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Study on the effect of non-cohesive material parameters on the pile profile

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ABSTRACT
This paper describes a method for estimating the angle of repose based on the analysis of the forces acting on each particle in a pile of noncohesive materials used in industrial applications. The theoretical model with sliding and rolling friction coefficients was used to predict the pile profile, and they are in good agreement with the results of previous studies. Several data sets can be obtained more easily, including the forces exerted by particles on the inside and on the surface of a stack of non-cohesive materials. This method helps to facilitate the understanding of the pile of noncohesive materials, significantly reduces the computational time and allows obtaining a large number of results. This method can be used to estimate DEM parameters from pile profile with artificial neural networks and optimization methods, and can also be applied to improve the accuracy of simulation related to the flow of bulk solid used in industrial applications.
KEYWORDS; *Curved slopes, Granular pile, Angle of repose*

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I. INTRODUCTION

The calibration of DEM parameters of noncohesive materials has been studied for a long time and many studies have been reported.

The results of the study showed that the profile of the pile is not really straight but has a curved shape. Y.Grasselli studied the particle piling morphology through experimental studies using glass spheres and described that the piling shape has a curved shape at the bottom and the inclination angle of the natural angle depends on the experimental parameters [1]. Topic et al. studied a relatively large pile and showed that the inclination angle of the natural angle varies with the distance from the center of the pile, and that the inclination angle of the natural angle at the bottom of the pile has a small value [2].

The profile of the pile has a curved shape and the equations of the curve have been studied. Hermann proposed an equation for the shape of the pile formed by particles falling at a certain height reflecting the coefficient of restitution of the particle [3]. Hermann found that the part formed by the particles rolled at the bottom of the pile satisfies the logarithmic function. Alain de Ryck proposed an equation for the pile profile based on the assumption that the tangential stress at the failure surface of the pile depends on the normal stress and the sliding frictional coefficient [4]. Fathan Akbar described a mathematical model of the shape of the pile by considering the particles on the surface as chains and estimating the sliding frictional coefficient of the material from the angle of inclination of the natural angle [5].

Wensrich et al. simulated the lifting cylinder test with varying sliding frictional coefficient and rolling frictional coefficient and found that the inclination angle of gravity depends on both the sliding frictional coefficient between particles and the rolling frictional coefficient [6]. Roessler et al. proposed a method to estimate the unique sliding and rolling friction coefficients by combining the natural inclination angles of piles in the upper and lower parts of the experimental apparatus through a downloading experiment on sand [7].

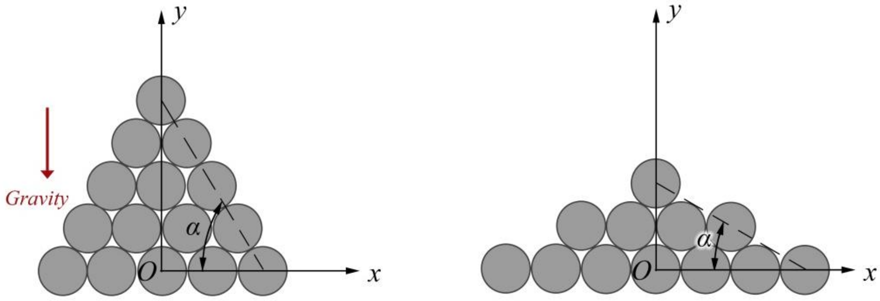
Albert R. et al. [8] mentioned stability criteria for particles lying on the surface of a pile in 2D and 3D, and stated that they can be used to calculate the maximum angle of stability of the pile. Zhou, Y.C. et al. analyzed the effects of particle characteristics, material properties, etc. on the angle of repose, and reported that sliding and rolling friction are the main causes of the pile formation. It is also noted that sliding and rolling friction are the main causes of pile formation and the effects of density, Poisson's ratio, damping coefficient and Young's modulus are not significant[9]. It was also noted that the angle of repose increases with increasing sliding and rolling friction coefficients and deviation from spheres, and decreases with increasing particle size and container thickness, and empirical equations for the effects of these factors were proposed [10].

The pile profile is actually dependent not only on the sliding frictional coefficient but also on the rolling friction coefficient. This paper describes a method for estimating the pile profile based on the analysis of the forces acting on the particles in a pile of noncohesive materials. The estimation of the pile profile reflects not only the sliding frictional coefficient but also the rolling friction coefficient.

