

# A Developed Model for the Effects of Axial Heat Conduction in Single-Phase Flow In Microchannels

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## -----ABSTRACT-----

*A model is developed to analyze the effect of axial conduction on heat transfer during single-phase flow in microchannels. The axial heat conduction in the wall introduces heat flow toward the inlet section resulting in an increase in the local fluid temperature and a corresponding increase in the wall temperature. Neglecting this effect while reducing the experimental data results in a lower value of the experimental Nusselt number. The model derived in this work takes into account this effect and offers a parameter to estimate the effect introduced by the axial heat conduction effect in the wall.*

**Keywords:** laminar flow, single-phase flow, model, axial conduction, heat transfer coefficient, microchannels.

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## Nomenclature

$A$  = area,  $m^2$

$A_{h,f}$  = convective heat transfer surface area from inlet to current location,  $m^2$

$A_{h,s}$  = channel wall cross-sectional area for axial conduction,  $m^2$

$A_f$  = fluid flow cross-sectional area,  $m^2$

$A^*$  = ratio of  $A_{h,s}$  to  $A_f$ , dimensionless

$c_p$  = heat capacity,  $J/kg\ ^\circ C$

$Bi$  = Biot number, dimensionless

$Cond$  = ratio of axial conduction to input heat, dimensionless

$D_h$  = hydraulic diameter,  $m$

$h$  = heat transfer coefficient,  $W/m^2\ ^\circ C$

$k$  = thermal conductivity of liquid,  $W/m\ ^\circ C$

$Kc$  = non-dimensional parameter in Eq. (14), dimensionless

$L$  = length,  $m$

$M$  = ratio of axial conduction to input heat, dimensionless

$\dot{m}$  = mass flow rate,  $kg/s$

$NTU$  = number of transfer unit, dimensionless

$Nu$  = Nusselt number, dimensionless

$Nu^*$  = ratio of  $Nu_{exp}$  to  $Nu_{th}$ , dimensionless

$P$  = perimeter,  $m$

$Pe$  = Peclet number, dimensionless

$Pr$  = Prandtl number, dimensionless

$q$  = heat transfer rate,  $W$

$q^n$  = heat flux,  $W/m^2$

$r$  = wall size ratio, dimensionless

$Re$  = Reynolds number, dimensionless

$St$  = Stanton number, dimensionless

$T$  = temperature,  $C$

$Tc$  = non-dimensional parameter in Eq. (15), dimensionless

$x$  = distance from the entrance

### Subscripts

$cond$  = conduction

$conv$  = convection

$f$  = fluid

$h$  = heating

$k$  = conductivity

$k0$  = neglecting the effect due to axial conduction

$s$  = surface

$th$  = theoretical

$w$  = wall

$x$  = stream-wise spatial coordinate

## I. Introduction

With the developments in the micro-electro-mechanical systems (MEMS) fabrication technology, microchannels can be manufactured directly on the backside of a silicon wafer. For this type of microchannels, the ratio of wall thickness to channel diameter is usually much larger than that for macro scale tubes. This ratio increases as the channel diameter decreases for a given silicon wafer thickness. Many experimental studies reported in the literature for microchannels revealed a significantly lower value of fully developed Nusselt number for laminar flow with a further decreasing trend at lower Reynolds numbers [1–7]. Some of the reasons proposed in the literature to explain this phenomenon are due to scale [5–7], aspect ratio [1], roughness [2], and axial conduction[4]. Scale effect should not be a reason for lower  $Nu$  in Refs.[5]–[7] since no microscale effects are expected for liquid flow in microchannels [8–11].

Rosa *et al.* [12] reviewed single-phase heat transfer in microchannels and observed that experimental data with single circular microchannels were usually more accurate and in good agreement with conventional correlations. For single commercial tubes, the axial conduction effect was much less since the heat transfer area for axial conduction was much smaller than that for microchannels on a silicon substrate [8–10,13–18].

Axial conduction effect on heat transfer was investigated by Davis and Gill [19] during Poiseuille–Couette flow between parallel plates. They concluded that axial conduction effect lowered the  $Nu$  value. The importance of axial conduction increased with increasing ratio of channel thickness to channel length, and wall to fluid thermal conductivity ratio, whereas increasing  $Pe (= Re \cdot Pr)$  reduced these effects. A few researchers [4,20–22] found that the fluid temperature along the channel length in a uniformly heated channel did not increase linearly due to the axial heat conduction. Guo and Li [20,22] numerically investigated the effect of axial heat conduction on  $Nu$ . A clear effect of axial conduction was revealed as a function of  $Re$  for different outer to inner diameter ratios. It was found that the  $Nu$  from the numerical simulation was significantly lower than the conventional channel predictions. The authors concluded that the lower  $Nu$  values can be attributed to the use of one-dimensional model without axial heat conduction in the channel wall. The deviation was found to increase with decreasing  $Re$ , and increasing external to internal diameter ratio or wall to fluid thermal conductivity ratio.

