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Heat Transfer on Unsteady MHD Oscillatory Visco-elastic flow through a Porous medium in a Rotating parallel plate channel

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ABSTRACT

The combined effect of a transverse magnetic field and radiative heat transfer on unsteady flow of a conducting optically thin visco-elastic fluid through a rotating parallel channel filled with saturated porous medium and non-uniform walls temperature has been discussed. It is assumed that the fluid has small electrical conductivity and the electromagnetic force produced is very small. The analytical solutions are obtained for the problem making use of perturbation technique. The effects of the radiation and the magnetic field parameters on velocity profile and shear stress for different values of the visco-elastic parameter with the combination of the other flow parameters are illustrated graphically, and physical aspects of the problem are discussed

Keywords: *Radiation, heat transfer, visco-elastic fluid, Magnetic field, porous medium, rotating parallel plate channel.*

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I. INTRODUCTION

In recent years, there has been a considerable interest in rotating hydro magnetic fluid flows due to possible applications to geophysical and astrophysical problems. The magnitude analysis shows that in the basic field equations, the Coriolis force is very significant as compared to the inertial force. Furthermore, it reveals that the Coriolis and magneto hydro dynamic forces are of comparable magnitude. It is generally admitted that a number of astronomical bodies (e.g. the Sun. Earth. Jupiter, Magnetic Stars, pulsars) possess fluid interiors and (at least surface) magnetic fields. Changes in the rotation rate of such objects suggest the possible importance of hydro magnetic spin-up. The flow of an electrically conducting fluid has important applications in many branches of engineering science such as magneto hydro dynamics (MHD) generators, plasma studies, nuclear reactor, geothermal energy extraction, electromagnetic propulsion, and the boundary layer control in the field of aerodynamics. In the light of these applications, MHD flow in a channel has been studied by many authors; some of them are Nigam and Singh [1], Soundalgekar and Bhat [2], Vajravelu [3], and Attia and Kotb [4]. A survey of MHD studies in the technological fields can be found in Moreau [5]. The flow of fluids through porous media is an important topic because of the recovery of crude oil from the pores of the reservoir rocks; in this case, Darcy’s law represents the gross effect. Raptis et al. [6] have analysed the hydro magnetic free convection flow through a porous medium between two parallel plates. Aldoss et al. [7] have studied mixed convection flow from a vertical plate embedded in a porous medium in the presence of a magnetic field. Makinde and Mhone [8] have considered heat transfer to MHD oscillatory flow in a channel filled with porous medium. Recently the combined effect of a transverse magnetic field and radiative heat transfer on unsteady flow of a conducting optically thin visco-elastic fluid through a channel filled with saturated porous medium and non-uniform walls temperature has been discussed by Rita Choudary and Utpal Jyothi Das [12]. In this study, an attempt has been made to extend the problem studied by Makinde and Mhone [8] to the case of visco-elastic fluid characterised by second-order fluid. The combined effect of a transverse magnetic field and radiative heat transfer on unsteady flow of a conducting optically thin visco-elastic fluid through a rotating parallel channel filled with saturated porous medium and non-uniform walls temperature has been discussed. It is assumed that the fluid has small electrical conductivity and the electromagnetic force produced is very small. The analytical solutions are obtained for the problem making use of perturbation technique.

II. MATHEMATICAL FORMULATION AND SOLUTION OF THE PROBLEM

Consider the flow of a conducting optically thin fluid in a channel filled with saturated porous medium under the influence of an externally applied homogeneous magnetic field and radiative heat transfer. It is assumed that the fluid has small electrical conductivity and the electromagnetic force produced is very small. The *x*-axis is taken along the centre of the channel, and the *z*-axis is taken normal to it. In the initial undisturbed state both the plates and the fluid rotate with the same angular velocity Ω . At $t > 0$, the fluid is driven by a constant pressure gradient parallel to the channel walls. Then, assuming a Boussinesq incompressible fluid model, the equations governing the motion are given by