II. Forces acting on particles on the surface in a pile

The profile of the heap can be considered to be determined by the particles placed on the surface. When the particles on the surface are in a steady state, the profile of the heap can be obtained by the position of those particles. Therefore, it is important to pay attention to the particles on the surface to obtain the profile of the heap.

The particles of noncohesive material can form a heap as shown in Fig. 1. The heap profile can be determined by the position of the particles on the surface in a heap of particles of equal diameter. If the frictional coefficient is large enough, the particles can form a heap in the same form as Fig. 1a, and if the frictional coefficient is small, a heap may be formed in the same form as Fig. 1b. And by the angle of the horizontal axis and the straight line through the center point of the particles lying on the surface of the heap, we can determine the angle of repose of the heap.



a) b)

Fig.1. The heap formed of particles of equal diameter and the angle of repose.
(a) large frictional coefficient, (b) low frictional coefficient

To obtain the shape of the heap, it is necessary to determine the forces acting on the particle. Assuming that the heap is formed, as shown in Fig. 1, the particle in the heap can be in contact with the surrounding six particles, as shown in Fig. 2, and the normal and tangential forces are applied at each contact point, respectively. Therefore, it is not straightforward to determine the forces acting between the particles. In general, if the forces acting on the particles in the heap are in equilibrium, we can write the following equations by considering $Q_{i,j}=\mu_{s.pp}N_{i,j}$.

$$P_{xi,j} = \left(N_{Ai,j} + N_{Bi,j} - N_{Ci,j} - N_{Di,j}\right)\left(sin\theta - \mu_{s.pp}cos\theta\right) + N_{Ei,j} - N_{Fi,j} \quad (1)$$

$$P_{yi} = \left(-N_{Ai} + N_{Bi} + N_{Ci} - N_{Di,j}\right)\left(cos\theta + \mu_{s.pp}sin\theta\right) + \mu_{s.pp}\left(N_{Ei,j} - N_{Fi,j}\right) - G \quad (2)$$

Here $\mu_{s.pp}$ is the coefficient of sliding friction between the particles and G is the weight of the particle. And i is the index that indicates the number of layer in which the particle is placed, and j means the particle's number in that layer. For the particle that is the top of the heap, i = 1, j = 1.

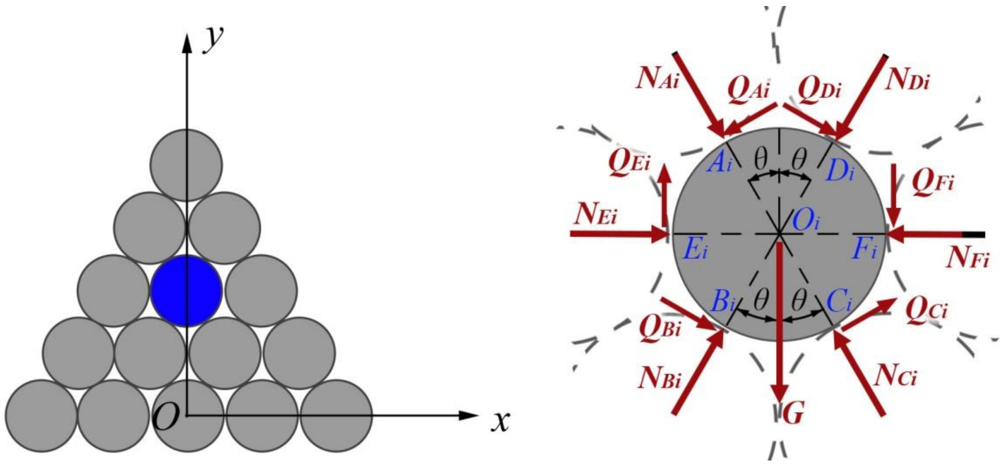


Fig. 2. Forces acting on the particle in the heap.

