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Comparative Analysis of Deterministic and Probabilistic Approaches to Selective Harmonic Elimination in Multilevel Inverter

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ABSTRACT
In this paper, both Newton Raphson iterative method and particle swarm optimization (PSO) technique are implemented for the computation of the optimal switching angles in an 11-level inverter. The switching angles are calculated offline to eliminate 5th, 7th, 11th and 13th harmonics that are more harmful and difficult to remove with filter while the fundamental output voltage is obtained as desired. Performances of the two methods are evaluated and compared in terms of speed, fundamental output voltage and total harmonic distortion (THD). Computational results are validated with MATLAB simulations, and both results are in close agreement.

Keywords- *Multilevel inverter, Selective Harmonics Elimination (SHE), Newton Raphson iterative method, PSO, THD*

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I. INTRODUCTION

A multilevel voltage source inverter is a power electronic system that synthesizes a near sinusoidal output voltage from several DC voltages using pulse width modulation technique. The concept of utilizing multiple small voltage steps to perform power conversion originated from the idea of step approximation of sinusoid [1]. The unique structure of multilevel converters enables the construction of relatively high power converters with improved harmonic spectrum using relatively low power semiconductor devices. This has resulted into the ability of the converters to meet stringent power quality and high power demands. There are several advantages to multilevel power conversion approach when compared with the traditional two-level power conversion. The smaller voltage steps yield lower switching losses, improved power quality, lower electro-magnetic interference (EMI), lower voltage change rate (dv/dt), and lower torque ripple [2].

Various pulse width modulation techniques used in conventional two-level inverter have been modified and deployed in multilevel inverters. These include sinusoidal pulse width modulation (SPWM), selective harmonic elimination (SHE) method, space vector control (SVC), and space vector pulse width modulation (SVPWM) [3]. Selective harmonics elimination-pulse width modulation (SHE-PWM) technique at fundamental switching frequency however, arguably gives the best result because of its high spectral performance and considerably reduced switching loss. The objective of SHE-PWM technique is to eliminate low order harmonics that are more harmful and difficult to remove with filter while the fundamental voltage is obtained at a desired value. The main challenge associated with the SHE-PWM technique is that it involves the solution of a specified number of transcendental nonlinear equations known as SHE equation that give the relation between the optimal switching angles and the harmonic components at different modulation indices. The available methods for solving SHE equations can be classified into two groups: The first group is based on deterministic approach using exact algorithms. Newton Raphson iterative method [4] is one of these algorithms. The main disadvantage of iterative methods is that they diverge if the arbitrarily chosen initial values are not sufficiently close to the roots. They also risk being trapped at local optima and fail to give all the possible solution sets. The theory of symmetric polynomials and resultants [5] has been proposed to determine the solutions of the SHE equations. A difficulty with this approach is that as the number of levels increases, the order of the polynomials becomes very high, thereby making the computations of solutions of these polynomial equations very complex. Another approach uses Walsh functions [6], [7], [8] where solving linear equations, instead of non-linear transcendental equations, optimizes the switching angle. The method results in a set of algebraic matrix equations and the calculation of the optimal switching angles is a complex and time-consuming operation.