Another numerical simulation work was performed by Maranzana *et al.* [21]; they investigated the axial heat conduction effect during water flow in a  $100 - \mu m$  channel formed by two  $10 - mm$  long and  $500 - \mu m$  thick silicon blocks. The channel was heated by a  $30 kW / m^2$  uniform heat flux on one wall and adiabatic boundary condition was imposed on the other wall. A normalized fluid temperature distribution along the channel length for several  $Re$  was clearly revealed. For  $Re$  higher than 500, the fluid temperature increased linearly while for low  $Re$  the temperature was higher than the linear variation, and the deviation increased with decreasing  $Re$ . Maranzana *et al.* [21] indicated that the axial conduction effect is negligible, if the ratio of axial conduction heat to input heat is less than 0.01. For the channels investigated, they obtained a lower  $Nu$  value, which decreased with decreasing  $Re$ .

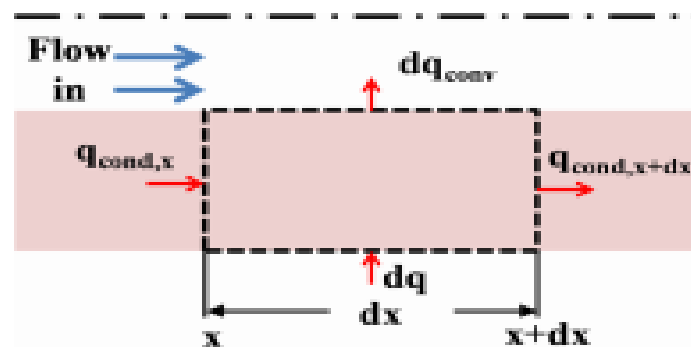
Tiselj *et al.* [4] conducted a systematic experimental and numerical investigation to study the effect of axial conduction through silicon wafer in the  $Re$  range from 3.2 to 84. The channels were fabricated on a  $530\ \mu m$  thick  $15 \times 15\ mm$  silicon substrate. The hydraulic diameter of the channel and the heating length were  $162\ \mu m$  and  $10\ mm$ , respectively. Both numerical and experimental results showed that the wall and fluid temperatures did not change linearly along the flow length. Numerical prediction of local  $Nu$  variation in the flow direction was influenced by axial conduction effects.

An exhaustive treatment of axial conduction effects and a comprehensive summary of research in this field are presented in Refs. [23] and [24]. They considered the effect of axial conduction in the fluid as well as in the wall. These effects were shown to be relevant in the entrance region of the microchannels. They also presented specific conditions under which the axial conduction effects need to be considered.

The work available in the literature clearly brings out the effect of axial conduction on heat transfer. Extensive numerical studies are available that confirm this effect. Although the axial conduction effect is found to be negligible in the fully developed region, experimental data from different investigators indicate significantly lower values of Nusselt numbers in microchannels. In the present work, a model is developed based on the axial conduction effects on the local fluid temperature, and consequently on the local wall temperature. The resulting model is used to compare the available experimental data for single-phase liquid flow.

## II. Theoretical Considerations

Consider a microchannel of uniform cross-section subjected to a constant heat flux wall boundary condition. Axial conduction effects are introduced due to the heat transfer in the fluid and in the wall in a direction opposite to the fluid flow. Following the analysis presented by Hetsroni *et al.* [23] and Yarin *et al.* [24], axial conduction in the fluid is important only in the entrance region for which  $x / Dh \cdot Pe \leq 10$ . For single-phase flow of water, considering a velocity of around  $1\ m/s$ , and a channel hydraulic diameter of  $200\ \mu m$ , the axial conduction effects are important only in the region  $x \leq 0.2\ \mu m$ . For under the same conditions, the axial conduction effects are important only in the region  $x \leq 200\ \mu m$  for a velocity of  $1\ m/s$ , and  $x \leq 20\ \mu m$  for a velocity of  $10\ m/s$ . From a practical standpoint, the effect of axial heat conduction in the fluid is insignificant for single-phase flow of both water and air in microchannels.



**Fig. 1 Flow configuration for the analytical model**

Detailed equations for analyzing the heat conduction effects have been presented by a number of investigators, e.g., Refs. [20]–[24]. In this work, the analysis is aimed at evaluating the local fluid temperature at any section in the fully developed region. The model is intended for reducing experimental data to obtain heat transfer coefficient and Nusselt number in the fully developed region. For this purpose, the flow configuration shown in Fig. 1 is used. Fluid flow is considered in the channel, and the tube wall is subjected to a constant heat flux on the outside surface. The following assumptions are made: steady state; incompressible, thermally, and hydrodynamically fully developed flow; and constant properties.

In the control volume

$$dq + q_{cond,x} + q_{cond,x+dx} - dq_{conv} = 0 \tag{1}$$

where  $dq$  is the heating power supplied from the outside wall,  $q_{cond,x}$  and  $q_{cond,x+dx}$  are the conduction heat transfer rates in locations  $x$  and  $x + dx$ , respectively, and  $dq_{conv}$  is the convection heat transfer rate to the fluid. The values can be calculated as the follows:

$$dq = q''P dx \tag{2}$$

$$q_{cond,x} = -k_s A_{h,s} \frac{dT_w}{dx} \tag{3a}$$

$$q_{cond,x+dx} = -k_s A_{h,s} \frac{dT_w}{dx} - k_s A_{h,s} \frac{dT_w}{dx^2} dx \tag{3b}$$

