

# Model for Vertical Temperature Profile in Waste Stabilization Ponds

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## ABSTRACT

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Thermal stratification is a natural phenomenon which occurs in waste stabilization ponds caused by vertical temperature difference which alters the flow patterns in the ponds, affects its performance and design, thereby reducing the efficiency of its operation. This study therefore developed a mathematical model for predicting the vertical temperature profile in WSPs. The solution of the model was obtained using a numerical solution method based on a fourth order Runge-Kutta method. The results were verified with data from several field ponds at different depths. Maximum difference of 19.05% between the calculated and predicted temperature profiles were obtained as shown in the Figures. The coefficients of correlation between the measured and the predicted values range from 0.6774 to 0.9990 as calculated.

**KEYWORD:** Thermal stratification, model, temperature profile, Runge-Kutta and waste stabilization ponds.

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## I. INTRODUCTION

Waste stabilization ponds (WSPs) are now regarded as the method of first choice for the treatment of wastewater in many parts of the world. In the United States, one third of all wastewater treatment plants are waste stabilization ponds serving populations upto 5000 as reported by (EPA, 1983; Boutin and others, 1987; and Bucksteeg, 1987) have also reported that wastewater stabilization pond are very widely used in small rural communities in Europe with population close to 2000. Larger systems exist in Mediterranean France and also in Spain and Portugal. However, in warmer climates (the middle East, Africa, Asia and Latin America) ponds are commonly used for large populations upto one million (Soares and others, 1996).

In the developing countries and especially in the tropical and equatorial regions, sewage treatment by wastewater stabilization ponds has been considered an ideal way of using natural processes to improve sewage effluent. The importance of waste stabilization ponds is well documented in the literature (Maraise, 1974; Mara and others, 1983; and Polprasert and Others, 1983). Its economic advantage over the conventional technologies and the abundance of sunlight and prevalent high ambient temperature in the tropical and subtropical regions, have made it so popular. The parameters used in judging the performance of wastewater stabilization ponds are bacteria rate of degradation, biochemical oxidation, dispersion, bacteria die-off rate and thermal stratification which are influenced by temperature gradient. Many models (Polprasert and others, 1983; Marais and others, 1961; Bowles, 1979; Klock, 1971; and Thirumurthi, 1969) have been proposed to describe the process of bacterial degradation, but none has been found acceptable. The fact that the largest relative difference in temperature appears close to the surface layer of the pond during thermal stratification processes can be related to the complex mechanisms of thermal energy transfer between the water surface and air. In this zone, wind gusts and surface waves play significant role in the transfer mechanisms but owing to their random behaviour it is difficult to predict their effects correctly. The phenomenon of thermal stratification, overturning of density stratification and complete mixing are natural processes which are rampant during summer.

The values and distribution of temperature in a mass of water play a fundamental role in the behaviour of these systems due to the following factors:

- [1] Physicochemical and biological processes depend greatly on temperature values and
- [2] The mixing phenomena are closely related with the thermocline, because it restricts the heat and matter transport between the different layers defined in the water column as a consequence of thermal stratification.

As in other water bodies (lakes, reservoirs, etc), the thermal stratification of a deep pond begins in spring and, starting from a homogeneous temperature, a stratification starts developing such that it eventually results in a well-defined thermocline. This thermocline separates two clearly differentiated layers: an aerobic superior one (epilimnion) and anaerobic inferior one (hypolimnion). This situation remains until when the surface cooling provokes a gradual deepening of the thermocline till isothermicity is re-established. This gradual deepening throughout the cycle causes the nutrient rich water of the hypolimnion to ascend to the surface layer that has become depleted as a result of the intense biochemical activity favoured by the higher temperature, the presence of solar radiation, and the photosynthetic and surface oxygenation. On the other hand, the biomass generated in the epilimnion settles down and, once its mineralization takes place, it constitutes an additional contribution of nutrients to the hypolimnion (Moreno, 1983; Schertzer and others, 1987; Simons and others 1987 and Soler and others, 1991). Stratification can be caused by temperature differences. Hence, the objective of this paper is to present a mathematical model for the prediction of vertical temperature profile in ponds. The model will be solved using Runge-Kutta method and verified with literature data (Silva, 1982; Vidal, 1983; Agunwamba, 1997; Moreno-Grau and others, 1983 and Kellner and Pires, 2002).

In the determination of the thermal flux provided by solar radiation and the air was primarily a function of the latitude of the pond site, the distance between the earth and the sun (varying along the year), the temperature of the air and the fraction of the cloud cover. The calculation of the thermal flux can be found in full detail in (Pires and Kellner, 1999 and Mayo, 1989). Although the approach by (Fritz, Meredith and Middleton 1980) was complicated, this study therefore tried to simplify their work by using Kreyszig (1993) who developed the rate constant equation based on solar radiation, the pond depth and a constant depending on the season of the year.

## II. METHODOLOGY

In order to verify the quality of the model, its results were compared with experimental data from other studies based on temperature measurement. The various researchers include (Silva, 1982; Vidal, 1983; Agunwamba, 1997; Moreno-Grau and others (1984). The pond studied by Silva is located in Campina Grande, para-lba, northeast Brazil (latitude  $7^{\circ}13'11'' \leq S$ ), which has a hot, dry climate, with small daily temperature variations, the pond studied by Vidal is located in Santa Fe do Sul, São Paulo, southeast Brazil (latitude  $20^{\circ}46'0'' \leq S$ ), an area with a higher daily temperature variation. Similarly, the pond studied by Agunwamba is located at the campus of the University of Nigeria, Nsukka in the southeastern part of Nigeria with small daily temperature variations; and the pond studied by Moreno-Grau *et al* is located at Cartagena-Spain. Silva (1982) measured the temperature variation with depth on 6-7 October 1987, in a facultative pond of length 25.70m, width 7.40m and depth 1.25m, located in Campina Grande. Vidal (1983) performed his measurements on a facultative pond in Santa Fê do Sul, with a length of 80m, a width of 66m and a depth of 1.5m. Agunwamba (1997) performed his measurements on a facultative pond in the University of Nigeria, Nsukka Campus measuring 120m length, 30m width and 0.2m deep.

## III. DEVELOPMENT OF MATHEMATICAL MODEL FOR VERTICAL TEMPERATURE PROFILE IN WSP

The general expression for the material balance can be written as:

The balance of thermal energy stored in the jth element = Solar radiation from sun + Local advection due to horizontal movement

– Vertical advection – Energy due to mixing (dispersion) - - - - (1)

According to Kellner and Pires (2002) the balance of thermal energy in a volume element can be expressed as:

$$\frac{\partial H_j}{\partial t} = (h_j - h_o + h_{sz}) - (h_{wj} - h_{wj} + 1) - (h_{dj} - h_{dj} + 1) \quad (2)$$

where:

$$H_j = C \bar{V}_j T_j \quad (3a)$$

$$h_i = C Q_j T_j \quad (3b)$$

$$h_o = C Q_o T_o \quad (3c)$$

$$h_{sz} = Z A_z \quad (3d)$$

$$h_{dj} = C D_z A_z \frac{\partial T}{\partial Z} \quad (3e)$$

$$Z = (1 - S_o) e^{-KZ} \quad (3f)$$

$$h_{wj} = C Q_z T_z \quad (3g)$$

and

- $H_j$  = the thermal energy contained in the jth element (J);
- $C$  = the specific heat of water,  $\text{Jkg}^{-1}\text{C}^{-1}$ ;
- $V_j$  = the specific mass of water  $\text{kgm}^{-3}$ ;
- $T_j$  = temperature in element j  $^{\circ}\text{C}$ ;
- $A_z$  = the direct insolation  $\text{J sec}^{-1}$ ;
- $h_i$  = the thermal energy introduced by the influent flow  $\text{J sec}^{-1}$ ;
- $h_o$  = the thermal energy removed by the effluent flow,  $\text{J sec}^{-1}$
- $h_{sz}$  = the direct insolation,  $\text{J Sec}^{-1}$ ;
- $Z$  = the flow of solar radiation at depth Z,  $\text{Jm}^{-2}\text{S}^{-1}$
- $S_o$  = the radiation rate absorbed in the liquid surface,  $\text{Cal m}^{-2} \text{day}^{-1}$ ;
- $h_{dj}$  = the heat transferred by diffusion along the vertical axis,  $\text{JS}^{-1}$
- $h_{wj}$  = the heat advected along the vertical axis;
- $D_z$  = the coefficient of vertical dispersion ( $\text{m}^2\text{S}^{-1}$ ) that comprises molecular and turbulent diffusions
- $K$  = the coefficient of light attenuation,  $\text{m}^{-1}$ .