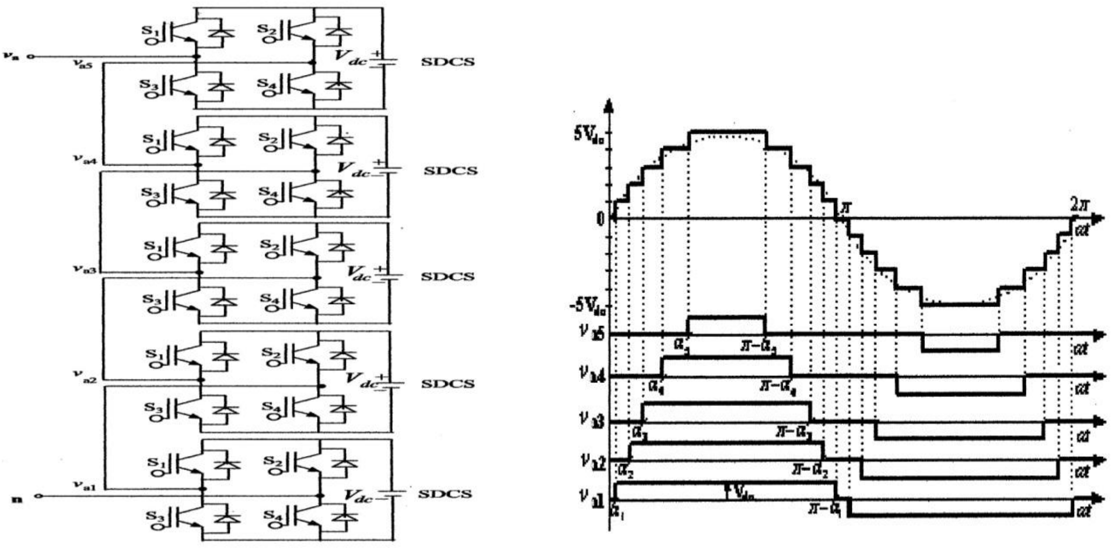
The second group is based on probabilistic approach using heuristics that minimize rather than eliminate the selected harmonics. Population-based evolutionary algorithms (EAs) such as genetic algorithm [9], particle swarm optimization [10], ant colony system [11] and bee algorithm [12] have been reported for computing the switching angles that eliminate 5^{th} and 7^{th} harmonics in 7-level inverter. The main benefits of EAs are improved convergence and the ability to find multiple solution sets over a wide range of modulation indices. These can be attributed to the parallel nature of EAs i.e. a search through a population of solutions rather than a sequential search for individual solutions, as in iterative method. EAs are derivative free and are successful in locating the optimal solution, but they are usually slow in convergence and require much computing time.

II. MULTILEVEL INVERTER

A. Multilevel Inverter Topologies

Basically, there are three main multilevel topologies. These are Diode-Clamped Multilevel Inverter [13], Capacitor-Clamped Multilevel Inverter [14], and Cascaded H-bridge Multilevel Inverter with separate DC sources [15]. Among the topologies, cascaded H-bridge inverter requires the least number of components, and its modular structure as well as circuit layout flexibility makes it suitable for high voltage and high power applications.

Cascaded H-bridge multilevel inverter is formed by connecting several single-phase H-bridge inverters in series as shown in Fig.1 for an 11-level inverter. The number of output voltage levels in a cascaded H-Bridge inverter is given by N =2S +1, where S is the number of H-bridges per phase connected in cascade. By different combinations of the four switches S_1, S_2, S_3, and S_4 shown in the Fig.1, each H-bridge switch can generate a square wave voltage waveform with different duty cycle on the AC side. To obtain $+V_{dc}$, switches S_1 and S_4 are turned on, whereas $-V_{dc}$ can be obtained by turning on switches S_2 and S_3. By turning on S_1 and S_2, or S_3 and S_4, the output voltage is zero.



(a) (b)

Figure1: (a) Single-phase structure of an 11-level cascaded multilevel inverter (b) Output voltage waveform of the 11-level inverter.

The outputs of H-bridge switches are connected in series such that the synthesized AC voltage waveform is the summation of all voltages from the cascaded H-bridge cells [4], [5].

B. Mathematical Model of SHE-PWM

The common characteristic of SHE-PWM method is that the analysis of the desired waveform is performed using Fourier theory. Generally, any periodic waveform such as the staircase waveform shown in Figure 2 can be shown to be the superposition of a fundamental signal and a set of harmonic components. By applying Fourier transformation, these components can be extracted since the frequency of each harmonic component is an integral multiple of its fundamental [16].

With the equal amplitude of all DC sources and quarter wave symmetry, the Fourier series expansion of the staircase output voltage waveform shown in Figure 1 (b) is given by equation (1).

$$v(\omega t)=\sum_{n=1,3,5...}^{\infty}\frac{4V_{dc}}{n\pi}(\cos(n\alpha_1)+\cos(n\alpha_2)+...+\cos(n\alpha_S))\sin n\omega t) \quad (1)$$