$$\frac{\partial u}{\partial t} - 2\Omega\, v = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu_1 \frac{\partial^2 u}{\partial z^2} + \nu_2 \frac{\partial^3 u}{\partial z^2 \partial t} - \frac{\sigma_e B_0^2 u}{\rho} - \nu_1 \frac{u}{K} + g\beta (T - T_0) \qquad (1)$$

$$\frac{\partial v}{\partial t} + 2\Omega\, u = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu_1 \frac{\partial^2 v}{\partial z^2} + \nu_2 \frac{\partial^3 v}{\partial z^2 \partial t} - \frac{\sigma_e B_0^2 v}{\rho} - \nu_1 \frac{v}{K} \qquad (2)$$

$$\frac{\partial T}{\partial t} = \frac{k}{\rho C_p}\frac{\partial^2 T}{\partial z^2} - \frac{1}{\rho C_p}\frac{\partial q_1}{\partial z} \qquad (3)$$

Corresponding boundary conditions

$$u = 0, v = 0, T = T_w \quad \text{on} \quad z = 1 \qquad (4)$$

$$u = 0, v = 0, T = T_0 \quad \text{on} \quad z = 0 \qquad (5)$$

Where *u* is the axial velocity, *t* is the time, *T* is the fluid temperature, *P* is the pressure, *g* is the gravitational force, q_1 is the radiative heat flux, β is the coefficient of volume expansion due to temperature, C_p is the specific heat at constant pressure, *k* is the thermal conductivity,
K is the porous medium permeability co-efficient, $B_0 (= \mu_e H_0)$ is electromagnetic induction, μ_e is the magnetic permeability, H_0 is the intensity of the magnetic field, σ_e is the conductivity of the fluid, ρ is fluid density, and $\nu_i = \mu_i / \rho, (i = 1, 2)$. It is assumed that both walls of temperature T_0, T_w are high enough to induce radiative heat transfer. Following Cogley et.al [11], it is assumed that the fluid is optically thin with a relatively low density and the radiative heat flux is given by

$$\frac{\partial q_1}{\partial z} = 4\alpha_1^2 (T_0 - T), \qquad (6)$$

Where α_1 is the mean radiation absorption co-efficient.

Combining equations (1) and (2) and let $q = u + iv$ and $\xi = x + iy$, we obtain

$$\frac{\partial q}{\partial t} + 2i\Omega\, q = -\frac{1}{\rho}\frac{\partial p}{\partial \xi} + \nu_1 \frac{\partial^2 q}{\partial z^2} + \nu_2 \frac{\partial^3 q}{\partial z^2 \partial t} - \frac{\sigma_e B_0^2 q}{\rho} - \nu_1 \frac{q}{K} + g\beta (T - T_0) \qquad (7)$$

The following non-dimensional quantities are introduced:

$$x^* = \frac{x}{a},\; y^* = \frac{y}{a},\; \xi^* = \frac{\xi}{a},\; z^* = \frac{z}{a},\; u^* = \frac{u}{U},\; v^* = \frac{v}{U},$$

$$q^* = \frac{q}{U},\quad t^* = \frac{tU}{a},\quad p^* = \frac{ap}{\rho \nu_1 U},\quad \theta = \frac{T - T_0}{T_w - T_0}$$

Making use of non-dimensional variables, the dimensionless governing equations together with appropriate boundary conditions (dropping asterisks) are

$$\mathrm{Re}\,\frac{\partial q}{\partial t} = -\frac{\partial p}{\partial \xi} + \frac{\partial^2 q}{\partial z^2} + \alpha \frac{\partial^3 q}{\partial z^2 \partial t} - (M^2 + 2iE^{-1} + S^2) q + Gr\, T \qquad (8)$$

$$Pe\,\frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial z^2} + R^2 \theta \qquad (9)$$