If the heap is symmetric about the y-axis as shown in Fig. 2, only the right part of the heap can be considered. Then, if i is odd, we set $j = 1$ for a particle whose center is on the y-axis, and $j = 1$ for a particle whose contour is tangent to the y-axis if i is even. Using the interaction relationship between the upper and left particles, the $N_{Ai,j}$, $N_{Di,j}$, $N_{Ei,j}$ on the blue particles in Fig. 2 can be expressed as

If i is odd,

$$N_{Ai,j} = N_{Ci-1,j-1}, \qquad N_{Di,j} = N_{Bi-1,j}, \qquad N_{Ei,j} = N_{Ei,j-1} \tag{3}$$

If i is even,

$$N_{Ai,j} = N_{Ci-1,j}, \qquad N_{Di,j} = N_{Bi-1,j+1}, \qquad N_{Ei,j} = N_{Ei,j-1} \tag{4}$$

The resultant force due to the particle weight G and the interaction forces between particles $N_{Ai,j}$, $N_{Di,j}$, $N_{Ei,j}$ can be considered as the external force acting on the particle, and $N_{Bi,j}$, $N_{Ci,j}$, $N_{Fi,j}$ as the corresponding reaction force. The x and y components of the resultant force by the weight of the particle G and the interaction forces between the particles $N_{Ai,j}$, $N_{Di,j}$, $N_{Ei,j}$ are

$$P'_{xi,j} = \left(N_{Ai,j} - N_{Di,j}\right)\left(sin\theta - \mu_{s.pp}cos\theta\right) + N_{Ei,j} \tag{5}$$

$$P'_{yi,j} = -\left(N_{Ai} + N_{Di,j}\right)\left(cos\theta + \mu_{s.pp}sin\theta\right) + \mu_{s.pp}N_{Ei,j} - G \tag{6}$$

It can be assumed that the following conditions should be satisfied to prevent particles from slipping or rolling in the heap:

$$\left|\frac{P'_{xi,j}}{P'_{yi,j}}\right| \leq \mu = \min\left(\mu_{s.pp}, \mu_{r.pp}\right) \tag{7}$$

If Eq. (7) is not satisfied, considering $N_{Fi,j}$, following conditions should be satisfied :

$$\left|\frac{P'_{xi,j} - N_{Fi,j}}{P'_{yi,j} - \mu_{s.pp}N_{Fi,j}}\right| \leq \mu \tag{8}$$

Considering $P'_{yi,j} < 0$ in Eq. (8)

$$N_{Fi,j} = \frac{P'_{xi,j} + \mu P'_{yi,j}}{1 - \mu_{s.pp}\mu} \tag{9}$$

If eq. (7) is satisfied, $N_{Fi,j}<0$ by eq. (9). In the case of $N_{Fi,j}<0$, the contact at point $F_{i,j}$ is lost, so $N_{Fi,j}=0$. Taking into account this case, Eq. (9) can be expressed as

$$N_{Fi,j} = \max\left(\frac{P'_{xi,j} + \mu P'_{yi,j}}{1 - \mu_{s.pp}\mu}, 0\right) \tag{10}$$

Obtaining $N_{Ai,j}$, $N_{Di,j}$, $N_{Ei,j}$, $N_{Fi,j}$ by Eqs. (3), (4), (10), we can obtain $N_{Bi,j}$, $N_{Ci,j}$ from Eqs. (1), (2).

$$N_{Bi,j} = \frac{1}{2} \cdot \frac{G\left(sin\theta - \mu_{s.pp}cos\theta\right) + \left(N_{Fi,j} - N_{Ei,j}\right)\left(2\mu_{s.pp}sin\theta - \mu_{s.pp}^2 cos\theta + cos\theta\right)}{\left(1 - \mu_{s.pp}^2\right)sin\theta cos\theta + \mu_{s.pp}\left(sin^2\theta - cos^2\theta\right)} + N_{Di,j} \tag{11}$$

$$N_{Ci,j}=\frac{1}{2}\cdot\frac{G(sin\theta-\mu_{s.pp}cos\theta)-(N_{Fi,j}-N_{Ei,j})(1+\mu_{s.pp}^2)cos\theta}{(1-\mu_{s.pp}^2)sin\theta cos\theta+\mu_{s.pp}(sin^2\theta-cos^2\theta)}+N_{Ai,j} \tag{12}$$

The forces acting on all particles in the heap can be determined using Eqs. (10)-(12). However, to use Eqs. (10)-(12), all forces of $N_{Ai,j}$, $N_{Bi,j}$, $N_{Ci,j}$, $N_{Di,j}$, $N_{Ei,j}$ and $N_{Fi,j}$ in at least one particle in the heap should be determined.

The particles lying on the surface in the heap are different from those in the heap. In Fig. 3, the blue particle at the top of the heap is in contact with the two particles below it at points B_1 and C_1, and there is no contact with the particles at other points. Therefore, N_{A1}, N_{D1}, N_{E1} and N_{F1} are equal to zero. And if the center of the particle that is top in the heap is located on the y-axis, and the points B and C are symmetric about the y-axis, and the N_{B1} and N_{C1} are assumed to be of equal size, they can be easily calculated. And using these forces, the forces acting on adjacent particles could be determined.