By substituting equations (3a) to (3g) into equation (2) yield the following results:

$$\frac{\partial(C \bar{V}_j T_j)}{\partial t} = (C Q_{ij} T_{ij} - C Q_{oj} T_j + z A_z) - (C Q_{zj} T_j - C Q_{zj+1} T_{j+1}) - \left( C D_j A_j \frac{\partial T}{\partial Z} \Big|_j - C D_{j+1} A_{j+1} \frac{\partial T}{\partial Z} \Big|_{j+1} \right) \quad (4)$$

The specific heat of water (C) is a constant and will cancel, out and equation (4) becomes:

$$\frac{\partial(\bar{V}_j T_j)}{\partial t} = (Q_{ij} T_{ij} - Q_{oj} T_j) + \frac{Z}{C} A_z - (Q_{zj} T_j - Q_{zj+1} T_{j+1}) - \left( D_j A_j \frac{\partial T}{\partial Z} \Big|_j - D_{j+1} A_{j+1} \frac{\partial T}{\partial Z} \Big|_{j+1} \right) \quad (5)$$

The impact of solar radiation is on the entire parameters in the pond such as temperature, specific heat of water (C), and the direct insolation (A). Thus, equation (5) gives:

$$\frac{\partial(\bar{V}_j S_o T_j)}{\partial t} = (Q_{ij} S_o T_{ij} - Q_{oj} S_o T_j)$$

$$+ \frac{1}{CS_o} S_o z S_o A_z - (Q_{zj} S_o T_j - Q_{zj+1} S_o T_{j+1}) - \left( D_j S_o A_j S_o \frac{\partial T}{\partial Z} \Big|_j - D_{j+1} S_o A_{j+1} S_o \frac{\partial T}{\partial Z} \Big|_{j+1} \right)$$

Thus, equation (6) can be related to the general word expression of equation (1). Therefore, after simplification and discretization, equation (6) becomes:

$$\begin{aligned} \frac{\partial T_j}{\partial t} = & \frac{1}{V_j} (Q_{ij} T_{ij} - Q_{oj} T_j) + \frac{1}{V_j} CS_o Z S_o A_j \\ & - \frac{1}{V_j} (Q_{zj} T_j - Q_{zj+1} T_{j+1}) - \frac{1}{V_j} \frac{D_j S_o A_j S_o T_{j+1}}{Z} \\ & + \frac{1}{V_j} \frac{D_j S_o A_j}{Z} S_o T_j + \frac{1}{V_j} \frac{D_{j+1} S_o A_{j+1}}{Z} S_o T_j \\ & + \frac{1}{V_j} \frac{D_{j+1} S_o}{Z} A_{j+1} S_o T_{j+1} \end{aligned} \quad \dots \quad 7$$

The solution to the partial differential equation give by equation (7) can be obtained by (Kreyszig, 1993; Micolescus, 1974 and Stroud, 1996) have given the basic numerical solution of the partial differential equation.

Therefore for the temperature increment a fourth order Runge-Kutta method will be applied as given in equation (8) thus:

$$T_j^{t+1} = T_j^t + \frac{1}{2} \frac{t}{6} \{ S_o O_{1,j} + 2S_o O_{2,j} + 2S_o O_{3,j} + O_{4,j} \} \quad \dots \quad 8$$

where:

$$O_{1,j} = f(T_j^t) \quad \dots \quad 9a$$

$$O_{2,j} = f\left(T_j^t + \frac{1}{2} t O_{1,j}\right) \quad \dots \quad 9b$$

$$O_{3,j} = f\left(T_j^t + \frac{1}{2} t O_{2,j}\right) \quad \dots \quad 9c$$

and

$$O_{4,j} = f\left(T_j^t + \frac{1}{2} t O_{3,j}\right) \quad \dots \quad 9d$$

Equation (8) can be simplified as:

$$T_j^{t+1} = T_j^t + \frac{t}{12} S_o \left\{ O_{1,j} + 2O_{2,j} + 2O_{3,j} + \frac{O_{4,j}}{S_o} \right\} \quad \dots \quad 10$$

From Mayo (1989) the rate constant K is given as:

$$K = 0.108 + f \times \frac{S_o}{H} \quad \dots \quad 11$$

where:

So = the solar radiation or the radiation rate aborbed in the liquid surface, Cal/m<sup>2</sup>/day

H = the pond depth in meter

f = a constant depending on the season of the year

Substituing equation (11) into equation (10) gives:

$$T_j^{t+1} = T_j^t + \frac{t}{12} \left( \frac{K - 0.108}{fxH^{-1}} \right) x \left\{ O_{1,j} + 2O_{2,j} + 2O_{3,j} + \left( \frac{FxH^{-1}}{K - 0.108} \right) O_{4,j} \right\} \quad - \quad 12$$

Hence, the final equation can be expressed as:

$$T_j^{t+1} = T_j^t + \lambda_y \{ O_{1,j} + 2O_{2,j} + 2O_{3,j} + \delta_y O_{4,j} \} \quad - \quad - \quad - \quad - \quad - \quad 13$$

where

$$\lambda_y = \frac{t}{1036800} \left( \frac{K - 0.108}{fxH_y^{-1}} \right) \quad - \quad - \quad - \quad - \quad - \quad - \quad 14$$

$$\delta_y = \left( \frac{fxH_y^{-1}}{K - 0.108} \right) \quad - \quad - \quad - \quad - \quad - \quad - \quad 15$$

and

$$O_{1,j} = f(T_j^t) \quad - \quad - \quad - \quad - \quad - \quad - \quad 16$$

$$O_{2,j} = f \left( T_j^t + \frac{t}{172800} O_{1,j} \right) \quad - \quad - \quad - \quad - \quad - \quad - \quad 17$$

$$O_{3,j} = f \left( T_j^t + \frac{t}{172800} O_{2,j} \right) \quad - \quad - \quad - \quad - \quad - \quad - \quad 18$$

$$O_{4,j} = f \left( T_j^t + \frac{t}{172800} O_{3,j} \right) \quad - \quad - \quad - \quad - \quad - \quad - \quad 19$$

where t is the adopted time interval in seconds.