With

$$q = 0,\quad \theta = 1 \quad \text{on} \quad z = 1 \qquad (10)$$

$$q = 0,\quad \theta = 0 \quad \text{on} \quad z = 0 \qquad (11)$$

where

$Re = \frac{Ua}{\nu_1}$ is the Reynolds number,

$M^2 = \frac{\sigma_e B_0^2 a^2}{\nu_1 U}$ is the Hartmann number, $E = \frac{\nu_1 U}{\Omega a^2}$ is the Eckmann number

$D = \frac{k}{a^2}$ is the Darcy number (or) $S = \frac{1}{D}$ is the porous medium shape factor parameter,

$\alpha = \frac{\nu_2 Re}{a^2}$ is the visco-elastic parameter,

$Gr = \frac{g\beta(T_w - T_0)a^2}{\nu_1 U}$ is the Grashoff number,

$Pe = \frac{Ua\,\rho C_p}{k}$ is Peclet parameter and

$R^2 = \frac{4\alpha_1^2 a^2}{k}$ is the Radiation parameter.

solving the equations (2.8) and (2.9) for purely oscillatory flow, Let

$$-\frac{\partial P}{\partial x} = \lambda e^{i\omega t}, \quad q(z,t) = q_0(z)e^{i\omega t}, \quad \theta(z,t) = \theta_0(z)e^{i\omega t} \tag{12}$$

where, λ is constant and ω is the frequency of oscillation.

Substituting the above expressions (12) into the equations (8) and (9), and making use of the corresponding boundary conditions (10) and (11), we obtain

$$(1 + i\alpha\omega)\frac{d^2 q_0}{\partial z^2} - m_1^2 q_0 = -\lambda - Gr\,\theta_0 \tag{13}$$

$$\frac{d^2\theta_0}{\partial z^2} + m_2^2\theta_0 = 0 \tag{14}$$

subjected to the boundary conditions

$$q_0 = 0, \quad \theta_0 = 1 \qquad \text{on} \qquad z = 1 \tag{15}$$

$$q_0 = 0, \quad \theta_0 = 0 \qquad \text{on} \qquad z = 0 \tag{16}$$

where $m_1 = \sqrt{M^2 + 2iE^{-1} + S^2 + i\omega Re}$ and $m_2 = \sqrt{R^2 - i\omega Pe}$

Equations (13) and (14) are solved; we obtained the solution for the fluid velocity and temperature as follows

$$q(z,t) = \left\{ a_1 e^{(m_1 z)/b} + \left(a_1 - \frac{\lambda}{m_1^2}\right)e^{-(m_1 z)/b} + \frac{\lambda}{m_1^2} + \frac{Gr\sin(m_2 z)}{(m_2^2 b + m_1^2)\sin(m_2)} \right\} e^{i\omega t} \tag{17}$$

$$\theta(z,t) = \frac{\sin(m_2 z)}{\sin(m_2)} e^{i\omega t} \tag{18}$$

where $a_1 = \frac{\left(\left(\frac{\lambda}{m_1^2}\right)e^{-m_1/b} - \left(\frac{\lambda}{m_1^2}\right) - \frac{Gr}{(m_2^2 b + m_1^2)}\right)}{\left(e^{m_1/b} - e^{-m_1/b}\right)}$, $b = 1 + i\alpha\omega$

The non-dimensional shear stress σ at the wall $z = 0$ is given by

$$\sigma = \frac{\sigma^*}{(\mu_1 U / a)} = \left(\frac{\partial u}{\partial z} + \alpha \frac{\partial^2 u}{\partial z \partial t} \right)_{z=0}$$

$$= \left[\frac{a_1 m_1}{b} - \left(a_1 - \frac{\lambda}{m_1^2} \right) + \frac{Gr}{(m_2^2 b + m_1^2) \sin(m_2)} \right] (1 + i\omega\alpha) \, e^{i\omega t} \quad (19)$$

The rate of heat transfer across the channel wall $z = 1$ is given as

$$Nu = -\left(\frac{\partial \theta}{\partial z} \right)_{z=1} = \frac{m_2 \cos(m_2)}{\sin(m_2)} e^{i\omega t} \quad (20)$$