$$dq_{conv} = hP dx (T_{w,x} - T_{f,x}) \tag{4}$$

where  $P, k_s, A_{h,s}, h$  and  $T_{w,x}$  in Eqs. (2)–(4) are channel perimeter, thermal conductivity of the channel walls, cross-section area of the channel, convection heat transfer coefficient, and local wall temperature, respectively. Equation (1) can then be rewritten as

$$q''P + k_s A_{h,s} \frac{d^2T_w}{dx^2} - hp (T_{w,x} - T_{f,x}) = 0 \tag{5}$$

Since the flow is fully developed, the fluid temperature in this region rises linearly for constant heat flux boundary condition. The wall temperature also follows the linear trend and the second derivative term in Eq. (5) becomes zero. To find local fluid temperature, consider the fluid segment from the inlet section  $i$  of the heated length to the current location  $x$ . An energy balance yields

$$\dot{m} c_p (T_{f,x} - T_{f,i}) = q''Px - q_{cond,x} \tag{6}$$

The heat supplied may be expressed in terms of the fluid temperature rise without the axial conduction effects, expressed with subscript  $k0$ .

$$\dot{m} c_p (T_{f,x} - T_{f,i}) = \dot{m} c_p (T_{f,x,k0} - T_{f,i}) + k_s A_{h,s} \frac{dT_w}{dx} \tag{7}$$

where  $\dot{m}$  and  $c_p$  are the mass flow rate and fluid heat capacity, respectively.  $T_{f,x}$  and  $T_{f,x,k0}$  are the local fluid temperatures with and without considering axial conduction effects.

The local fluid temperature without axial conduction may be derived from Eq. (7)

$$T_{f,x,k0} = T_{f,x} - \frac{k_s A_{h,s}}{\dot{m} c_p} \frac{dT_w}{dx} \tag{8}$$

In fully developed flow,  $\frac{dT_f}{dx} = \frac{dT_w}{dx}$

$$q''P dx = \dot{m} c_p dT_f \tag{9}$$

$$\frac{dT_f}{dx} = \frac{q''P}{\dot{m} c_p} \tag{10}$$

The heat transfer coefficients for the two cases,  $h_x$  and  $h_{x,k0}$ , with and without considering axial heat conduction effects, respectively, are given by

$$h_x = \frac{q''}{T_{w,x} - T_{f,x}} \tag{11}$$

$$h_{x,k0} = \frac{q''}{T_{w,x} - T_{f,x,k0}} \tag{12}$$

The ratio of the two heat transfer coefficients may be written as

$$\frac{h_{x,ko}}{h_x} = \frac{T_{w,x} - T_{f,x}}{T_{w,x} - T_{f,x,ko}} = \frac{T_{w,x} - T_{f,x}}{T_{w,x} - T_{f,x} + \frac{k_s A_{h,s}}{\dot{m} c_p} \frac{dT_w}{dx}} = \frac{q''}{\frac{q''}{h_x} + \frac{k_s A_{h,s}}{\dot{m} c_p} \frac{q'' P}{\dot{m} c_p}} = \frac{1}{1 + \frac{k_s A_{h,s}}{\dot{m} c_p} \frac{h_x P dx}{\dot{m} c_p}} \quad (13)$$

where  $\frac{k_s A_{h,s}}{\dot{m} c_p} \frac{h_x P dx}{\dot{m} c_p}$  represents the ratio of axial conduction (taking its absolute value, since heat conduction is opposite to the fluid flow direction), to convective heat transfer and is termed as  $Kc$

$$Kc = \frac{k_s A_{h,s}}{\dot{m} c_p} \frac{h_x P dx}{\dot{m} c_p} = \frac{k_s A_{h,s}}{\dot{m} c_p} \frac{dT_w}{dT_f} = \frac{dq_{cond}}{dq} = 4 \frac{k_s}{k_f} \frac{1}{Re Pr} \frac{A_{h,s}}{A_{h,f}} \quad (14)$$

Where  $k_f$  and  $A_{h,f}$  are the fluid thermal conductivity and convection heat transfer area. The ratio  $\frac{h_x P dx}{\dot{m} c_p}$  is termed as  $Tc$  and can be expressed in the following form:

$$Tc = \frac{h_x P dx}{\dot{m} c_p} \frac{\left( \frac{h_x D_h}{k_f} \right) A_{h,f}}{\left( \frac{\dot{m} D_h}{A_f \mu_f} \right) \left( \frac{\mu_f c_p}{k_f} \right) A_f} = \frac{Nu_{th}}{Re Pr} \frac{A_{h,f}}{A_f} = St \frac{A_{h,f}}{A_f} \quad (15)$$

where  $A_f$  is the cross-sectional area of fluid flow in channel.  $Nu$  this the theoretical laminar fully developed flow  $Nu$ , which is a function of surface geometry and thermal boundary condition. The ratio of  $Nu$  without axial conduction to  $Nu$  with axial conduction can be determined from Eq. (13) and rewritten as

$$\frac{Nu_{ko}}{Nu_{th}} = \frac{h_{x,ko}}{h_x} = \frac{1}{1 + KcTc} = \frac{1}{1 + 4 \frac{k_s}{k_f} \frac{A_{h,s}}{A_f} \frac{Nu_{th}}{(Re Pr)^2}} \quad (16)$$

where  $Nu_{ko}$  is the Nusselt number neglecting the effect of axial heat conduction. The term  $KcTc$  is always a positive value; hence,  $\frac{Nu_{ko}}{Nu_{th}}$  is always less than 1 for nonzero values of  $KcTc$ .