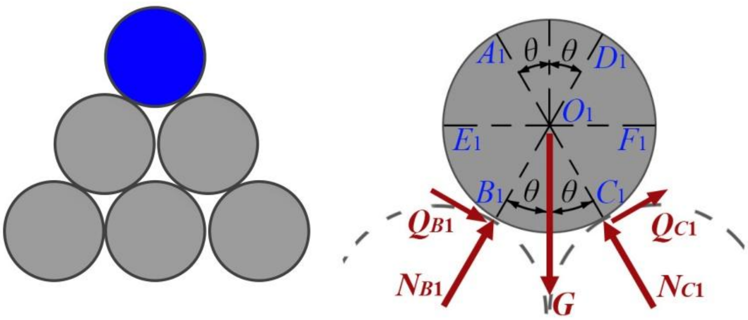


Fig. 3. Forces acting on particle on the top of the heap

For many particles lying on the surface of the heap, including the particles lying below the top particle, forces can be considered as acting as in the blue particle in Fig. 4. It is assumed that the blue particle is located on the surface of the heap, it is in contact with the top particle at point $A_{i,j}$, and the lower particles at points B_i and C_i. And since there are no particles in contact at point $D_{i,j}$, $N_{Di,j}$ = 0.

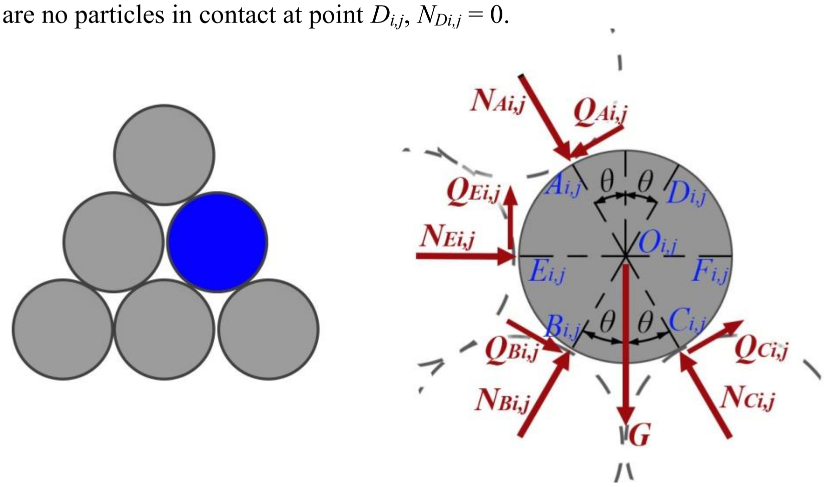


Fig. 4. The forces acting on the particles on the surface of the heap.

According to Eq. (10), for $N_{Fi,j}$ = 0, the forces acting on the particles in the lower layer are calculated assuming that the particle is in a steady state. In the case of $N_{Fi,j}$>0, it means that the particle can be in a stable state only when it is forced in the horizontal direction. In this case, as shown in Fig. 5, the yellow particles can be added to the heap to allow the blue particles to be in stable state. Assuming that the yellow particles added to the heap are in contact with the blue particles at point $F_{i,j}$.

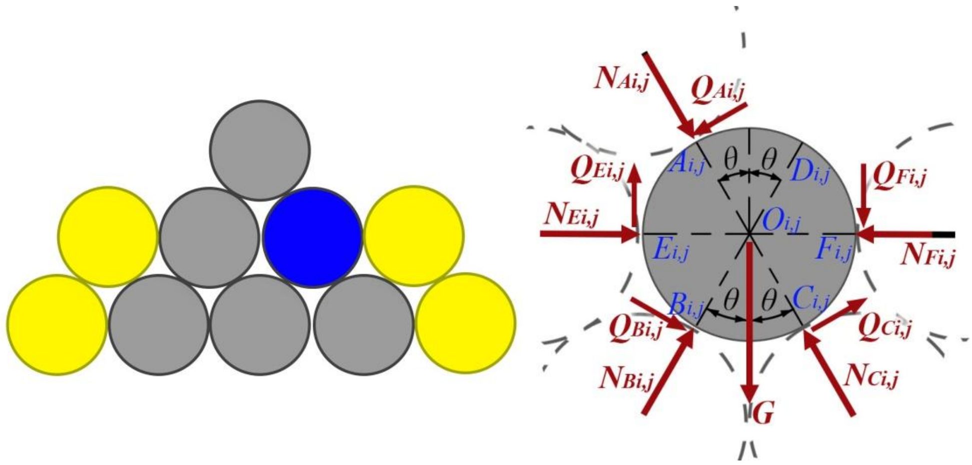


Fig. 5. The force on the blue particles when yellow particles were added to the heap.