III. RESULTS AND DISCUSSION:

The flow governed by the non dimensional parameters, Re is the Reynolds number, M^2 is the Hartmann number, E Eckmann number, D is the Darcy number (or) S is the porous medium shape factor parameter, α is the visco-elastic parameter, R is the Radiation parameter with fixed values of Gr the Grashoff number, Pe Peclet parameter. We have considered the real and imaginary parts of the results *u* and *v* throughout for numerical validation. The velocity profiles for the components against *z* is plotted in Figures 1–12 while figure 13-18 to observe temperature profiles on the visco-elastic effects and other parameters for various sets of values of Hartmann number *H*, porous parameter *S* and radiation parameter *R*, with fixed values of other flow parameters, namely, Pe = 2, t = 0.1, Gr = 2, λ = 1, and ω = 1.

It is evident from Figures (1–12) that the velocity profiles is parabolic in nature, and the magnitude of velocity *u* and *v* increase with the increasing values of the Reynolds number Re, Porous parameter *S*, the visco-elastic parameter $|\alpha|$, Radiation parameter R and (Figure 1, 2, 5-12). It is also noted from the figures (3-4) that the magnitude of the velocity component *u* experiences retardation and the behaviours of the velocity component *v* remains the same with the increasing values of the Hartmann number. We observe that lower the permeability of the porous medium lesser the fluid speed in the entire fluid region. The resultant velocity *q* enhances with increasing the parameters Re, D, α , R and experiences retardation with increasing the intensity of the magnetic field. It is evident that the temperature profiles exhibit the nature of the flow on governing parameters. The magnitude of the temperature increases with increasing Re, D, α , R, E and experiences retardation with increasing the magnetic field parameter (Hartmann number *M*) (Figures 13-18).

The shear stress on the wall and the rate of heat transfer evaluated analytically and computationally discussed with reference to governing parameters (Tables 1-2). It is evident that the shear stress enhances with increasing Re, *M* and α and reduces with increasing S. We also noted that it is increases firstly and then decreases with the increasing values of radiation parameter R. Table. 2 depict that the Nusselt number (*Nu*) increases with the increasing values of the visco-elastic parameter $|\alpha|$ and the magnitude of the heat transfer increases with increasing Re, S, α , R, E retardation with increasing the magnetic field parameter. It has also been observed that the temperature field is not significantly affected by the visco-elastic parameter.

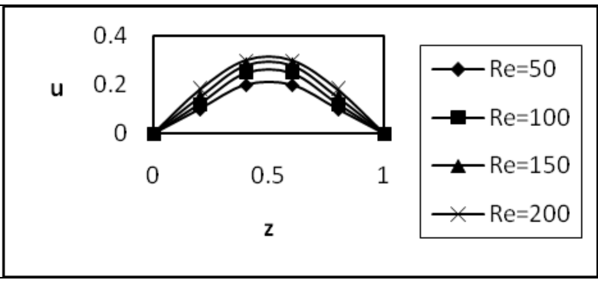


Fig. 1 The velocity profile for *u* on Reynolds number Re with $M = 2, \; S = 1, \; \alpha = -0.1, E = 0.01, R = 1.5$

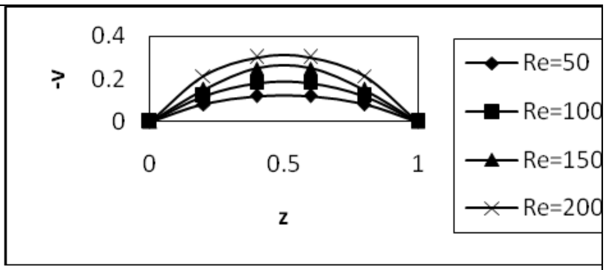


Fig. 2: The velocity profile for *v* on Reynolds number Re with $M = 2, \; S = 1, \; \alpha = -0.1, E = 0.01, R = 1.5$