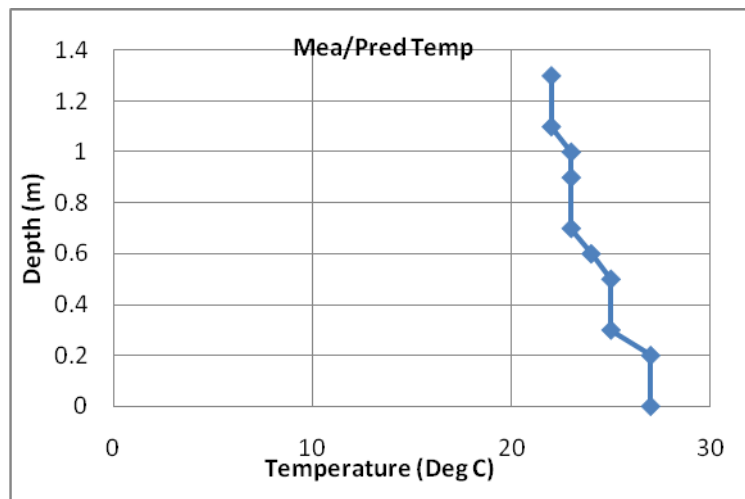
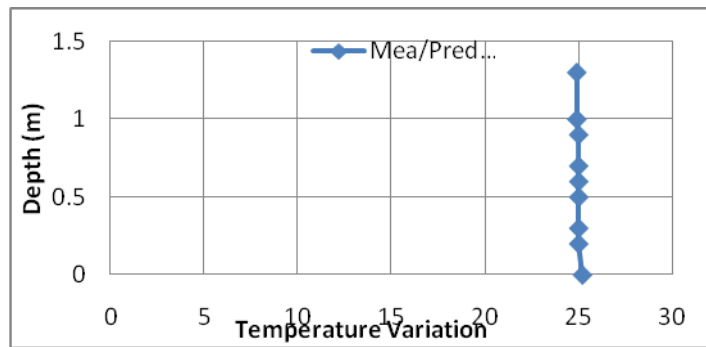
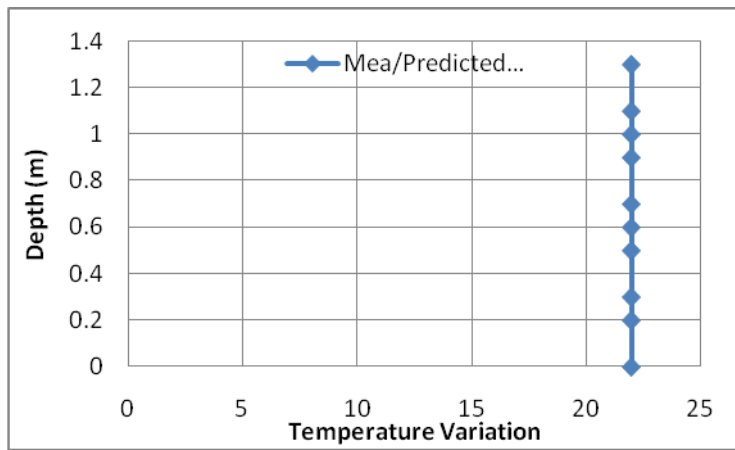
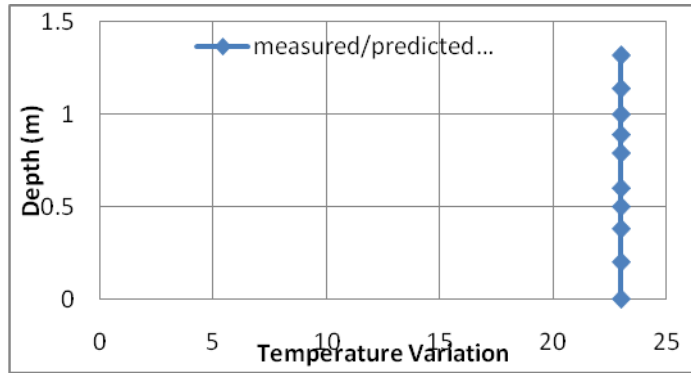
#### IV. DISCUSSION OF RESULTS

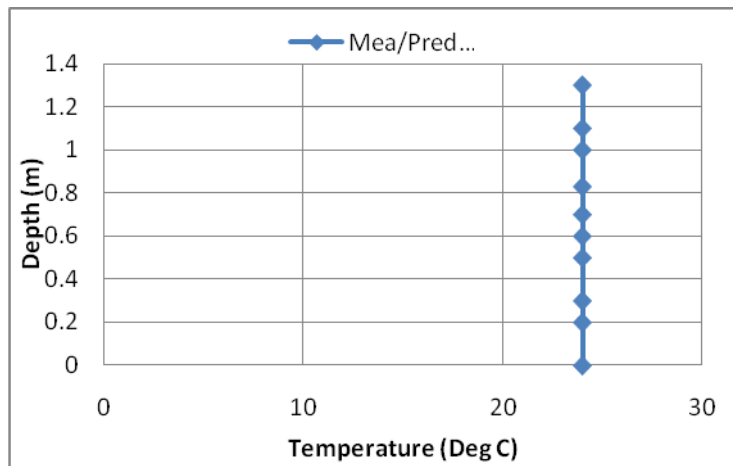
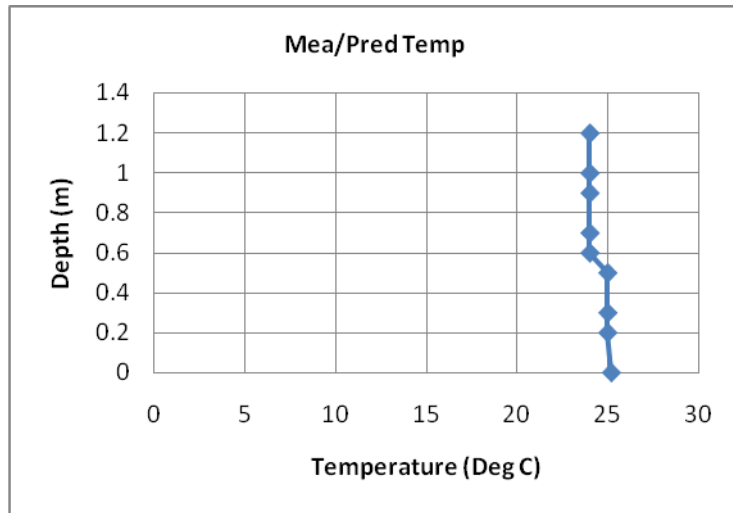
The data from several field wastewater stabilization ponds obtained from literature were used to evaluate the response of vertical temperature profile model of equation (13) developed in this study. Comparisons of the measured experimental temperature data and those predicted from the model were made to show the responses accuracy and sensitivity. The plot of predicted temperature values and measured temperature values versus depth are shown in Figures 1, 2, 3, 4, 5 and 6, respectively. Comparing the measured temperature values with the predicted values gives a maximum difference of 19.05% between the calculated and observed temperature profiles. The coefficients of correlation between the measured and the predicted values range from 0.6774 to 0.990 and the standard error of estimate ranges from 0.00315 to 0.382 indicating that the model of vertical temperature profile developed in this study performed with a high degree of accuracy.

#### V. CONCLUSION

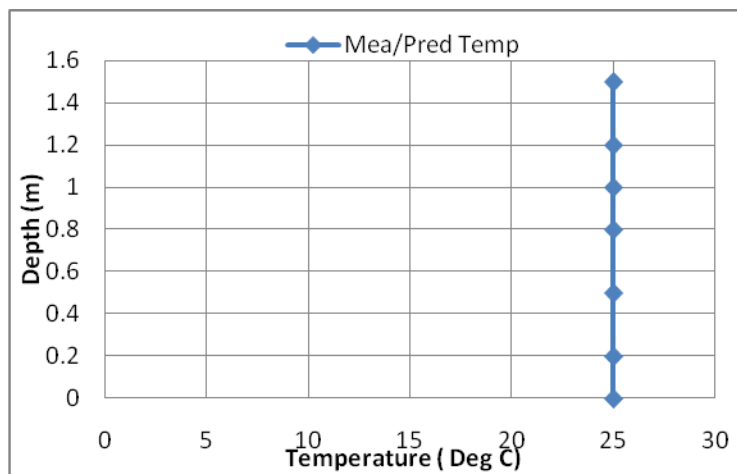
- [1] The proposed model for the determination of vertical temperature profile in waste stabilization ponds developed in this study performed with a high degree of accuracy.
- [2] The proposed model indicates the complete thermal cycle such as complete mixing, overturning of density stratification and thermal stratification.
- [3] Complete mixing occurs when the temperature profiles are in uniform; and
- [4] Thermal stratification are caused by temperature differences.