The particle added to the heap can be considered to be located on the surface of the heap, and the forces acting on the particle (blue) are shown in Fig. 6. $N_{Ai,j}$ and $N_{Di,j}$ are equal to zero because this particle has no particle in contact at points $A_{i,j}$ and $D_{i,j}$. In this case, we continue to add the particle until $N_{Fi,j} = 0$ by Eq. (10).

Fig. 6. Forces acting on particles added to the heap

In Fig. 6, when the blue particle is in a steady state, the particle below it is in the same state as in Fig. 4. Let n_i be the number of particles in each layer of particles from the y-axis to the particle lying on the surface of the heap, and the position of the particle in the state shown in Fig. 4 is $n_{i-1}+1$ for i odd and n_{i-1} for i even. In each particle layer, all particles up to the particle front in the same state as Fig. 4 are considered to be in the heap. And from the particles in the state shown in Fig. 4, we add the particles until the $N_{Fi,j} = 0$ obtained by Eq. (10). If $N_{Fi,j} = 0$, then we consider the particle to be located on the surface and perform the calculations for the next layer. Connecting the center of the last particles in each particle layer, we can obtain the heap profile. The position of the center of each particle in the heap is

If i is odd,

$$x_{i,j} = 2rj - r \tag{13}$$

If i is even,

$$x_{i,j} = 2rj \tag{14}$$

$$y_i = H - 2r(i-1)\cos\theta \tag{15}$$

Where H is the height of the heap and r is the radius of the particle.

III. RESULT VIEW

Considering the sliding frictional coefficient and rolling friction coefficient, the shape of the heap can be obtained. The results were obtained by taking the sliding frictional coefficient from 0 to 0.5 and rolling frictional coefficient from 0 to 0.1 for a density of material ρ = 2500 kg/m^3 and a diameter of d = 10 mm. The heap profiles from Eqs. (10)-(15) are shown in Fig. 7. The heap profiles are similar to linear and as Wensrich et al. [6] and Zhou, Y.C. et al. [10] mentioned, the slope of the profile increased as the rolling frictional coefficient increased.

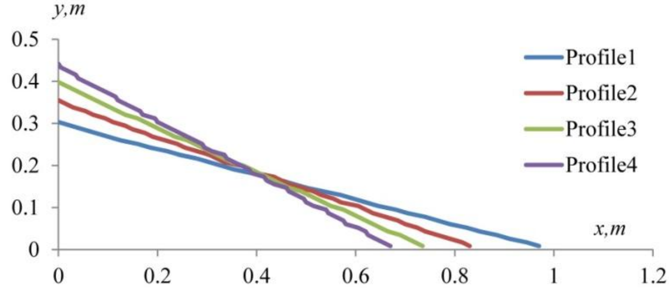


Fig. 7. Profile of heap($\mu_{s.pp}$=0.3)

Profile1-$\mu_{r.pp}$=0.025, Profile2-$\mu_{r.pp}$=0.05, Profile3-$\mu_{r.pp}$=0.075, Profile4-$\mu_{r.pp}$=0.1, In Fig. 8, the upper part of the heap profile is shown with the centers of the particles. The center of the particles in the upper part of the heap is indicated by the red dot, and the dotted line connecting the points means the shape of the surface when the particles are stacked as shown in Fig. 1. The center of the particle, which lies at the top of the heap, is located on the y-axis and decreases the y-coordinate of the center towards the x-axis. And the horizontal points represent the centers of the yellow particles added to the surface as shown in Fig. 5.