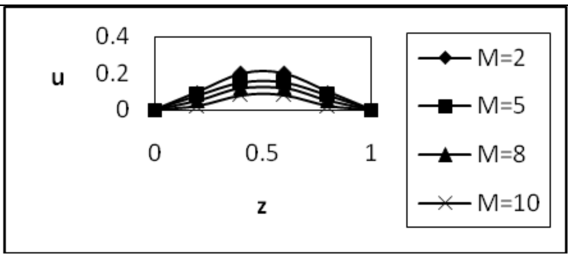


Fig. 3: The velocity profile for *u* on Hartmann number M with $\mathrm{Re} = 50,\ S = 1,\ \alpha = -0.1, E = 0.01, R = 1.5$

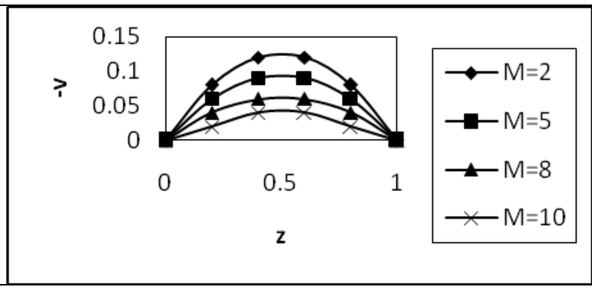


Fig. 4: The velocity profile for *v* on Hartmann number M with $\mathrm{Re} = 50,\ S = 1,\ \alpha = -0.1, E = 0.01,\ R = 1.5$

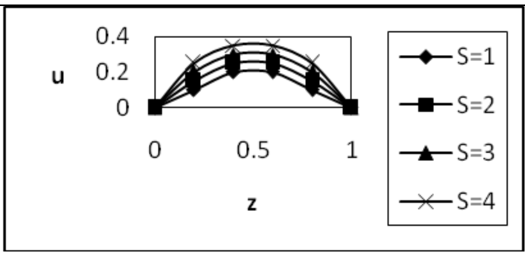


Fig. 5: The velocity profile for *u* on Porous parameter S with $\mathrm{Re} = 50,\ M = 2,\ \alpha = -0.1, E = 0.01, R = 1.5$

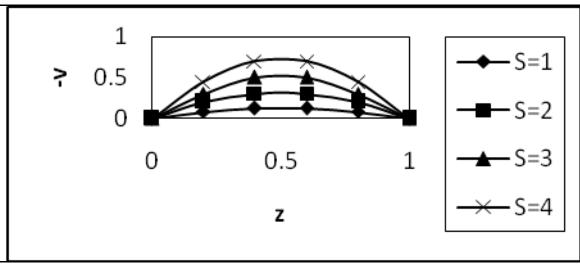


Fig. 6: The velocity profile for *v* on Porous parameter S with $\mathrm{Re} = 50,\ M = 2,\ \alpha = -0.1, E = 0.01, R = 1.5$

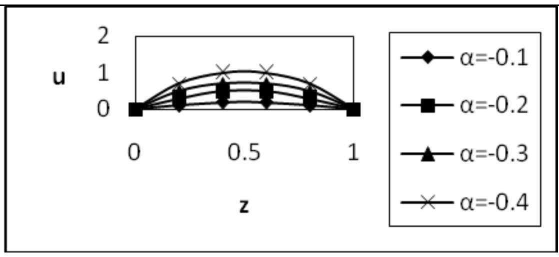


Fig. 7: The velocity profile for *u* on visco-elastic parameter α with $\mathrm{Re} = 50,\ M = 2,\ S = 1,\ E = 0.01, R = 1.5$