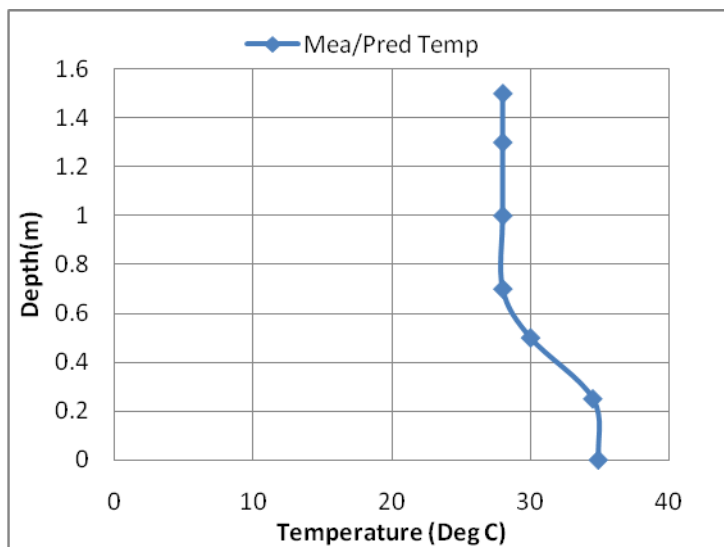
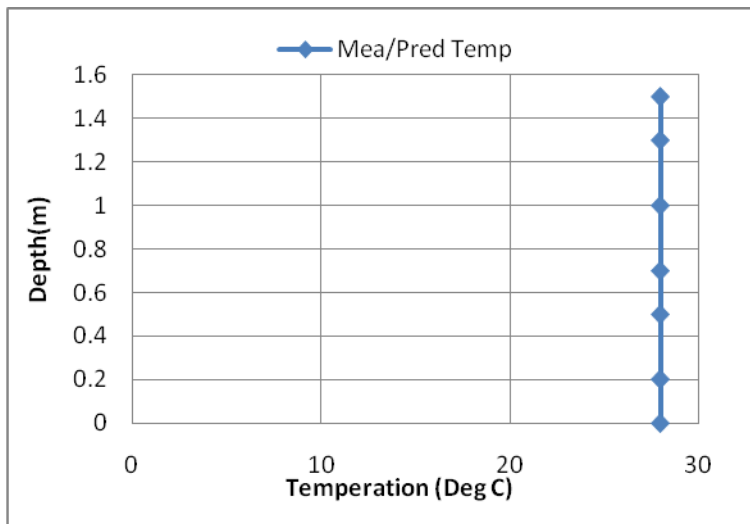
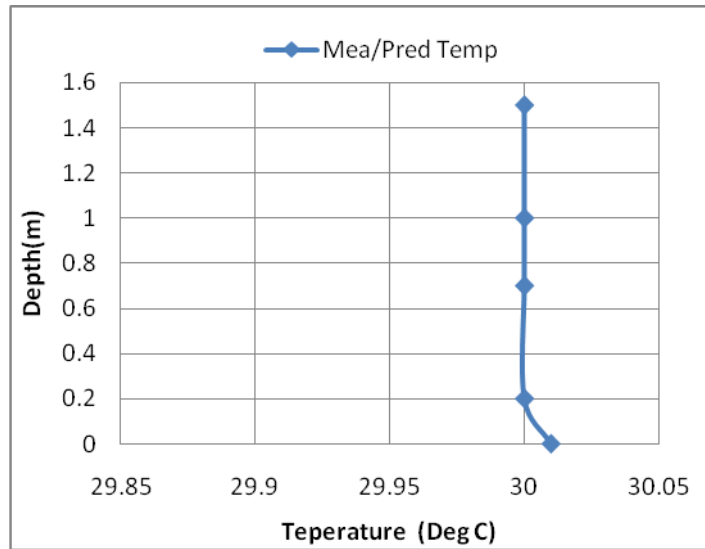
This study therefore recommends that waste stabilization pond should be sited in the direction of wind in order to reduce the temperature increase caused by heat, thereby avoiding overturning and subsequent stratification in the pond.



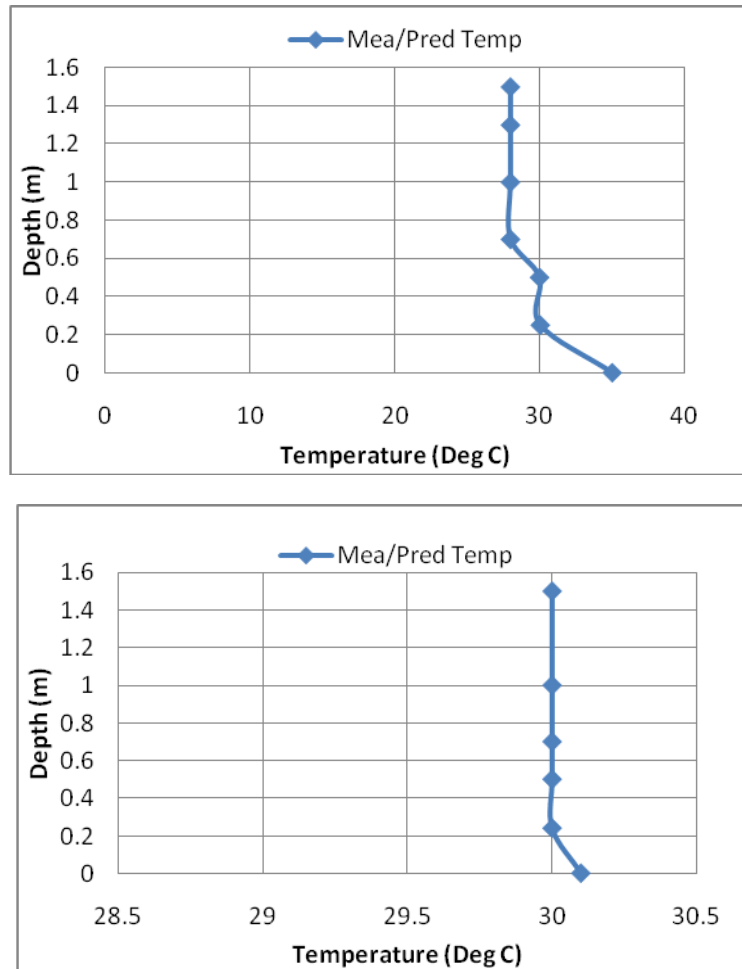


**Fig.1:** Measured and Predicted Temperature Variation with Depth Using Silver (1982) Experimental Data For 6:00, 10:00, 12:00, 14:00 and 22:00 Hours.