In literature, different terminologies are used to express the ratio  $\frac{q_{cond}}{q_{conv}}$ . Maranzana *et al.* [21] defined  $\frac{q_{cond}}{q_{conv}}$  as

$M$  number and expressed it as  $M = \frac{r^2 NTU}{Bi}$ , where  $r, NTU$ , and  $Bi$  are wall size ratio, number of transfer units and Biot number. Guo and Li [22] termed the ratio  $\frac{q_{cond}}{q_{conv}}$  as “Cond” and expressed it as:

$Cond = \frac{k_s}{k_f} \cdot \frac{1}{Re Pr} \cdot \frac{A_{h,s}}{A_f} \cdot \frac{D_f}{L_{h,x}}$ . The axial conduction effects decrease with decreasing values of  $M$  and  $Cond$  [21,22].

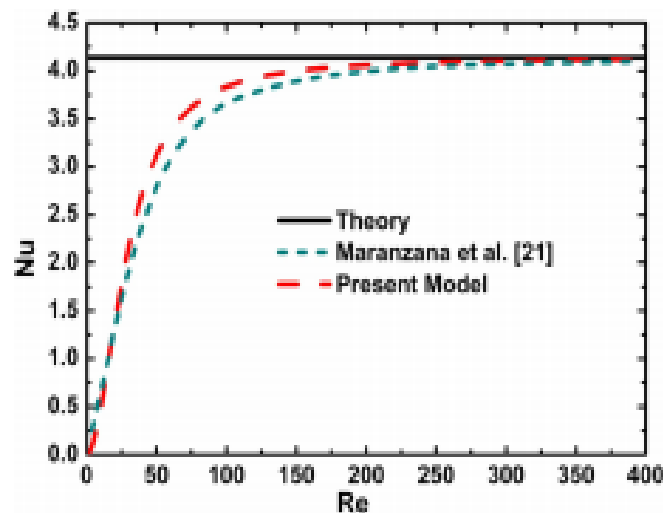
The physical meaning of  $\frac{A_{h,s}}{A_f}$ ,  $\frac{1}{Re Pr}$  and  $\frac{k_s}{k_f}$  may be explained as follows:

1.  $\frac{A_{h,s}}{A_f}$ : This parameter represents the ratio of axial conduction heat transfer area to fluid flow cross-sectional area. The axial conduction heat transfer increases with the increasing  $A_{h,s}$ . From Eq. (3), one can easily observe that the higher the  $A_{h,s}$  is in relation to  $A_f$ , the higher axial conduction heat transfer rate will be.
2.  $\frac{1}{Re Pr}$ : Reducing the flow rate results in a lower value of  $Re$  and a higher value of  $Pr$ . Since the flow rates in microchannels are generally low due to pressure drop considerations, the effect of heat conduction will be quite significant.  $Pr$  is a fluid property; the higher the value the better the convection heat transfer will be. For a fluid with a higher  $Pr$ , the axial heat conduction effect becomes less significant.
3. Ratio  $\frac{k_s}{k_f}$ : This parameter is the ratio of thermal conductivities of the channel material and the fluid. The higher the  $k_s$  value, the higher will be the effect due to axial conduction. The channel walls used in heat transfer experiments reported in literature include glass, stainless steel, silicon, and copper. The  $k_s$  value differs over a wide range from 1.4 to  $400 W / m^{\circ}C$  for these materials. The fluid conductivity values  $k_f$  for gases and liquids are also significantly different. For instance,  $k_f$  for air and water are  $0.026 W m^{\circ}C$  and  $0.61 W m^{\circ}C$  at room temperature, respectively. Thus the results for heat transfer with gas flow would experience much larger axial heat conduction effects as compared to the liquid flow.

### III. Model Verification

#### 3.1 Comparison of the Model to Available Numerical Simulations.

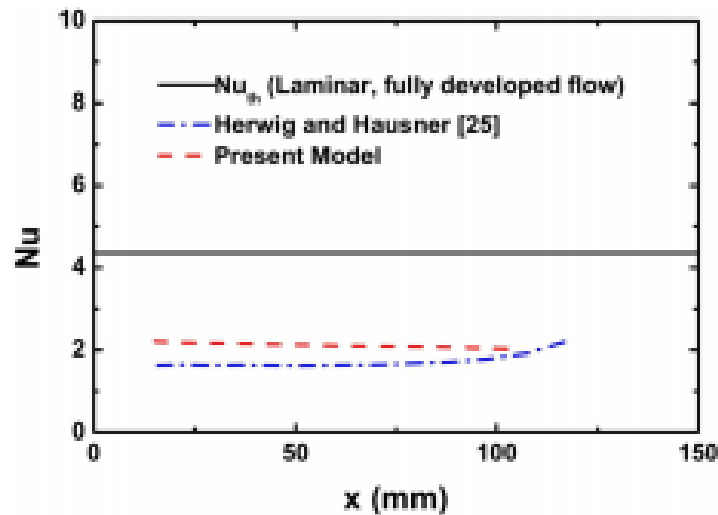
Available literature results are compared to the present model to validate its applicability under different operating conditions. Maranzana *et al.* [21] investigated the axial conduction effect using numerical simulation for the following case: water flow in a  $100 - \mu m$  channel formed by two  $10 mm$  long and  $500 \mu m$  thick silicon blocks. The channel was heated by a  $30 kW / m^2$  uniform heat flux on one wall, and the other wall was applied an adiabatic boundary condition.