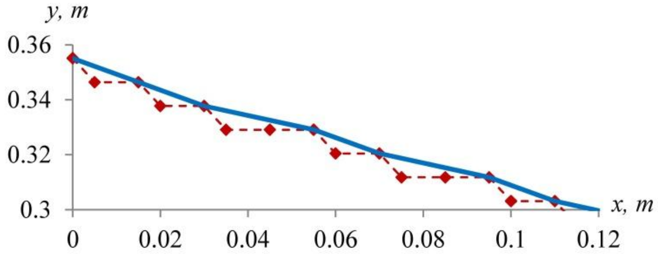


Fig. 8. The upper part of the heap profile($\mu_{s.pp}$=0.3, $\mu_{r.pp}$=0.05)

In Fig. 8, the curve (blue) can be obtained by connecting the points at the right end of the horizontal line. This curve represents the shape of the heap and can be used to determine the angle of repose. The angle of repose can be easily obtained from the intersections with the x and y axes because the heap profile is similar to linear,.

The forces acting on the contact points on the particles lying on the surface of the heap are shown in Fig. 9, as a ratio of the particle's weight. At the contact points $B_{i,j}$, $C_{i,j}$ with the below particles, the forces on the particles are equal to each other when x = 0. This corresponds to the top particle of the heap. And for other particles, the normal force acting on point $C_{i,j}$ is greater than the one on point $B_{i,j}$, and difference seems to be related to the force $N_{Ei,j}$, where the particle in contact at point $E_{i,j}$ acts horizontally..

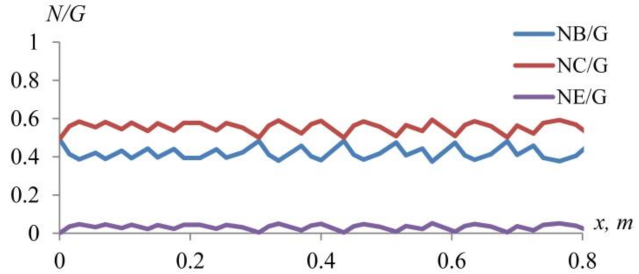
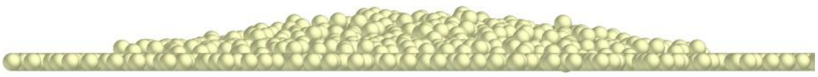


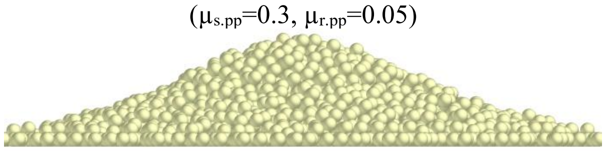
Fig. 9. Forces on a surface particle of heap($\mu_{s.pp}$=0.3, $\mu_{r.pp}$=0.05)

The lifting cylinder test was simulated by using EDEM. The material properties used for the tests were set as described above. Some of the heaps from the simulation are shown in Fig. 10.



($\mu_{s.pp}$=0.2, $\mu_{r.pp}$=0.025)

($\mu_{s.pp}$=0.3, $\mu_{r.pp}$=0.05)



($\mu_{s.pp}$=0.5, $\mu_{r.pp}$=0.075)

Fig. 10. Some of the heaps from the simulation

The angle of repose predicted by varying the sliding and rolling friction coefficients is shown in Fig. 11, with the ones from the simulation.

Fig. 10. angle of repose with varying frictional coefficient

lin1 ~ lin4 are curves obtained using Eqs. (10) ~ (15). $\mu_{r,pp}$=0.025, line2: $\mu_{r,pp}$=0.05, line3: $\mu_{r,pp}$=0.075 , line4: $\mu_{r,pp}$=0.1

1in5 ~ line8 is a curve obtained from DEM simulation. line5: $\mu_{r,pp}$=0.025, line6: $\mu_{r,pp}$=0.05, line7: $\mu_{r,pp}$=0.075 , line8: $\mu_{r,pp}$=0.1

As the friction coefficients increase, the angle of repose will increase. Compared with the DEM simulation results, the slope angle of the natural angle is slightly different for the sliding friction coefficient of 0.1-0.2, but it has similar values for the values of 0.3-0.5.

IV. CONCLUSION

Assuming that all particles on the surface in the pile of noncohesive materials were in contact, the forces acting on each particle were analyzed. And we estimated the shape of the pile from the conditions under which the forces acting on each particle are balanced, and they are in relatively good agreement with the experimental results. By estimating the pile profile, it can be seen that the pile profile has a relatively convex shape, which is consistent with previous studies. The results of the study suggest that the formation of piles can be further improved by reflecting the coefficient of friction of rolling together with the coefficient of friction of sliding. The method of estimating the pile profile from the sliding and rolling friction coefficients will be used in the future to obtain a relatively accurate pile profile of bulk solid and to estimate and calibrate the particle sliding and rolling friction coefficients.

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