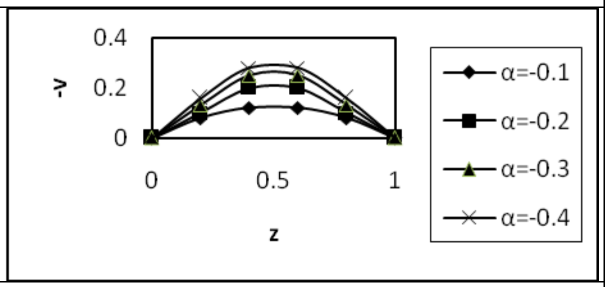


Fig. 8: The velocity profile for *v* on visco-elastic parameter α with $\mathrm{Re} = 50,\ M = 2,\ S = 1, E = 0.01, R = 1.5$

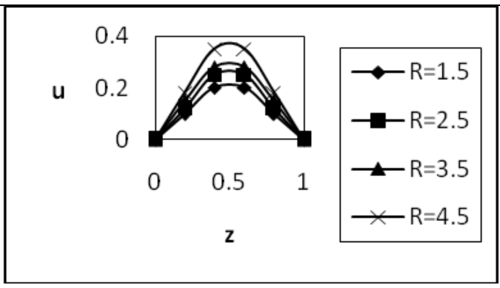


Fig. 09: The velocity profile for *u* on Radiation parameter R with $\mathrm{Re} = 50,\ M = 2,\ S = 1, E = 0.01, \alpha = -0.1$

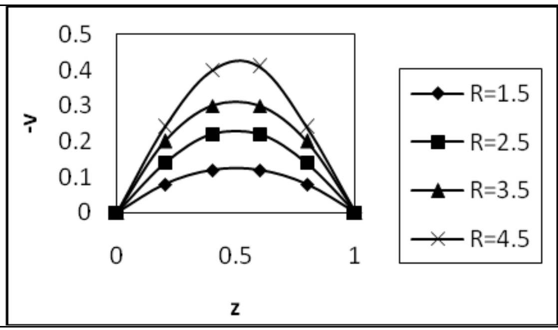


Fig. 10: The velocity profile for *v* on Radiation parameter R with $\mathrm{Re} = 50,\ M = 2,\ S = 1,\ \alpha = -0.1, E = 0.01$

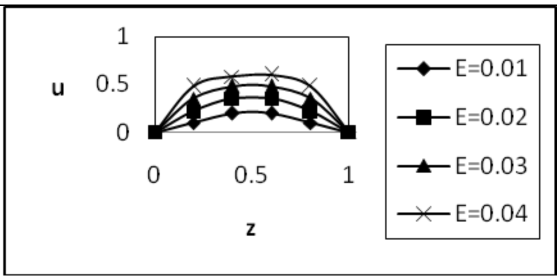


Fig. 11: The velocity profile for *u* on Eckmann number *E* with $\mathrm{Re}=50,\ M=2,\ S=1,\ E=0.01,R=1.5,\ \alpha=-$

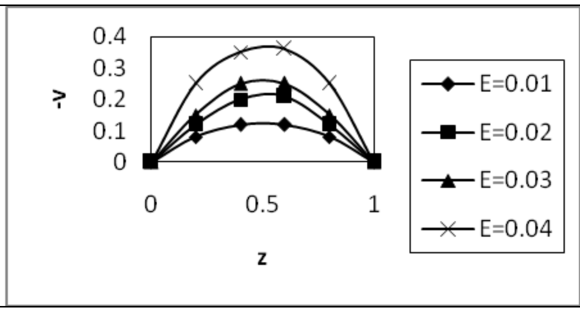


Fig. 12: The velocity profile for *v* on Eckmann number *E* with $\mathrm{Re}=50,\ M=2,\ S=1,E=0.01,R=1.5,\ \alpha=-$

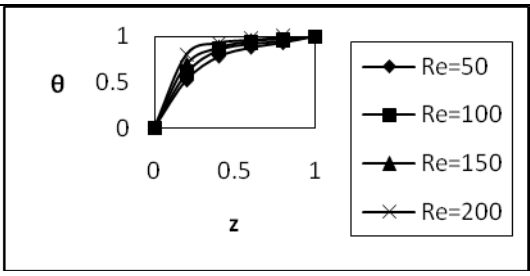


Fig. 13: The temperature profile for θ on Reynolds number Re with $M=2,\ S=1,\ \alpha=-0.1,E=0.01,R=1.5$