Subject to $0<\alpha_1<\alpha_2<...\alpha_S\leq \pi/2$

Where *S* is the number of switching angles and *n* is the harmonic order. For 7-level, 11-level and 15-level inverters, the numbers of SDCSs required are three, five, and seven, respectively. In three-phase power system, the triplen harmonics in each phase need not be cancelled as they automatically cancel in the line-to-line voltages as a result only non-triplen odd harmonics are present in the line-to-line voltages [5]. Generally, for *S* number of switching angles, one switching angle is used for the desired fundamental output voltage V_1 and the remaining (*S*-1) switching angles are used to eliminate certain low order harmonics that dominate the total harmonic distortion (THD) such that equation (1) becomes

$$V(\omega t)=V_1\sin(\omega t) \quad (2)$$

From equation (1), the expression for the fundamental output voltage V_1 in terms of the switching angles is given by

$$V_1=\frac{4V_{dc}}{\pi}\left(\cos(\alpha_1)+\cos(\alpha_2)+...+\cos(\alpha_s)\right) \quad (3)$$

The relation between the fundamental voltage and the maximum obtainable fundamental voltage V_{1max} is given by modulation index. The modulation index, m_i, is defined as the ratio of the fundamental output voltage V_1 to the maximum obtainable fundamental voltage V_{1max}. The maximum fundamental voltage is obtained when all the switching angles are zero [4]. From equation (3),

$$V_{1\max}=\frac{4sV_{dc}}{\pi} \qquad \therefore m_i=\frac{V_1}{V_{1\max}}=\frac{\pi V_1}{4sV_{dc}} \Rightarrow V_1=m_i\left(\frac{4sV_{dc}}{\pi}\right) \quad \text{for } 0<m_i\leq 1 \quad (4)$$

To develop an 11-level cascaded multilevel inverter, five SDCSs are required. The appropriate modulation index and switching angles that yield the required fundamental harmonic coupled with the elimination of 5th, 7th, 11th, and 13th harmonics can be found by solving the following SHE equations that characterize the selected harmonics:

$$\begin{aligned}
&\cos(\alpha_1)+\cos(\alpha_2)+\cos(\alpha_3)+\cos(\alpha_4)+\cos(\alpha_5)=5m_i\\
&\cos(5\alpha_1)+\cos(5\alpha_2)+\cos(5\alpha_3)+\cos(5\alpha_4)+\cos(5\alpha_5)=0\\
&\cos(7\alpha_1)+\cos(7\alpha_2)+\cos(7\alpha_3)+\cos(7\alpha_4)+\cos(7\alpha_5)=0\\
&\cos(11\alpha_1)+\cos(11\alpha_2)+\cos(11\alpha_3)+\cos(11\alpha_4)+\cos(11\alpha_5)=0\\
&\cos(13\alpha_1)+\cos(13\alpha_2)+\cos(13\alpha_3)+\cos(13\alpha_4)+\cos(13\alpha_5)=0
\end{aligned} \quad (5)$$

Generally equation (5) can be written as

$$F(\alpha)=B(m_i) \quad (6)$$

The Total Harmonic Distortion (THD) is computed as shown in equation (7):

$$THD=\sqrt{\sum_{i=5,7,11,13,...}^{49}\left(\frac{V_i}{V_1}\right)^2} \quad (7)$$

III. OPTIMIZATION TECHNIQUES

A. Newton Raphson Iterative Method

Newton-Raphson (NR) method is one of the fastest iterative methods. This method begins with an arbitrary initial approximation and generally converges at a zero of a given system of nonlinear equations. In Newton Raphson method, the following matrices are created from equation (6)

1. The switching angle matrix

$$\alpha = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \\ \alpha_5 \end{bmatrix} \tag{9}$$

2. The nonlinear system matrix and the Jacobian matrix

$$F = \begin{bmatrix} \cos(\alpha_1) & \cos(\alpha_2) & \cos(\alpha_3) & \cos(\alpha_4) & \cos(\alpha_5) \\ \cos(5\alpha_1) & \cos(5\alpha_2) & \cos(5\alpha_3) & \cos(5\alpha_4) & \cos(5\alpha_5) \\ \cos(7\alpha_1) & \cos(7\alpha_2) & \cos(7\alpha_3) & \cos(7\alpha_4) & \cos(7\alpha_5) \\ \cos(11\alpha_1) & \cos(11\alpha_2) & \cos(11\alpha_3) & \cos(11\alpha_4) & \cos(11\alpha_5) \\ \cos(13\alpha_1) & \cos(13\alpha_2) & \cos(13\alpha_3) & \cos(13\alpha_4) & \cos(13\alpha_5) \end{bmatrix} \tag{10}$$