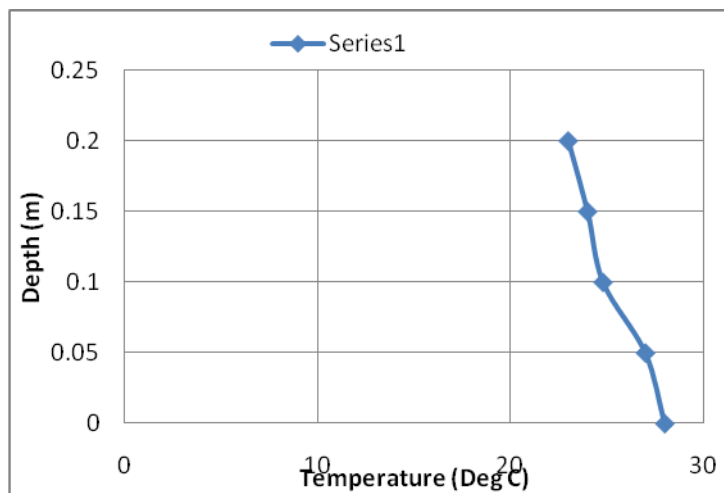


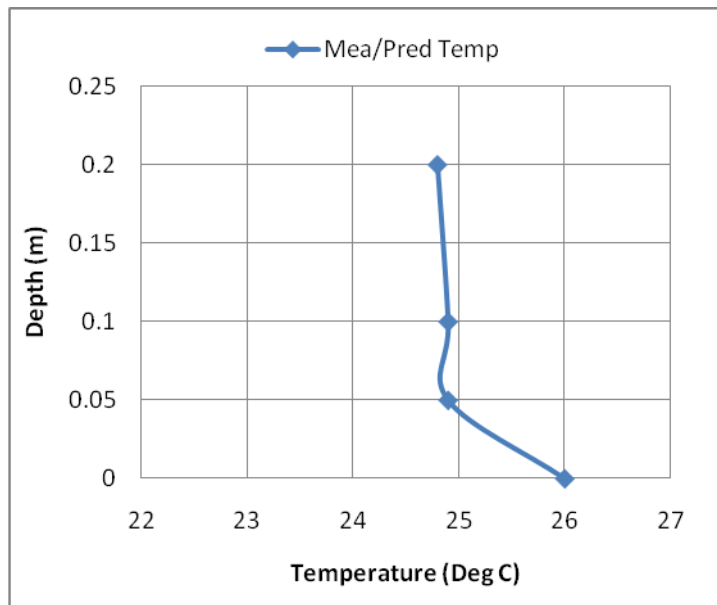
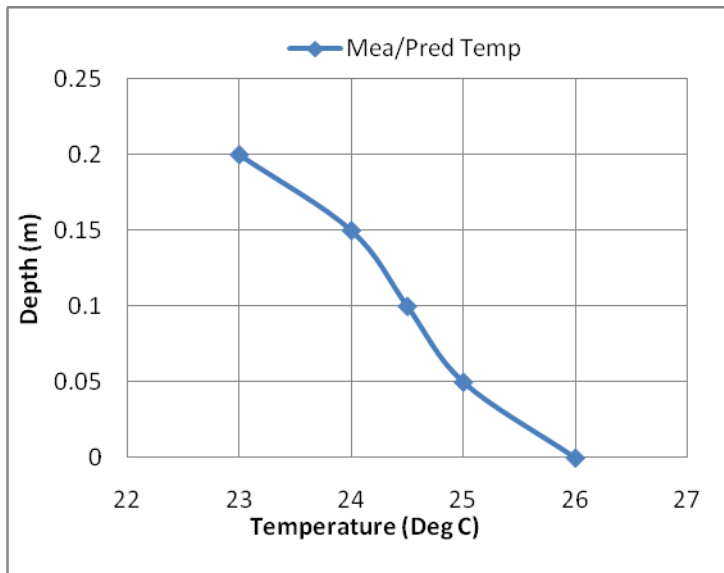
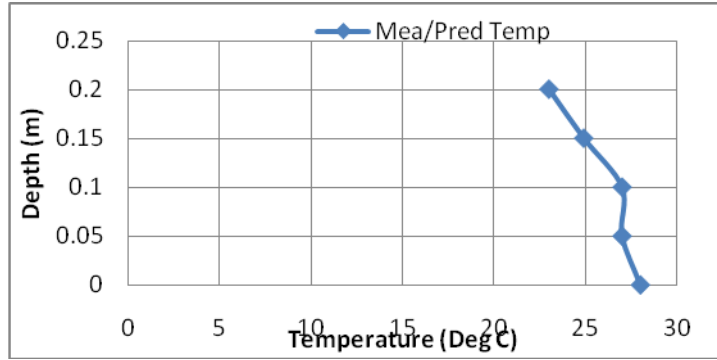


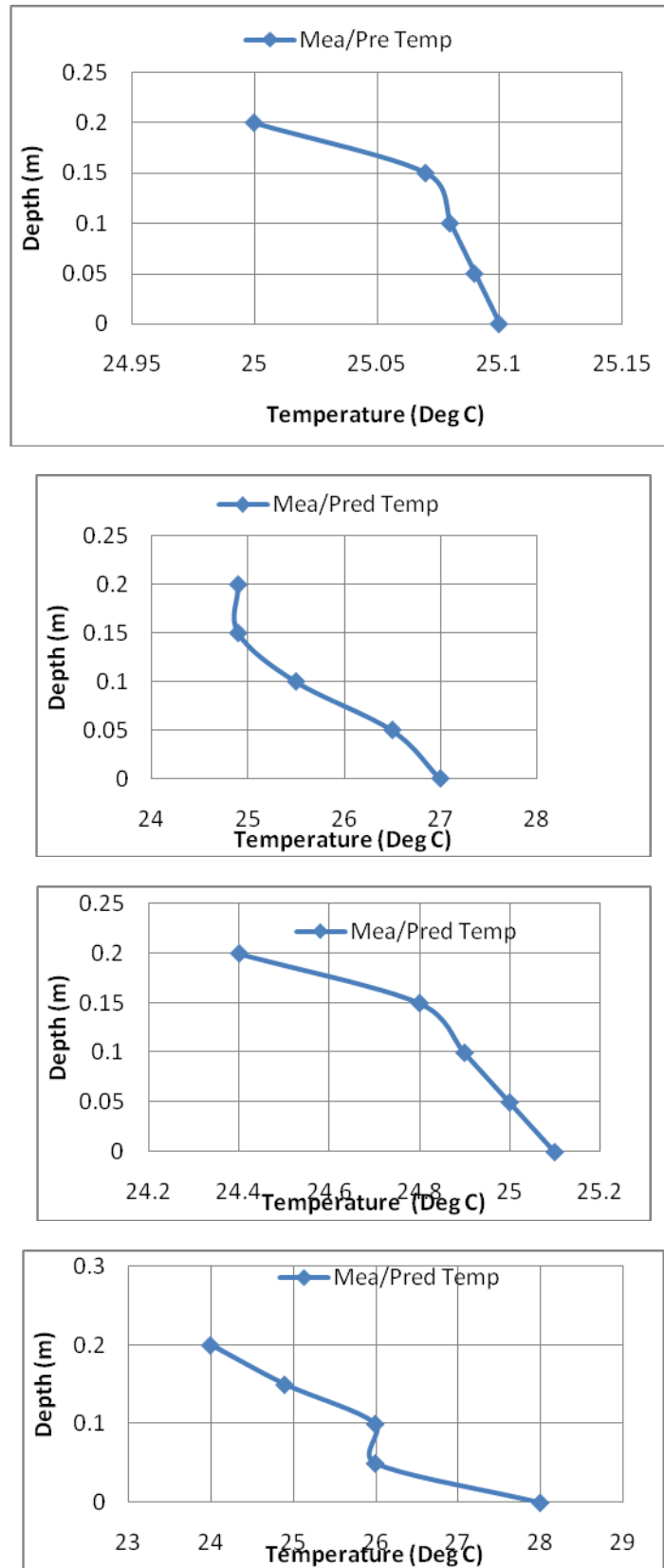




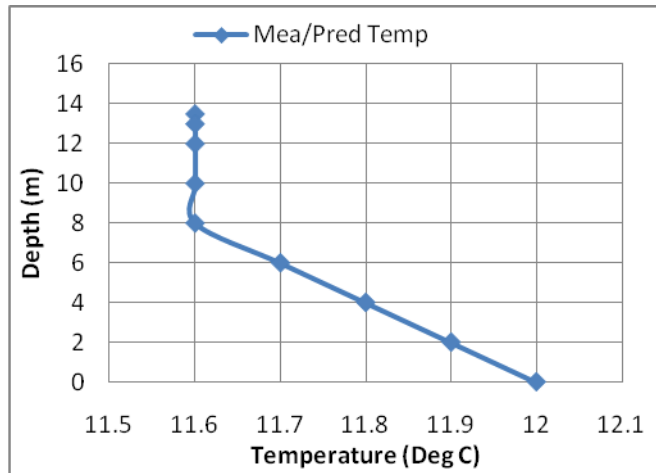
**Fig.2:** Measured and Predicted Temperature Variation with Depths Using Vidal (1983) Experimental Data for 6:00,10:00,12:00,14:00,16:00 and 22:00 Hours



