a_{11} + a_{12}y = p_1$
$a_{21} + a_{22}y = p_2$
$a_{11}x + a_{12}y - p_1 = 0$ ________(i) x a_{22}
$a_{21}x + a_{22}y - p_2 = 0$ ________ (ii) x a_{12}
$a_{11}a12x + a_{12}a_{22}y - a_{22}p_1 = 0$ ________ (iii)
$a_{12}a_{21}x + a_{12}a_{22}y - a_{12}p_2 = 0$ ________ (iv)

Subtracting equation (iv) from equation (iii)
$a_{11}a_{22}x - a_{12}a_{21}x - a_{22}p_1 + a_{12}p_2 = 0$
$x (a_{11}a_{22} - a_{12} a_{21}) = -a_{21}p_2 + a_{22}p_1$
$x = \frac{- a_{12}p_2 + a_{22}p_1}{a_{11}a_{22} - a_{12}a_{21}}$

$a_{11}x + a_{12}y - p_1 = 0$ ________ (i) x a_{21}
$a_{21}x + a_{22}y - p_2 = 0$ ________ (ii) x a_{11}
$a_{11}a_{21}x + a_{12}a_{21}y - p_1a_{21} = 0$ ________ (iii)
$a_{11}a_{21}x + a_{12}a_{22}y - p_2 a_{11} = 0$ ________ (iv)

Subtracting equation (iii) from (iv)
$a_{11}a_{22}y - a_{12}a_{21}y - p_2a_{11} + p_1a_{21} = 0$
$y(a_{11}a_{22} - a_{12}a_{21}) = - p_1a_{21} + p_2a_{11}$
$y = \frac{- p_1a_{21} + p_2a_{11}}{a_{11}a_{22} - a_{21}a_{12}}$

The matrix aij has the ith row and jth column by adding the 11th row and jth column.

So, if i + j = Even (assign the positive symbol (+)
i + j = Odd (assign the negative symbol (-1)
a_{11} = 1 + 1 =2 (positive), a14 = 1 + 4 = 5 (negative)
a_{21} = 2 + 1 = 3 (negative) a24 = 2 + 4 = 6 (positive)
a_{31} = 3 + 1 = 4 (positive) a34 = 3 + 4 = 7 (negative)

\+ - + -
a_{11} a_{12} a_{13} = p_1
\- + - +
a_{21} a_{22} a_{23} = p_2
\+ - + -
a_{31} a_{32} a_{33} = p_3

Since the coefficient p_1 contains negative sign which occurs after equal to sign, it will be included in our final answer. $-\frac{\Delta_1}{\Delta}$

Since the coefficient p_2 contains positive sign which occur after equal to sign, it will be included in our final answer. For p_2, $y = \frac{\Delta_2}{\Delta}$

While for p3, $z = -\frac{\Delta_3}{\Delta}$

GENERAL FORMULAR

x = 1 ⁿ∑ Ajjkj, y= 1 k_j Aij – 2 unknown
A j = 1 A

x = -1 ⁿ∑ Ajikj, y= 1 ⁿ∑ Ajikj, z = –1 ⁿ∑ Aji k – 3 unknown
A j=1 A j=i A j = 1

x = 1 ⁿ∑ Ajikj, y= 1 ⁿ∑ Ajikj, z = 1 ⁿ∑ Aji kj, P = 1 ⁿ∑ Ajikj, - unknown
A j=1 A j=i A j = 1 A j=1

So for an even number i.e two unknown, four unknown e.t.c x, y is all positive while for odd numbers i.e 3 unknown, 5 unknown, 7 unknown e.t.c x,y , z, p, t follow (-, + -, +, -) sign respectively.

NOTE

MUTIPLICATION OF MATRICES

C = AB

Cij = aii bij +$a_{12}b_2$j+------+a_1q bqj = a a_1k bki
y = a π ∑k = 1
Integer Equation

Cofactor of aij = x ij = $(-1)^{1+j}$ Mij

Mij is the minor of aij

$$\text{Det } A = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$a_{23} = (-1)^{2+3} M_{23}$

$= (-1)^3 M_{23}$

$$= (-1)\begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix}$$

<u>The General nth order determinant</u>

Det A ⁿ∑ aij& ij, with I chosen for one row.
j =1

Det A ⁿ∑ aij& ij, with j chosen for one row.
l =1

y = A π

Vector y may be a scalar, multiple of x

Assume y = Aλ, A is a scalar

Aλ = Aλ

Aλ = Aλ = (λI – A) = 0

I = identify matrix

Det (λI – A) = 0

For every possible value A, (I = 1,2,3 ……n) of nth order characteristic equation we can write,

$(\lambda iI - A)\, x_i = 0$