$$J = dF = \begin{bmatrix} -\sin(\alpha_1) & -\sin(\alpha_2) & -\sin(\alpha_3) & -\sin(\alpha_4) & -\sin(\alpha_5) \\ -5\sin(5\alpha_1) & -5\sin(5\alpha_2) & -5\sin(5\alpha_3) & -5\sin(5\alpha_4) & -5\sin(5\alpha_5) \\ -7\sin(7\alpha_1) & -7\sin(7\alpha_2) & -7\sin(7\alpha_3) & -7\sin(7\alpha_4) & -7\sin(7\alpha_5) \\ -11\sin(11\alpha_1) & -11\sin(11\alpha_2) & -11\sin(11\alpha_3) & -11\sin(11\alpha_4) & -11\sin(11\alpha_5) \\ -13\sin(13\alpha_1) & -13\sin(13\alpha_2) & -13\sin(13\alpha_3) & -13\sin(13\alpha_4) & -13\sin(13\alpha_5) \end{bmatrix} \tag{11}$$

3. The solution matrix

$$B(m_i) = \begin{bmatrix} 5m_i \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{12}$$

For each iteration loop,

$$\alpha_{new} = \alpha_{old} + \Delta(\alpha) \tag{13}$$

Where $\Delta(\alpha) = J^{-1}(\alpha)\left[B(m_i) - F(\alpha)\right]$ (14)

The steps that are involved in the implementation of NR method are as follows:

1. Randomly generate initial values of switching angles without any constraint violation
2. Set $m_i = 0$
3. Calculate $F(\alpha_0), B(m_i),$ and Jacobian $J(\alpha_0)$
4. Compute the correction factor, $\Delta\alpha = J^{-1}(\alpha_0)\left[B(m_i) - F(\alpha_0)\right]$
5. Update the switching angles such that $\alpha(k+1) = \alpha(k) + \Delta\alpha(k)$
6. Transform the switching angles $\alpha(k+1) = \cos^{-1}\left[abs[(\cos(\alpha(k+1))]\right]$
7. Repeat steps (3) to (6) until solutions are found or iteration limit is reached

8. Increment modulation index by a fixed step
9. Repeat steps (2) to (8) until any of the stopping criteria is met
10. Plot solution sets and THD of each set as a function of modulation index.

B. Particle Swarm Optimization (PSO)

Particle swarm optimization is a swarm intelligence based algorithm that was inspired by the social behavior in a flock of birds or a school of fish. In PSO, an initial population of potential solutions to the optimization problem called particles is randomly generated. Each particle in a swarm searches for the best position in the search space, while the social behavior that is modeled in PSO guide the swarm to the optimal region. Each particle in the search space is assigned a randomized position and velocity. During successive iteration, the current position of each particle in the swarm is evaluated with an objective function, and each particle keeps track of its coordinates in the problem space, which are associated with the best solution (fitness) that it has achieved so far.

Based on the fitness evaluation of all the particles, the best position so far of the i^{th}particle in a d-dimensional space is called personal best (*Pbest*), and is denoted by $P_i = [p_{i1}, p_{i2}, ..., p_{id}]$, while the overall best position obtained so far by any particle in the swarm is called global best (*Gbest*), and is denoted by $P_g = [p_{g1}, p_{g2}, ..., p_{gd}]$. This implies that each particle has a memory, which enables it to update its current position and velocity according to the distance between its current position and *Pbest*, as well as the distance between its current position and *Gbest*. If the velocity and position vectors of the i^{th} particle at iteration k are represented as $V_i = [v_{i1}, v_{i2}, ..., v_{id}]$and $X_i = [x_{i1}, x_{i2}, ..., x_{id}]$, respectively, then the velocity and position of the particle in the next iteration are determined as follows:

$$v_i(k+1) = wv_i(k) + c_1 r_1 [p_i(k) - x_i(k)] + c_2 r_2 [p_g(k) - x_i(k)] \quad (15)$$

$$x_i(k+1) = x_i(k) + v_i(k+1) \quad (16)$$

Where w is the inertia weight parameter that provides the balance between global exploration and local exploitation capabilities of the particle, $v_i(k)$ is the velocity of the particle at iteration k; $x_i(k)$ is the position theparticle at iteration k; c_1 and c_2 are constants known as cognitive and social coefficients, respectively; r_1 and r_2 are random values uniformly distributed within [0, 1] [17].