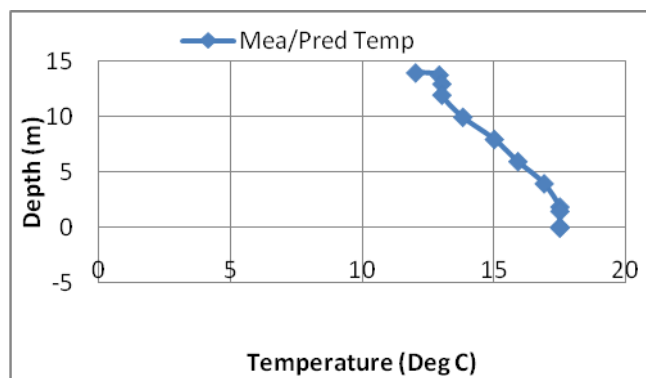




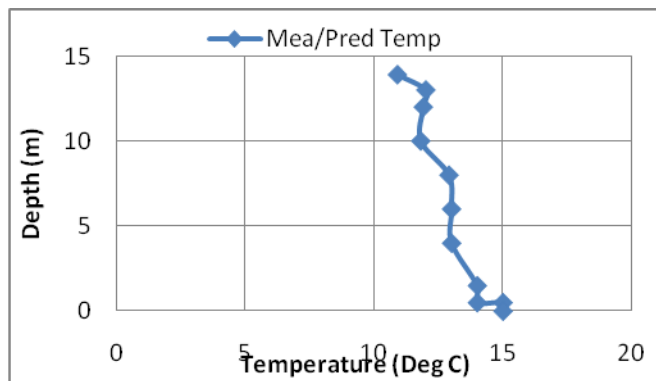
**Fig.3:** Measured and Predicted Temperature Variation with Depth Using Agunwamba (1997) Experimental Work at Nsukka WSP



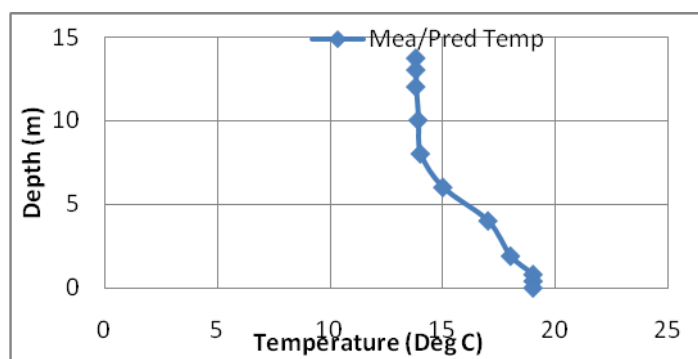
Feb. 13, 1980



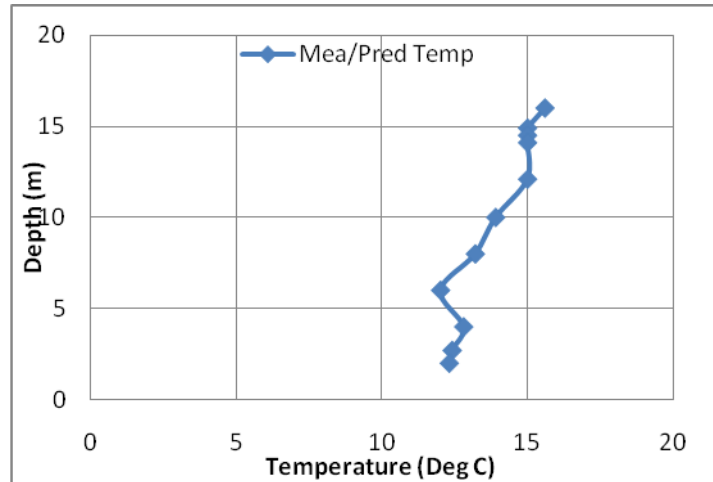
April 23, 1980



March 12, 1980

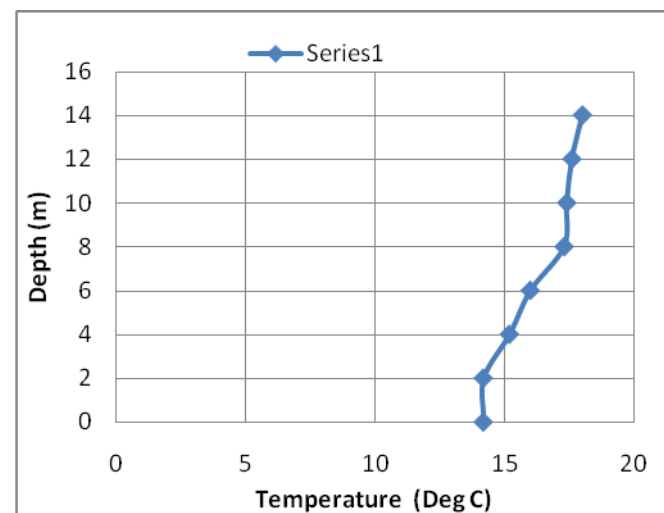
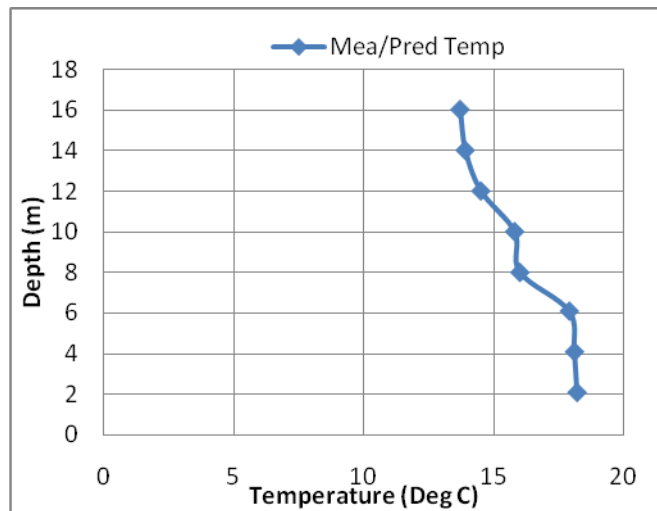


