

## Theoretical Formulaion to Evaluate Capacitance for Before And After Touch Point Mems Capacitive Pressure Sensors

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### Abstract

*Micro-electromechanical systems (MEMS) have received a great deal of attention in recent years. This is due to the great promise of increased miniaturization and Performance of MEMS devices over conventional devices. MEMS pressure sensors currently dominate the market for greater than atmospheric pressure sensors. In this paper, a theoretical and finite elements analysis (FEA) solution for Micro-electromechanical systems (MEMS) pressure sensor to evaluate capacitance for before and after touch point is proposed. By looking at MEMS devices, when the diaphragm starts touching the fixed electrode by applying loads, it will have a major effect on the overall of the capacitance. Therefore, one should consider the effect of touch mode capacitance value in the system to evaluate good linearity, large operating pressure range and large overload protection at output. As of so far the evaluation for capacitance value of touch point and after touch point has not been evaluated in the literatures. This paper presents the new analytical formula to approach for including the touch-down effect capacitance value of Microsystems. The proposed MEMS capacitive pressure sensor demonstrated diaphragm with radius of  $180\mu\text{m}$ , the gap depth of  $0.5\mu\text{m}$  and the sensor exhibit linear response with pressure from 0.01 Mpa to 1.7 Mpa.*

**Keywords:** MEMS Pressure sensor; Touch mode; Circular Diaphragm.

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### I. Introduction

Pressure sensors are required in many high performance applications such as biomedical, industrial, automotives, power stations and environmental monitoring. One of the main mechanisms behind of using capacitive pressure sensor is to achieve high pressure sensitivity, low noise and low turn-on temperature drift, and a minimum dependence on side stress [2], [3]. However its output is nonlinear with respect to input changes and sensitivity in the near-linear is not high enough to ignore many stray capacitance effects. Touch mode is used to achieve linearity characteristics, high range pressure sensing and overload protection. The performance of a capacitive pressure sensor depends on the absolute capacitance and how to apply uniform pressure. To study the effect of touched and after touched point capacitors in MEMS design is important as the capacitance value, sensitivity, linearity constant needed to be clearly understood. In the following discussion, some theoretical expressions for before and after touched capacitors derived to evaluate capacitance between two electrodes before and after touch point (contact area). Finally the FEA results for evaluating the capacitance for pressure ranges form 0.01Mpa-1.7Mpa at various temperatures are discussed. Figure 1 presents a cross-sectional views of a touch mode and normal mode operation of capacitive pressure sensor with a circular diaphragm and clamped edges suspended over sealed cavity. As shown in figure 1(a), in touch mode, when external pressure is applied, the diaphragm will deflect toward inside and the diaphragm starts touching the bottom electrode (is known as of substrate) with a distance of insulator in between. In normal mode operation, the diaphragm is kept distance away from the substrate as shown in figure 1(b) [4].

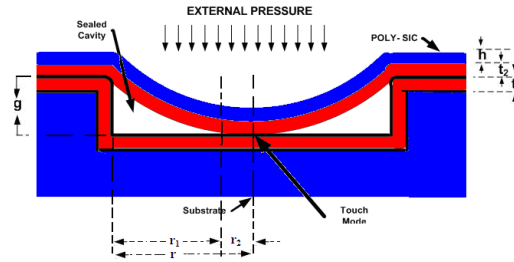


Figure 1(a): Cross-section view of touch mode Of capacitive pressure sensor.

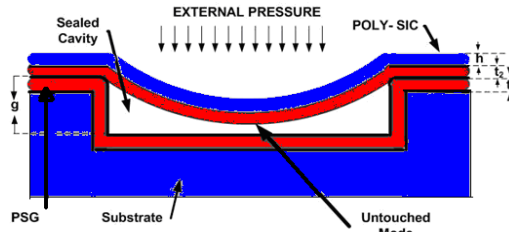


Figure 1(b): Cross-section view of normal mode of Capacitive pressure sensor.

## II. Behavioral Model Of Diaphragm Before And After Touch Mode Effect

In the analysis of MEMS, touch mode capacitance effect is the most difficult subject due to deflecting of the top diaphragm, and the movement of top diaphragm should be considered whenever it touches the bottom electrode with an insulator in between. There have been some suggestions in the literatures for solving the touch-down problems on deflection, but as of today none of the literatures has come up to a theoretical solution for calculating the touch-down capacitance value [8, 9]. For theoretical analysis several assumptions have been made for small deflection: 1) stretching of the plate has been neglected, since the gap between two plates is much less than the thickness of diaphragm. 2) Electric field fringing effects have been neglected, since the gap between two electrodes is small. With the given conditions the deflection  $w(r)$  before touch of the circular plate as a function of radius is given by (1) and maximum center deflection  $w_0$  is defined by (2). The deflection for any point of the plate at ( $0 < r < a$ ) would be:

$$w(r) = \frac{Pa^4}{64D} \left( 1 - \left( \frac{r}{a} \right)^2 \right)^2 \quad (1)$$