The steps that are involved in the implementation of PSO algorithm are as follows:

1. Randomly generate an initial population of particles subject to eqn. (11).
2. Set $m_i = 0$
3. Perform the fitness evaluation of the particles
4. Update the personal best position Pbest and global best position Gbest.
5. Evaluate the velocity of each particle
6. Compute new position of each particle using the updated velocity.
7. Repeat steps (3) to (6) until solutions are found or generation limit is reached
8. Increment modulation index by a fixed step
9. Repeat steps (2) to (7) until any of the stopping criteria is met
10. Plot solution sets and THD of each set as a function of modulation index.

IV. IMPLEMENTATION

The solution set at each step of the PSO algorithm is evaluated with the fitness function. The objective here is to determine the switching angles such that the selected low order harmonics are either eliminated or minimized to an acceptable level while the fundamental voltage is obtained at a desired value. For each solution set, the fitness function is calculated as follows [12]:

$$f = \min_{\alpha_i}\left[\left(100\frac{V_1^* - V_1}{V_1^*}\right)^4 + \sum_{s=2}^{5}\frac{1}{h_s}\left(50\frac{V_{hs}}{V_1}\right)^2\right]$$

Where i= 1, 2 …5 subject to $0 \le \alpha_1 < \alpha_2 ... < \alpha_5 \le \pi/2$ (17)

Where V_1^* is the desired fundamental output voltage, S is the number of switching angles, h_s is the order of the s^{th} viable harmonic at the output of a three phase multilevel converter. For example, $h_2 = 5$, $h_3 = 7$. It should be noted that different weight are assigned to different harmonics. The fitness function assigns Eq. (17) assigns higher importance to the low order harmonics, which are more harmful and difficulty to remove with filter.

Using MATLAB software, the both NR and PSO algorithms were implemented to eliminate 5th, 7th, 11th, and 13th harmonics that constitute the low order harmonics in an 11-level inverter. The choice of PSO algorithm's parameters is a tradeoff between accuracy and convergent speed. Due to the inexact nature of PSO algorithm, its parameters were chosen on trial and error basis. Based on the performance evaluation after several trials, the population size in this work is 40, and the values of both cognitive and social coefficients are chosen to be 1.5. Sometimes, solutions are found before 100 iterations are completed. In order to improve the convergent speed, iterations are stopped if the result remains unchanged for 50 iterations. The solutions were computed for both NR and PSO by incrementing the modulation index, m_i in steps of 0.001 from 0 to 1. A personal computer (2.66 GHz Intel Core i7 processor with 4GB Random Access Memory) running MATLAB R2014b on OS X Yosemite version 10.10was used to carry out the computations.

The observed analytical results were validated with modelling and simulation of an 11-level single-phase Cascaded H-Bridge inverter using SimPower System block set in MATLAB-SIMULINK. In each of the five H-Bridges in the 11-level single-phase Cascaded H-Bridge inverter, 12V dc source is the SDCS, and the switching device used is Insulated Gate Bipolar Transistor (IGBT). Simulations were performed at the fundamental frequency of 50 Hz using the solution sets of both NR and PSO at the same arbitrarily chosen modulation index, m_i of 0.63. The solution sets at the chosen modulation index are $[22.1086^\circ\ 38.9973^\circ\ 52.6843^\circ\ 59.1740^\circ\ 70.8701^\circ]$ and $[9.6351^\circ\ 33.6343^\circ\ 43.0780^\circ\ 60.9859^\circ\ 83.2735^\circ]$ for NR and PSO, respectively. In order to demonstrate the accuracy and global search capability of PSO, simulation was performed with solution set found with PSO at modulation index, m_i of 0.915: $\alpha_1 = 5.8656^\circ$, $\alpha_2 = 6.9384^\circ$, $\alpha_3 = 20.2539^\circ$, $\alpha_4 = 25.6308^\circ$, and $\alpha_5 = 41.6664^\circ$. This is an infeasible region for NR to find solution. Fourier Transform analyses of the simulated phase voltage waveforms were done using the FFT block to show the harmonic spectra of the synthesized AC voltages.

V. RESULTS

NR with average execution time of 3.24s is much faster than faster than PSO with the average execution time of 453.02s. The operating range of modulation index of NR is [0.441 0.846] compared with PSO with operating range of [0.261 0.929]. Shown in Figure 2 are the plots of switching angles and harmonic profiles against modulation index for NR method.