May 15, 1980



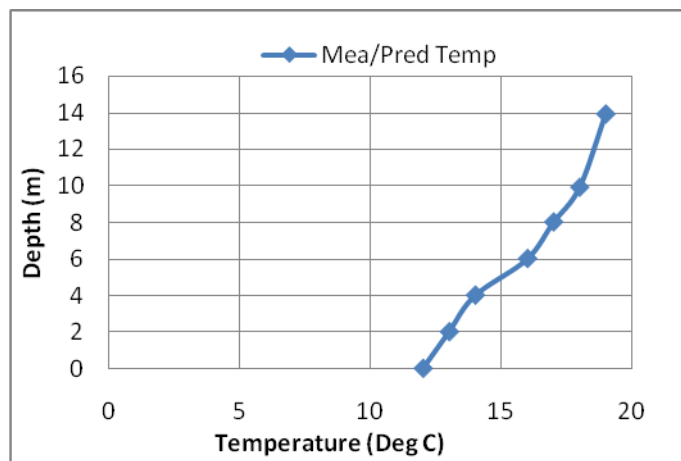
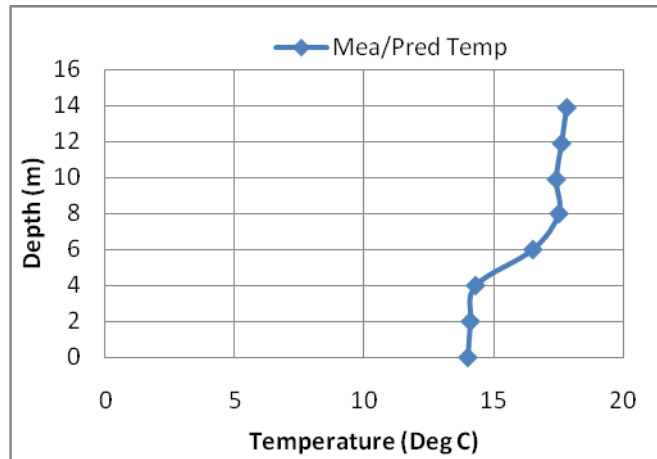
April 19, 1980

**Fig.4:** Measure and Predicted Temperature Variation with Depths Using Moreno- Grau & others (1984) Experimental Data for February 13, March12, April23, and May 15, (1980)



March 26, 1980

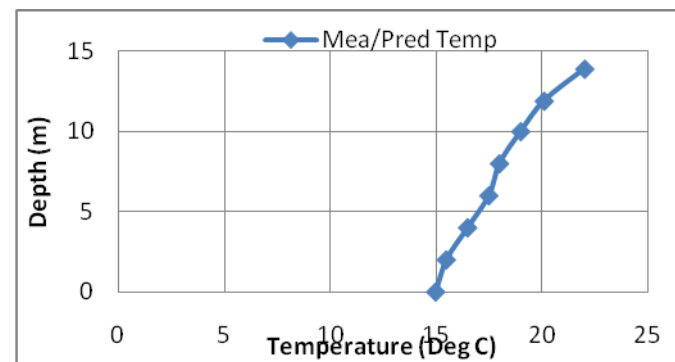
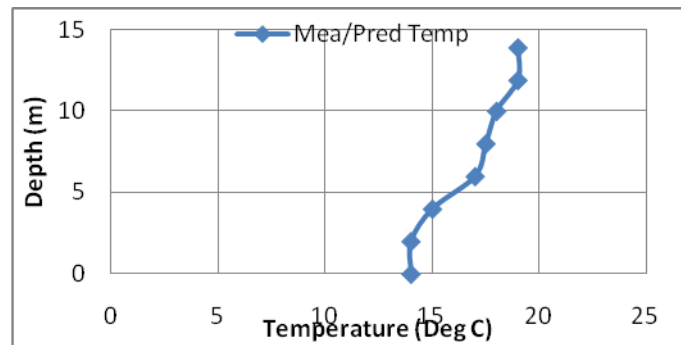
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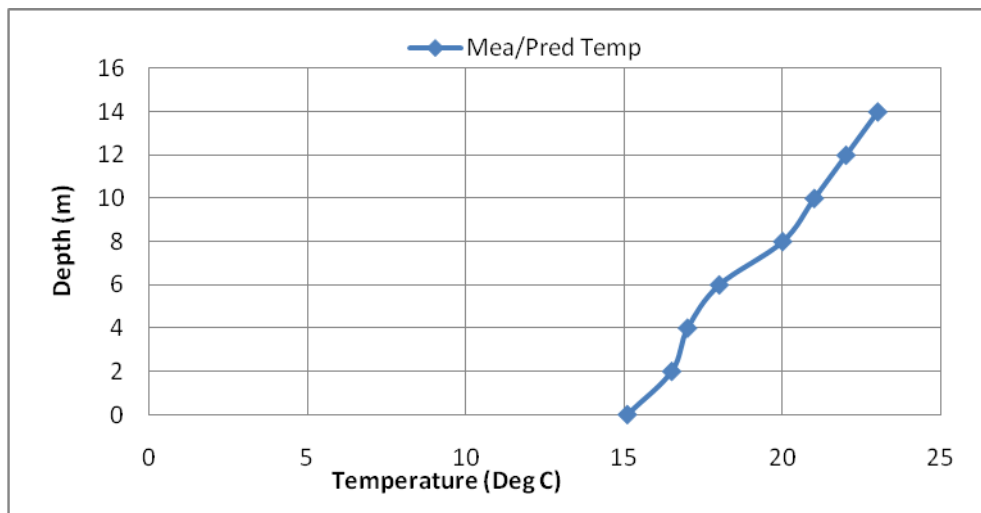
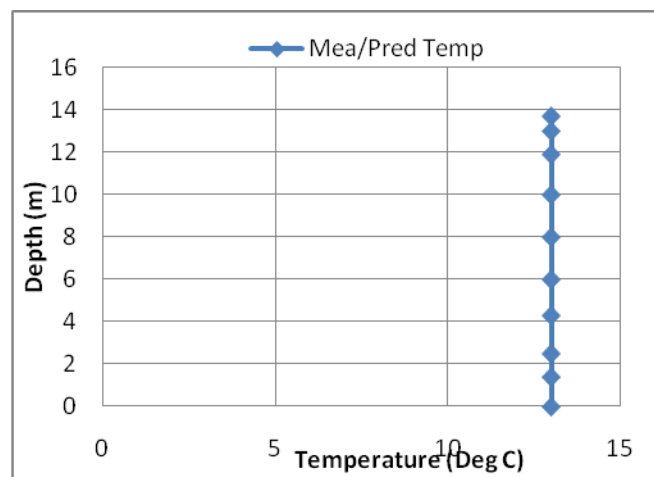
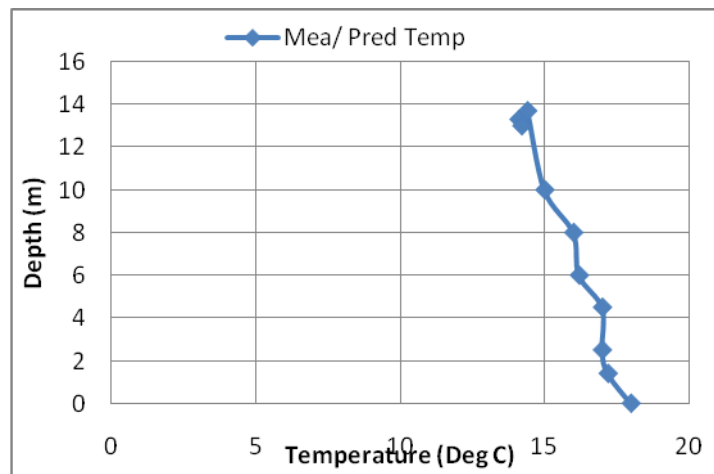
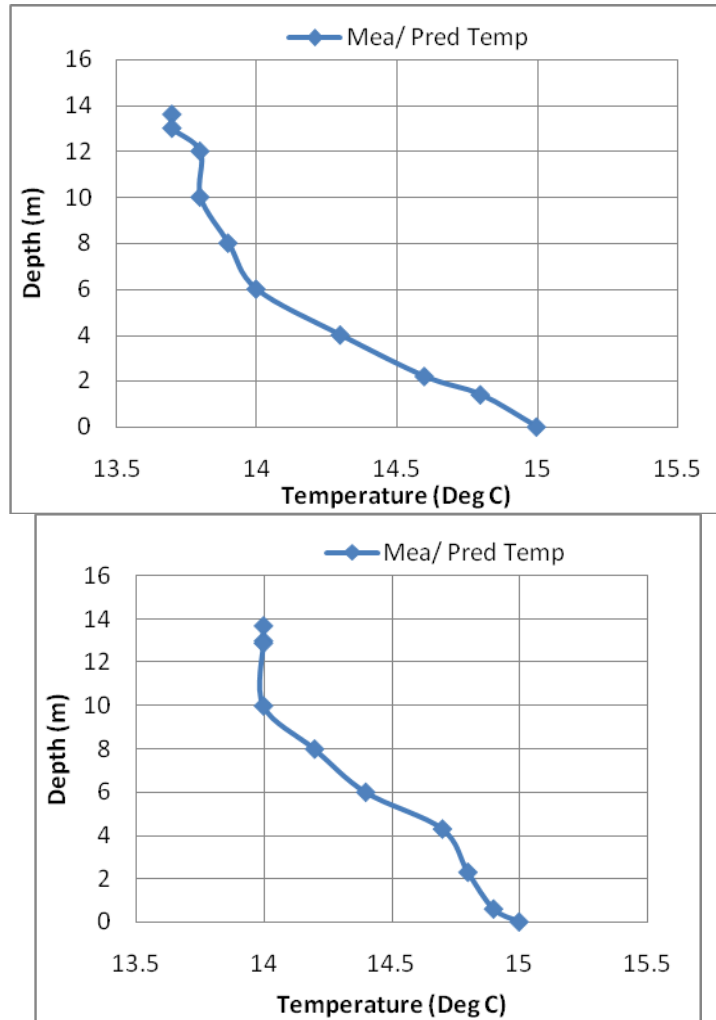