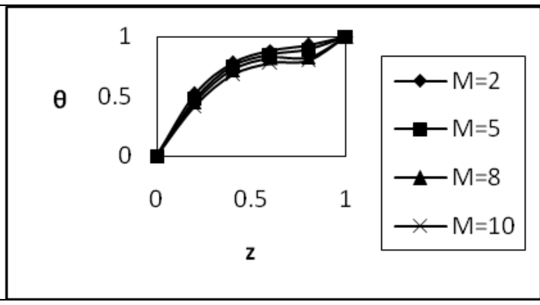


Fig. 14: The temperature profile for θ on Hartmann number M with $\mathrm{Re}=50,\ S=1,\ \alpha=-0.1,E=0.01,\ R=1.5$

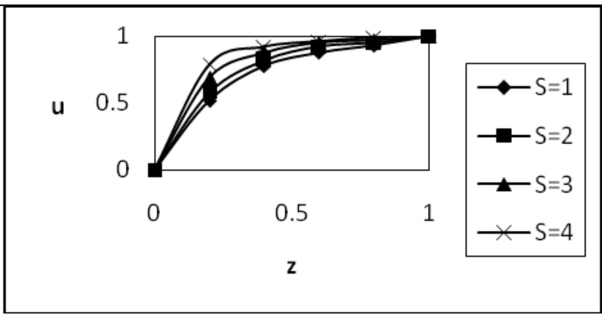


Fig. 15: The temperature profile for θ on Porous parameter S with $\mathrm{Re}=50,\ M=2,\ \alpha=-0.1,\ E=0.01,R=1.5$

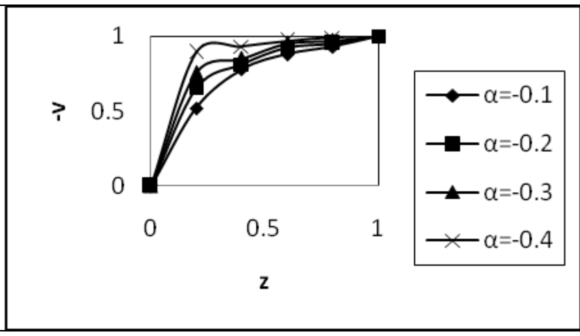
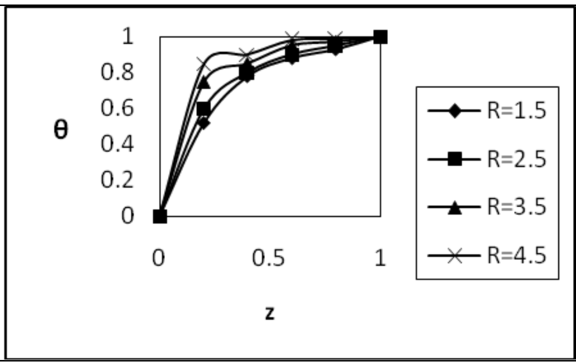


Fig. 16: The temperature profile for θ on visco-elastic parameter α with $\mathrm{Re}=50,\ M=2,\ S=1,\ E=0.01,R=1.5$



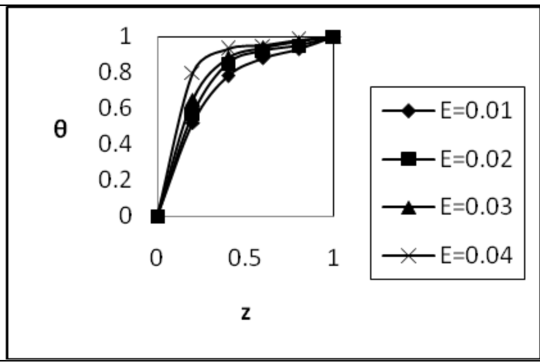


Fig. 17: The temperature profile for θ on Radiation parameter R with $\text{Re}=50,\ M=2,\ S=1,\ \alpha=-0.1, E=0.01$	Fig. 18: The temperature profile for θ on Eckmann number E with $\text{Re}=50,\ M=2,\ S=1,\ \alpha=-0.1, E=0.01, R=$