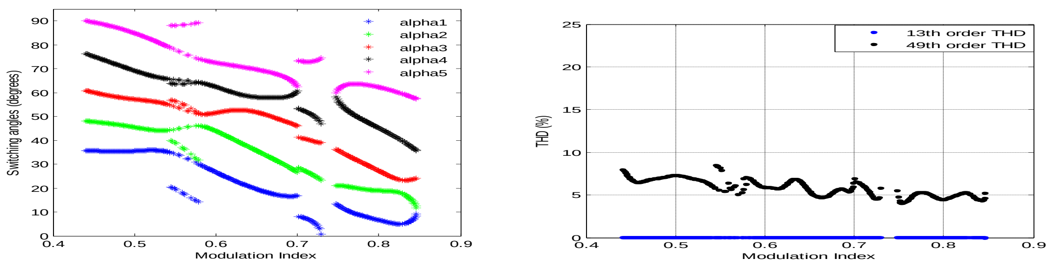


Figure 2: Plots of (a) Switching angles (b) THD versus modulation index for 11-level CMLI using NR method

Variation of the fitness function with modulation index over the range of 0.1 to 1.0 is shown in Figure 3 (a) for PSO. Solution sets with fitness value greater than 10^{-2} are rejected. When the fitness function at a modulation index is 10^{-2} or less, the corresponding switching angles are considered as a solution set.

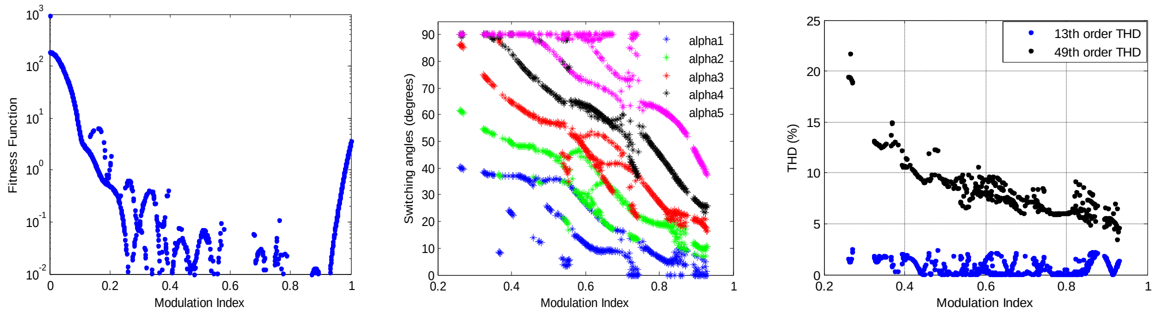
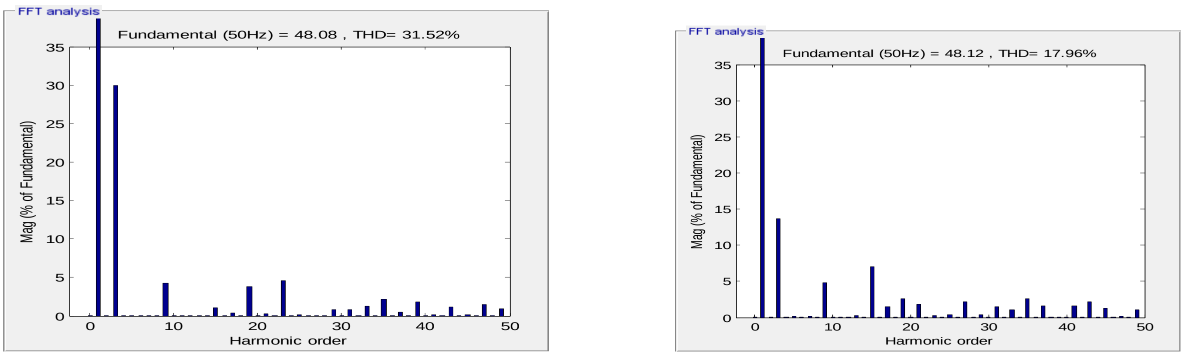


Figure 3: Plots of (a) Fitness function (b) Switching angles (c) THD versus modulation index for 11-level CMLI using PSO