Fig.5: Measure and Predicted Temperature Variation with Depths Using Moreno- Grau & Others (1984) Experimental Data for June, 1981.





**Fig.6:** Measure and Predicted Temperature Variation with Depths Using Moreno-Grau & others (1984) Experimental Data.

### REFERENCES

- [1] Agunwamba, J.C. (1997). Prediction of bacterial population in Waste Stabilization Ponds using multiple depth layer model, I. E (1) Journal EN, Vol. 77, pp. 42-48.
- [2] Boutin, P., Vachon, A., and Racault, Y. (1987). Waste Stabilization Ponds in France; an overall view. Water Science and Technology, 19(12), pp. 25-31.
- [3] Bowles, D.S. (1979). Coliform Decay Rates in Waste Stabilization Ponds. Journal Water Pollution Control Federation, 51, pp. 87-99.
- [4] Bucksteeg, K. (1987). German experiences with sewage treatment ponds. Water Science and Technology, 19(12), pp. 17-23.
- [5] EPA (1983). Design Manual: Municipal Wastewater Stabilization Ponds. Report No. EPA-625/1-83-015. Cincinnati: Environmental Protection Agency, Centre for Environmental Research Information.
- [6] Fritz, J.J., Meredith, D.D. and Middleton, A.C (1980). Non-steady state bulk temperature determination for Stabilization Ponds, Water Resources, 14, pp. 413-420.
- [7] Kellner, E., and Pires, E. C (2002). The influence of thermal stratification on the hydraulic behaviour of Waste Stabilization Ponds. School of Engineering of Sao Carlos, Hydraulics and Sanitary Eng. Dept. Brazil, Water Science and Technology, Vol. 45, No. 1, pp. 41-48.
- [8] Klock, J.W. (1971). Survival of Coliform bacteria in waste water treatment Lagoons. Journal of Water Pollution Control, Fed, 43, pp. 2071.
- [9] Kreyszig, E. (1993). Advanced Engineering Mathematics, John Wiley and Sons, Inc. Toronto, Singapore, pp. 1034-1082.
- [10] Mara, D.D., Pearson, H.W., and Silva, S.A., (1983). Brazilian stabilization pond research suggest low-cost urban applications, world water 6, pp. 20-24.
- [11] Marais, G.V.R. (1974). Faecal Bacterial Kinetics in Stabilization Ponds. Journal of Environmental Engineering Decision, ASCE, Vol. No. 1, pp. 119-129.
- [12] Marais, G.V.R. and Shaw, V.A. (1961). A rational theory for the Design of Sewage Stabilization Ponds in Central and South Africa. Trans. S. Afr. Inst. Civil Eng. 13, pp. 20.



- [13] Mayo, A.W. (1989). Effect of pond depth on bacterial mortality rate. *Journal Environmental Engineering Division, ASCE*, 115, (5), pp. 964-977.
- [14] Moreno, M.D. (1983). Application of deep waste stabilization ponds. Doctoral Thesis, University of Murcia, Spain.
- [15] Moreno-Grau, M.D., Soler, A., Saez, J and Moreno-Clavel, J. (1983). Thermal simulation of deep stabilization ponds, *Trib. Cebedicau*, No. 491, 37, pp. 403-410.
- [16] Pires, E.C. and Kellner, E. (1999). Thermal Stratification Model for Wastewater Stabilization Ponds (in Portuguese). 15<sup>th</sup> Brazilian Congress of Mechanical Engineering, November 22-26, Brazil.
- [17] Polprasert, C., Dissanayake, M.G. and Thanh, N.C. (1983). Bacterial die-off Kinetics in Waste Stabilization Ponds. *Journal Water Pollution Control Fed.* 55(3), pp. 285-296.
- [18] Schertzer, W.H. Saylor, J. H., Boyce, F. M., Robertson, D. G., and Rosa, D. (1987). Seasonal thermal cycle of Lake Erie. *Journal of Grate Lakes Res.* 13(4), pp. 468-486.
- [19] Silva, S.A. (1982). Treatment of Domestic Sewage in Waste Stabilization Ponds on North East Brazil. Ph.D Thesis, University of Dundee, Dundee, Scotland.
- [20] Simons, J. J. and Schertzer, W.M. (1987). Stratification, currents and upwelling in Lake Ontario, Summer 1982, *Can J. Fish. Aquat. Sci.*, 44(12), pp. 2047-2058.
- [21] Soares, J., Silva, S.A., De Oliverira, R., Araujo, A. L. C., Mara, D.D., and Pearson, H.W. (1996). Ammonia removal in a Pilot-Scale Waste Stabilization Pond Complex in Northeast Brazil. *Water Science and Technology*, 33(7), pp. 165-171.
- [22] Soler, A., Saez, J., Llorens, M; Martinez, I., Berna, L. M., and Torrella, F. (1991). Changes in Physicochemical parameters and Photosynthetic microorganisms in a deep wastewater self-depuration Lagoon. *Water Resources*, 25(6), pp. 689-695.
- [23] Stroud, K.A. (1996). *Further Engineering Mathematics*, Macmillan Press Ltd, Hound-Mills, Basingstoke, Hampshire RG 216 x 5 and London. Third edition, pp. 251-263.
- [24] Thirumurthi, D. (1969). Design of Principle of Waste Stabilization Ponds. *Journal Sanit. Eng. Div., Proc. Amer. Soc. Of Civil Eng.*, 95, No. SAZ, Proc. Paper, 6515, pp. 311.
- [25] Vidal, W.L. (1983). The study of vertical temperature profile in facultative pond in Santa Fe do Sul, Brazil.