Where,  $P$  applied pressure acting in the same direction as  $Z$ ,  $a$  is the radius of the plate, and  $r$  is the radial distance from the center. The maximum deflection of the plate is located in the center point of the plate ( $r = 0$ ), and is defined by:

$$w_0 = \frac{Pa^4}{64D} \quad (2)$$

$D$  is the flexural rigidity of the plate is given by:

$$D = \frac{Eh^3}{12(1-\nu^2)} \quad (3)$$

Where  $E$  is Young's Modulus,  $h$  is the thickness of diaphragm,  $\nu$  and is the poison's ratio.

As shown in Figure 1(a). Equation (1) is valid for before the diaphragm touches the bottom electrode with the insulator in between and then the shape of the bending line deviates extremely from expression (1) and for every pressure, the radius of the diaphragm that barely contacts the bottom is called untouched radius ( $r_2$ ) and touched radius ( $r_1$ ) will be calculated by subtracting the total radius,  $a$  from  $r_2$  (4, touched- point radius  $r_1$  is given by:

$$r_1 = a - r_2 \quad (4)$$

### III. Theoretical Formulation For Evaluation Of Capacitance Value For Untouched And Touched Points

The concept of parallel plate capacitor is expressed and is given in (5)

$$C = \epsilon_0 \epsilon_r \frac{A}{d} \quad (5)$$

Where  $\epsilon_0$  is the permittivity of the media between the two plates,  $\epsilon_r$  is the dielectric constant of the material between the plates of the capacitance.  $A$  is the area of the electrode, and  $d$  is the gap between two plates. The concept of the capacitance element of the sensor requires a change in the capacitance as a function of some applied pressure load. A realization function of this concept would be the plates of the capacitor could move under pressure load, for example if the plates move closer together, the gap height,  $g$ , would decrease, resulting an increase in capacitance of the sensor. As the external pressure applied, the diaphragm will deform up to designed area of bottom contact which is known as of substrate with an insulator in between, the more pressure applied on the diaphragm, the bigger touched radius ( $r_1$ ) will get, and at the same time the untouched radius ( $r_2$ ) will get smaller, and the value of capacitance will increases nearly linearly with increasing pressure, before touch point the radius ( $r_1$ ) is zero. As shown in Figure 1(a).  $r$ ,  $r_1$ ,  $r_2$  are defined radial distance from center, touched-point radius, and untouched-point radius respectively.  $t_1$ ,  $t_2$  are defined the thickness of dielectrics respectively.  $g$  is the distance between the un-deformed diaphragm and the bottom electrode.  $h$  is the thickness of the diaphragm [5]. Since in this paper small deflection is considered, then the diaphragm deflection will be small compare to the diaphragm thickness, so the electric field fringing effect will be neglected, the electric field lines are perpendicular to the surface of fixed electrode and the diaphragm of a capacitive pressure sensor displaces down under external pressure and then the capacitance between the diaphragm and the bottom plate increases due to reduction of the gap between them. Therefore, a general expression for the change in capacitance as a result of pressure,  $P$ , on a circular plate is given by:

$$\frac{dC(r)}{dP} = \frac{\partial C(r)}{\partial d(r)} \cdot \frac{\partial d(r)}{\partial P} \quad (6)$$

#### 3.1 Capacitance evaluation Before Touch Point

As shown in Figure 1(b), the curvature of the plate  $w(r)$ , requires the integration over the total surface of the plate, the capacitance value variation with applied pressure is a function of the diaphragm deflection and the bottom plate (substrate), therefore the capacitance can be expressed as a function of the shape function  $w(r)$  of the diaphragm and the resultant sensor capacitance before touch can be obtain by integrating (7):

$$dC(r) = \frac{\epsilon_0 \epsilon_d r dr d\theta}{d} \quad (7)$$

$$d = d_1 - w(r) \quad (8)$$

$$C_{sensor} = C(r) = \epsilon_0 \int_0^{2\pi} d\theta \int_0^a \frac{r dr}{(d_1 - w(r))} \quad (9)$$

$$C_{sensor} = \epsilon_0 \int_0^{2\pi} \int_0^a \frac{r dr d\theta}{(d_1 - w(r))} \quad (10)$$

$$d_1 = g + \frac{t_1 + t_2}{\epsilon_r} \quad (11)$$

Where  $C_{sensor}$  is the capacitance under a differential applied pressure ( $P$ ),  $C_0$  is the Zero-pressure capacitance,  $d$  is the distance between two plates,  $t_1$  and  $t_2$  are defined the thickness of dielectric,  $\epsilon_d$ , is the permittivity of dielectric constant of material. Solve the integral and find the capacitance for un-deflected diaphragm or at zero pressure is given by (12):

$$C_0 = \frac{\epsilon_0 \pi a^2}{d_1} \tag{12}$$

The formula could be written as of **Taylor series** expansion of the ratio of the maximum deflection of the diaphragm to the gap before touch as shown in figures 1(b) and the solution of the (13)  $C_{sensor}$  can be described as follows [11]:

$$C_{sensor} = C_0 \sum_{n=0}^{\infty} \frac{1}{(2n+1)} \left[ \frac{Pa^4}{64Dd_1} \right]^n \tag{13}$$