**Fig. 2 Comparison of numerical simulation from Maranzana *et al.* [21] and the present model, Eq. (16)**

Figure 2 shows the numerical simulation results by Maranzana *et al.* [21] plotted as  $Nu$  versus  $Re$ . The present model, Eq. (16), is also plotted in Fig. 2 for comparison with the simulation results. It is found that the trends match quite well and the estimated  $Nu$  in Ref. [21] and the present equation are in very good agreement. In the high  $Re$  range ( $Re > 400$ ), the estimated values were the same as the theoretical values for

fully developed flow, indicating that the axial conduction effects are negligible for these cases. Confirming with other investigators' findings, the estimated  $Nu$  values were lower than the theoretical values in the low  $Re$  range and decreased further for lower  $Re$  cases.



**Fig. 3 Comparison of numerical simulation by Herwig and Hausner [25] to the present model, Eq. (16)**

Another numerical simulation work was performed by Herwig and Hausner [25]. Their numerical model was based on the experimental conditions of Tso and Mahulikar [26], who investigated heat transfer with water flow in an aluminum plate with 25 circular multi channels heated with constant heat flux. The calculated  $Nu$  was significantly lower than the conventional value; they explained the results by a nonconventional model. However, Herwig and Hausner [25] indicated that the reason of  $Nu$  departure from conventional values of Ref. [26] was the result of neglecting the axial heat conduction. They established a numerical model by using a commercial Computational fluid dynamics(CFD) code based on the test section and experimental condition similar to those in Ref. [26]. The hydraulic diameter, channel length, channel wall thickness, and channel pitch were  $0.730\text{ mm}$ ,  $115.5\text{ mm}$ ,  $1.414\text{ mm}$  and  $1.970\text{ mm}$ , respectively. The fluid bulk temperature varied nonlinearly from inlet to outlet. By calculating  $Nu$  with an interpolated fluid temperature, the calculated  $Nu$  was significantly lower than theoretical value as shown in Fig. 3. The calculated value was about 2.0, while for circular channel the theoretical value for laminar fully developed flow with constant heat flux heating was 4.364. As to the prediction by the present equation, the predicted values are also about 2.0, which is in good agreement with the simulation work. Since the flow is in laminar fully developed region, there is no significant variation of  $Nu$  along the flow direction.

### 3.2 Comparison of the Model with Available Experimental Data.

Besides simulation work, a few researchers have experimentally investigated axial conduction effect on fluid flow in microchannels. As mentioned in Sec. 3.1, Tso and Mahulikar [26] experimentally investigated convective heat transfer in aluminum micro channels. The experiments were performed with four different flow rates and heat fluxes. Local wall temperatures along the channel were recorded, and the local  $Nu$  was calculated. Local fluid bulk-mean temperature was derived by interpolation from the inlet and outlet fluid temperatures. This confirms that the  $Nu$  was derived based on the assumption of constant heat flux and linear fluid temperature rise along the channel length.

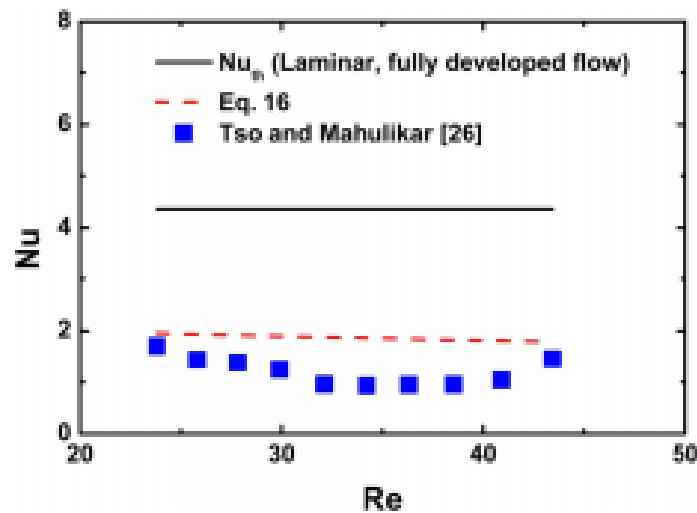


Fig. 4 Comparison of experimental data [26] to the present equation

Figure 4 shows a comparison of the experimental data in Ref. [26] to the theoretical laminar flow values and predictions by Eq. (16). The experimental data were significantly lower than the theoretical value of 4.364 for circular channel with constant heat flux in laminar fully developed flow, while it was found to have a better agreement with the predicted values from Eq. (16). The results indicate that the axial conduction effects reduce the local heat transfer coefficient and are well represented by the present model.