Re	I	II	III	IV	V	VI	VII	VIII	IX	X	XI
50	0.0245	1.8554	3.8809	0.0126	0.0052	0.0546	0.0785	0.0854	0.0569	0.1055	0.2551
100	0.1446	2.8847	5.8824	0.1222	0.0145	0.1545	0.1884	0.1885	0.1254	0.2526	0.5887
150	0.2114	5.9968	8.1055	0.1884	0.1225	0.2405	0.2887	0.2995	0.2254	0.5855	0.7886
200	0.2889	9.5548	12.658	0.2254	0.1889	0.2998	0.3225	0.3825	0.2996	0.6335	0.8888

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI
M	2	5	8	2	2	2	2	2	2	2	2
S	1	1	1	2	3	1	1	1	1	1	1
α	-0.1	-0.1	-0.1	-0.1	-0.1	-0.2	-0.3	-0.1	-0.1	-0.1	-0.1
R	1.5	1.5	1.5	1.5	1.5	1.5	1.5	2.5	3.5	1.5	1.5
E	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.02	0.03

Table.1 The shear stresses (τ) at the wall $z=0$

Re	I	II	III	IV	V	VI	VII	VIII	IX	X	XI
50	1.2552	0.9885	0.6585	2.3314	3.9985	2.8845	3.1145	2.0147	3.0054	1.8225	1.9969
100	2.0458	1.5665	1.1125	3.2214	4.2256	3.2251	4.6556	3.1022	4.0552	2.2258	2.5566
150	2.9985	2.4458	1.8447	3.9985	6.5582	4.8859	5.9985	3.5229	4.8002	3.2556	2.2985
200	3.6687	3.1205	2.7195	4.8556	7.5222	5.6648	6.7808	4.2245	5.1566	3.8853	4.1205

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI
M	2	5	8	2	2	2	2	2	2	2	2
S	1	1	1	2	3	1	1	1	1	1	1
α	-0.1	-0.1	-0.1	-0.1	-0.1	-0.2	-0.3	-0.1	-0.1	-0.1	-0.1
R	1.5	1.5	1.5	1.5	1.5	1.5	1.5	2.5	3.5	1.5	1.5
E	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.02	0.03

Table.2 The Rate of heat transfer (Nusselt number) at the wall $z=1$

IV. CONCLUSIONS:

1. The magnitude of velocity *u* and *v* increase with the increasing values of the Reynolds number Re, Porous parameter *S*, the visco-elastic parameter α, Radiation parameter R and Eckmann number *E*.
2. The magnitude of the velocity component *u* experiences retardation and the behaviours of the velocity component *v* remains the same with the increasing values of the Hartmann number.
3. Lower the permeability of the porous medium lesser the fluid speed in the entire fluid region.
4. The resultant velocity *q* enhances with increasing the parameters Re, *D*, α, *R*, *E* and experiences retardation with increasing the intensity of the magnetic field.
5. The magnitude of the temperature increases with increasing Re, *D*, α, *R*, *E* and experiences retardation with increasing the magnetic field parameter (Hartmann number *M*)
6. The shear stress enhances with increasing Re, *M* and α and reduces with increasing S. We also noted that it is increases firstly and then decreases with the increasing values of radiation parameter R.
7. The Nusselt number increases with the increasing values of the visco-elastic parameter $|\alpha|$ and the magnitude of the heat transfer increases with increasing Re, *S*, *R*, *E* retardation with increasing the magnetic field parameter *M*.

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