Shown in Figures 2 (a) and 3 (b) are the plots of switching angles that minimize 5^{th}, 7^{th}, 11^{th}, and 13^{th} harmonics in an 11-level inverter for NR and PSO, respectively. It can be seen from figures that there are multiple solution sets at some modulation indices. In the case of multiple solution sets, the set with the least THD is chosen. As can be observed from the THD curves of the solution sets plotted in Figure 3 (c), values of the 49^{th} order THD are higher at lower modulation indices while they are considerably reduced at the upper end of modulation index. The plot of 13^{th} order THD shows how efficiently the selected harmonics are minimized. For all values of modulation index that solutions are found with NR method, 13^{th} order THD is zero as shown in Figure 2 (b). Figure 3 (c) shows that 13^{th} order THD is not zero for all values of modulation index that solution are found with PSO indicating that low order harmonics are minimized rather than eliminated in some intervals.

The analytically computed peak value of the fundamental output voltage given by eqn. (4) is $V_1 = m_i\left(\frac{4sV_{dc}}{\pi}\right) = 0.63\left(\frac{4\times5\times12}{\pi}\right) = 48.13V_{(peak)}$. This value closely agrees with the simulation values of 48.08V and 48.12V shown in Figure 4 (a) and (b) for NR and PSO, respectively. The harmonic spectra of the synthesized voltage waveforms shown in Figure 4 reveal the complete elimination of the $5^{th}, 7^{th}, 11^{th}$ *and* 13^{th} harmonics as their values tend towards zero.

The THD values in line-to-line voltage as computed analytically with eqn. (7) are 6.79% and 9.47% for NR and PSO, respectively. The simulation values of THD are 6.81% and 5.47% NR and PSO, respectively. It should be noted that the simulation values of THD for NR and PSO are shown in Figures 4 (a) and (b) as 31.52% and 17.96%, respectively. These values are for the phase voltages, which include triplen harmonic components while analytical values are for line voltages which exclude the triplen harmonics.



\ (a) (b)

Figure 4: Harmonic spectrum of (a) NR and (b) PSO solution set at the same modulation index, $m_i = 0.63$

As shown in Figure 5, the simulation value of the fundamental output voltage corresponds exactly with the analytically computed value of 69.9V. The harmonic spectrum of the synthesized voltage waveform shown in Figure 5 reveals the complete elimination of the selected harmonics as their values tend towards zero. Simulation THD value of the line voltage is 4.74% compared to the phase THD value of 16.44% shown in Figure 5.

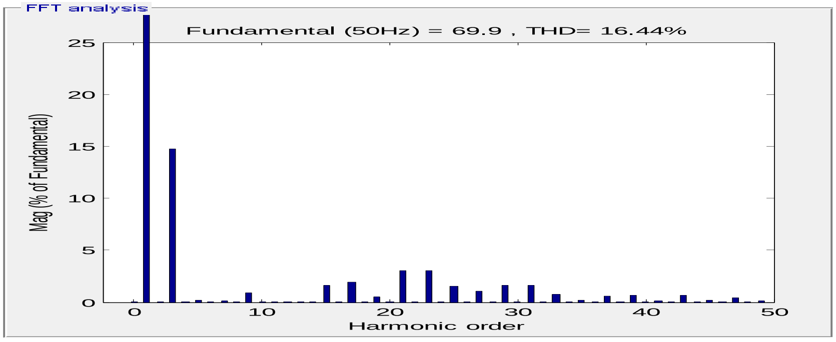


Figure 5: Harmonic spectrum of PSO solution set at modulation index, $m_i = 0.915$

VI. CONCLUSION

Both derivative dependent method (NR) and derivative free algorithm (PSO) with random initial values have been successfully implemented for solving the transcendental nonlinear equations characterizing the selected harmonics in an 11-level inverter. The results of NR method closely agree with the related works [4], [5] reported in literature. It has been shown that NR is vey fast and accurate. Selected harmonics are completely eliminated at all values of modulation index that solution sets are found. PSO has been observed to be accurate with global search capability. Solution sets with well-attenuated low order harmonics have been found with PSO in the regions that are infeasible for analytical methods to find solutions. Analytical results are validated with simulation results and both are in close agreement. The absence of the selected harmonics in the synthesized output phase voltages validates the analytically computed results.

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