In Fig. 4, it was found that the predicted  $Nu$  from Eq. (16) decreases slightly with increasing  $Re$ . This phenomenon was different from the simulation results shown in Fig. 2. The calculated  $Nu$  and  $Re$  were local values along the flow direction for the same flow rates. The fluid temperature increased along the flow direction. With the increasing fluid temperature the viscosity and  $Pr$  decreased, while the  $Re$  increased. In Fig. 4, the  $Re$  values from low to high represent the local values at measurement locations from inlet to outlet.

Tiselj *et al.* [4] investigated axial conduction effects on heat transfer performance of water flow in a  $530\ \mu m$  silicon chip with 17 triangular microchannels over  $Re$  from 3 to 85. The hydraulic diameter of channel, channel width, channel pitch, and heating length were  $162\ \mu m$ ,  $310\ \mu m$ ,  $620\ \mu m$ , and  $10\ mm$ , respectively. From the experimental and simulation results, they found that the fluid temperature as well as the wall temperature changed nonlinearly along the length, and the wall temperature distribution agreed with their numerical predictions that accounted for the axial conduction effects in the wall. A table listed in their paper reported the raw data for the inlet and outlet fluid temperatures, flow rates, and wall temperatures in a few locations for three (low, middle, and high) heat fluxes.

The raw data presented by Tiselj *et al.* [4] are used to calculate the  $Nu$  using Eq. (16). The input heat to the fluid was calculated from energy balance; heat flux was assumed to be uniform and calculated by the input heat divided by the heat transfer area; heat transfer coefficient was calculated from the measured local wall temperature and the local fluid temperature, which was obtained from the inlet and outlet fluid temperatures by assuming a linear fluid temperature profile along the flow direction.



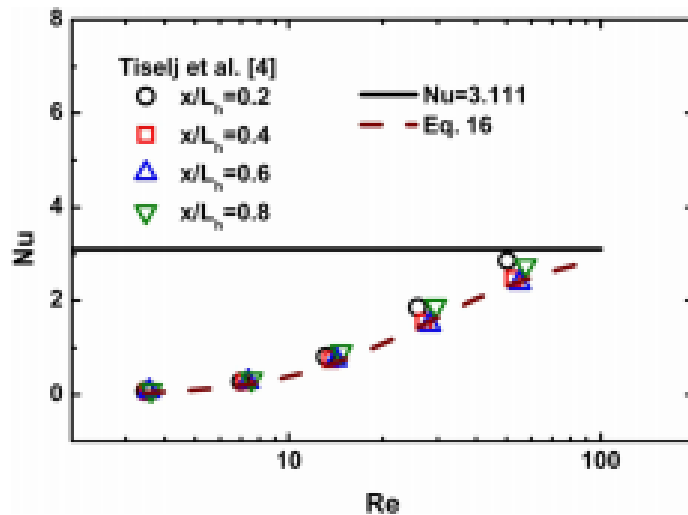


Fig. 5 Comparison of experimental data [4] and the prediction by Eq. (16)

Figure 5 shows the calculated  $Nu$  from Ref. [4] plotted as a function of  $Re$  in different heating lengths. It was found that the data can be predicted very well by Eq. (16). The  $Nu$  is a function of  $Re$  and increases with it. For an  $Re$  of about 100, the experimental data approach the  $Nu$  value for the theoretical laminar fully developed flow. The theoretical laminar fully developed  $Nu$  is 3.111 and is independent of  $Re$ . With the decreasing  $Re$ , the  $Nu$  becomes lower than the theoretical value and the deviation increases for lower  $Re$  values. The  $Re$  in their research was from 3 to 85; the theoretical fully developed lengths were between 0.13 mm and 3.3 mm, while the channel length was 10 mm.

Another experimental data set used to compare with the present model was by Harms *et al.* [27]. They studied water flowing through silicon chip with 68 rectangular microchannels. The hydraulic diameter of the channel, 68 channel width, channel height, channel pitch, and heating length were 404  $\mu m$ , 251  $\mu m$ , 1030  $\mu m$ , 370  $\mu m$  and 25 mm, respectively.

In the experiments of Harms *et al.* [27], the flow was in the developing flow regime due to the short heating length employed. As a first approximation, predicted  $Nu$  values were calculated by the present equation, Eq. (16), with slight modification for the two cases. In one case, the laminar fully developed values were used, while in the other case the developing flow values were used. For the aspect ratio of the channel, the laminar fully developed flow  $Nu$  is about 5.33. This value was used as fully developed  $Nu_{th}$  for all  $Re$  values. The developing  $Nu_{th}$  was calculated from the conventional developing flow correlation. The  $Nu$  values were found to be a function of  $Re$  and increased with it.

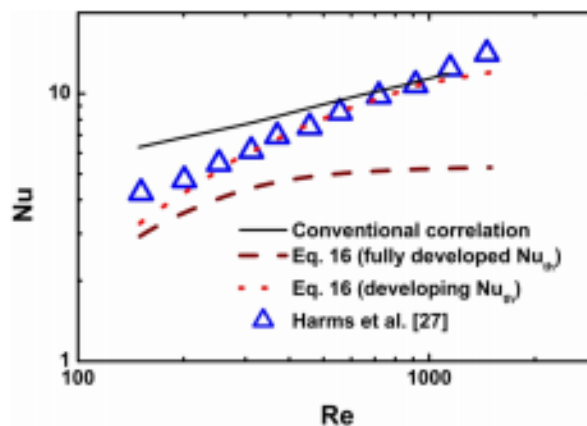


Fig. 6 Comparison of experimental data [27] and prediction of proposed model  $\frac{x}{L_h} = 0.25$

Figure 6 shows the experimental data of Ref. [27] and the prediction by conventional developing flow correlation and the present equation. In the high  $Re$  range, the data points approach the prediction of conventional correlation. The  $Nu$  was lower than the prediction of conventional correlation in the low  $Re$  range and the departure increases for lower  $Re$ . The authors indicated that the phenomenon was due to the flow bypass in the manifold. Another possible explanation is attributed to the axial conduction effects. Compared to the prediction of Eq. (16) using laminar fully developed values for  $Nu_{th}$ , the experimental data can be predicted well in the low  $Re$  range. However, for higher  $Re$ , the data were under predicted since the flow regime was in the developing region. Comparing with the prediction of Eq. (16) by using developing flow values for  $Nu_{th}$ , the experimental data can be predicted very well over the entire  $Re$  range. Although the present correlation was derived from the assumption of fully developed flow, it seems that Eq. (16) can predict the developing flow data well by using developing flow  $Nu$  value as  $Nu_{th}$ .

#### IV. Parametric Study of Axial Conduction Effects in Microchannels

In this section, axial heat conduction effects modeled by Eq. (16) are represented by  $\frac{Nu_{k0}}{Nu_{th}}$ . The ratio  $\frac{Nu_{k0}}{Nu_{th}}$

is denoted by  $Nu^*$  and the ratio  $\frac{A_{h,s}}{A_f}$  is denoted by  $A^*$  in the following discussion.  $Nu^*$  is always less

than 1 with axial heat conduction effects. As  $Nu^*$  approaches 1, the axial conduction effects become negligible. The effects of wall thickness for water and airflow, and wall material thermal conductivity and Reynolds number on  $Nu^*$  with water flow.

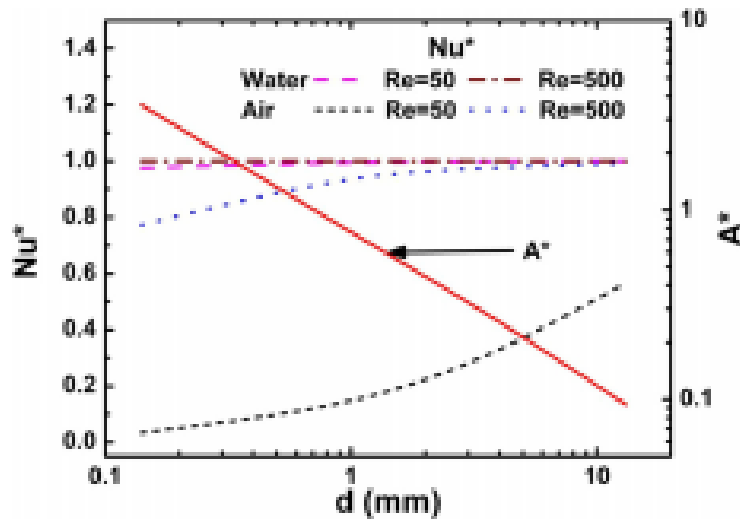


Fig. 7 Axial conduction effects of fluid flow in commercial stainless steel 304 tubes

The wall thickness of commercial macro tubes is usually small compared to the tube diameter; however, it becomes comparable to the tube diameter for small diameter tubes. Figure 7 shows the comparison of air and water flow in commercial tubes as a function of  $A^*$  (= ratio of wall cross-sectional area to fluid flow cross sectional area) and  $Nu^*$ . In larger diameter tubes,  $A^*$  is much less than 1, the tube wall cross-section area is much smaller compared to the flow cross-sectional area. From this figure, it is observed that the axial conduction effect is negligible for water flow in commercial stainless steel 304 tubes with  $Re$  higher than 50, while for air flow the axial conduction effect is not negligible and the  $Nu^*$  decreases significantly for smaller diameter tubes.

In literature, it was found the data for macro tubes were in good agreement with conventional correlation for commercial stainless steel tubes [8–10,13–18]. The experimental  $Re$  in literature was usually higher than 50 and the fluids used in these investigations were all liquid. As seen from Fig. 7, the axial conduction is found to be negligible for these cases.

The prediction in Fig. 7 shows a significant axial heat conduction effect for air flow in commercial tube. However, there is no data available for heat transfer with gas flow in microchannels. Such experiments are difficult to conduct due to severe heat losses(as compared to the convective heat transfer to the gas).

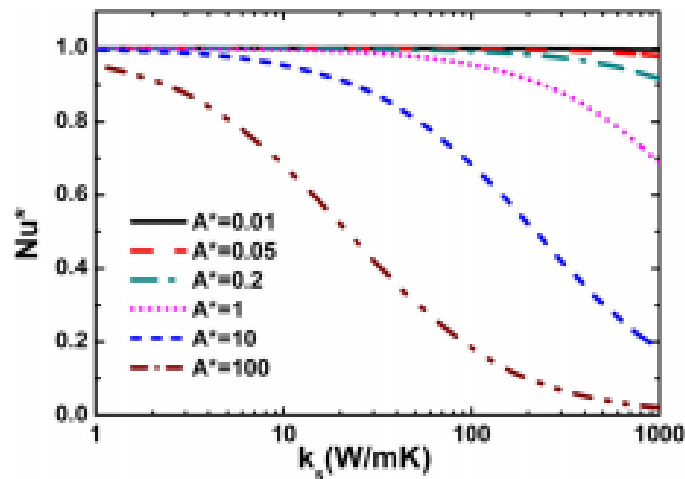


Fig. 8 Axial heat conduction effects of water flow in channelin Re = 50

Although from the prediction of Eq. (16) axial heat conduction is found to be negligible for water flow in commercial stainless steel tubes, it may not be negligible for water flow in high thermal conductivity or high  $A^*$  channels. Figure 8 shows axial conduction effects with  $Nu^*$  plotted as a function of channel thermal conductivity for water flow at  $Re = 50$  in different  $A^*$  channels. The axial heat conduction effect increases with increasing channel thermal conductivity and  $A^*$ . The thermal conductivity of stainless steel 304 is  $14.9 W/m^{\circ}C$  and the  $A^*$  value for commercial tube is less than 6. This results in the axial conduction effect of less than 5%. However, if the tube material is changed from stainless steel 304 to aluminum ( $k_s = 401 W/m^{\circ}C$ )  $Nu^*$  for  $A^* = 6$  is 0.67, while for copper tubes ( $k_s = 401 W/m^{\circ}C$ )  $Nu^*$  becomes only 0.48.

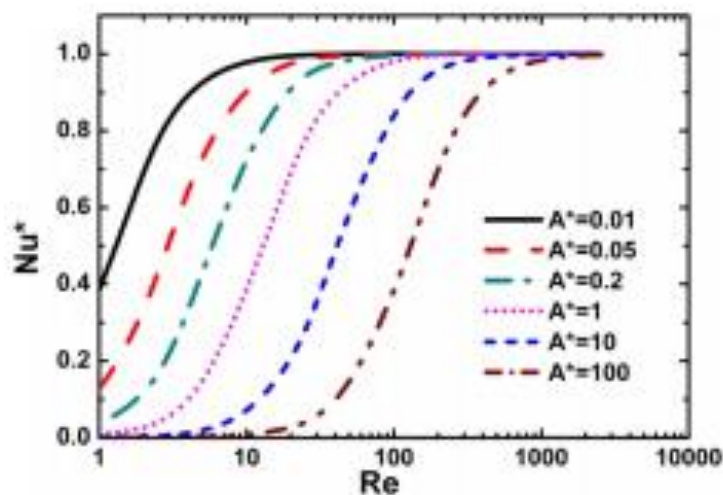


Fig. 9 Axial conduction effects of water flow in silicon channel 4 Parametric Study of Axial Conduction Effect in Microchannels

Etching channels in silicon wafer is a very common method for making microchannels. For these types of channels the diameter can be much smaller than conventional commercial tubes, and the wall thickness is usually comparable to channel diameters. Figure 9 shows the  $Nu^*$  calculated from the present equation as a

function of  $Re$  for water flowing in silicon microchannels. For very low  $Re$  values,  $Nu^*$  values are all significantly less than 1, which means that the axial conduction is not negligible even for  $A^*$  as low as 0.01.  $Nu^*$  increases with increasing  $Re$  and approaches 1 for high  $Re$ . For  $A^* = 0.01$  axial conduction is less than 5% for  $Re$  higher than 6. While for  $A^* = 0.05, 0.2, 1, 10$  and  $100$  the  $Re$  value corresponding to an axial heat conduction effect less than 5% are 13, 26, 57, 180, and 570, respectively.

## V. Conclusions

Effect of axial heat conduction in the fully developed flow is postulated to be a result of an increase in the fluid temperature at any section. The temperature gradient in the wall causes a heat flow in the wall opposite to the flow direction. This effect becomes important, while reducing the experimental data in which the wall temperature is measured, but the local fluid temperature is obtained from the conventional energy balance equation, which does not consider the additional heat transfer to the fluid. A new model is developed to account for this effect. The effect of axial heat conduction in the fluid is shown to be negligible for air and water flow in microchannels for conditions generally encountered in cooling applications.

The results of the model are compared with the numerical simulation results [21,25] and the experimental data [4,26,27] available in the literature. The comparisons show that the model results are in good agreement with the data, and the trends with  $Re, Pr$ , and other geometrical and flow parameters are also correctly predicted.

A parametric analysis using the model shows that the axial conduction effects in the wall are severe for gas flow in any tube material. Axial conduction effects in the wall are negligible for water flow in commercial stainless-steel tubes, while for higher conductivity channel material, or thicker channel walls, the effects of axial heat conduction in the wall are not negligible even for water. It is also noted that it is very difficult to obtain accurate experimental data as the heat losses from the test section, especially the non uniform heat losses along the channel length, make it very difficult to evaluate the heat transferred to the fluid along the length in calculating accurate fluid temperature profile.

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