### 3.2 Capacitance evaluation After Touch Point

Figure 2 shows the cross-view of a touch mode diaphragm suspended over sealed cavity. The total capacitance in touch mode evaluated as of; one is the capacitance introduced by the touch-down of diaphragm noted as  $C_t$ , and the other two  $C_{d2}$ ,  $C_{d3}$  are in serial connection of the air cavity capacitance  $C_{ut}$  and parallel with underlying insulation capacitance  $C_{d1}$ , the result Could be approximated to  $C_{ut}$ . With known touch-point radius  $r_1$  obtained from the FEA simulator, the capacitance  $C_t$  can be calculated directly by using the concept of a parallel capacitor (5).

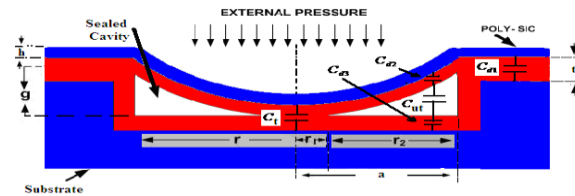


Figure 2: Cross-Sectional View of Touch Mode Capacitance

Next for evaluating  $C_{ut}$ , since the plates are not moving in parallel, so they can not be calculated as of parallel plate concept, due to diaphragm deflection gradient and non-uniform boundary conditions based on applied load, which is normal in electrostatic field. One of the best alternatives for solving  $C_{ut}$  is to simply observe the physical aspects of the axial-symmetry of boundary conditions and to make approximation accordingly [12]. On physical aspect it will look at the fundamental physical laws describing the electric field, using this law will define the definition of capacitance followed by a method to measure the capacitance in a practical situation. The process will lead to a time dependent electromagnetic field and the capacitance could still be determined by using a quasi-static approximation. Since the gap between two electrodes is small, then electric field fringing effects will be neglected. Due to axisymmetry of the diaphragm and underlying substrate, electrical field flux lines are approximated as directional Arcs as shown in Figure3:

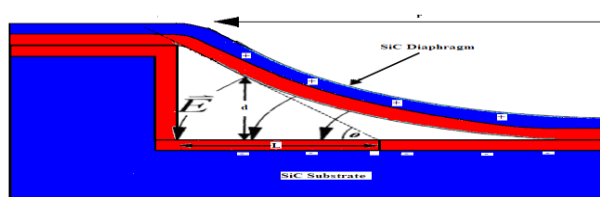


Figure 3: Cross-Sectional View of the Electric Flux lines between two Electrodes

From a physical point of view a capacitive sensing elements is very attractive compared to other types of sensing elements because the physical laws that characterized its behavior are so fundamental. Using Gauss's law defines that the electric displacement  $\bar{D}$  is related to the surface charge density  $\rho$  which is total electric charge density. The electric charge density  $\rho(C/cm^3)$  at a given point on one of the electrodes, given by [12]:

$$\rho = \bar{D} = \epsilon_0 \bar{E} \quad (14)$$

Where,  $\bar{E}$  is the electric field intensity and  $\epsilon_0$  is the permittivity of the air.

Since the electric flux lines are approximated as directional arcs, the electric field intensity is:

$$\bar{E} = \frac{V}{\theta \cdot L} \quad (15)$$

Where  $V$  is defined the applied voltage between the two electrodes.  $\theta$  Is the angle between the tangent through the given point on the diaphragm and the horizontal line  $L$  and is given by:

$$L = \frac{d}{\sin \theta} \quad (16)$$

Where,  $d$  is the vertical distance between a point on the diaphragm and the SiC substrate and  $w(r)$  is the deflection of the given point at radius  $r$  on the diaphragm:

$$d = g - w(r) \quad (17)$$

Using last three equations, the total untouched capacitance (cavity area)  $dC_{ut}$  and underlying dielectric capacitances are approximately equal to  $dC_{ut}$  and evaluated by (18):

$$dC_{ut} = \frac{\rho \cdot A}{V} = \frac{\epsilon_0 \cdot \epsilon_d \cdot A}{\theta \cdot L} = \frac{\epsilon_0 \sin \theta \cdot A}{\theta \cdot d} = \frac{\epsilon_0 \sin \theta \cdot 2\pi r dr}{\theta \cdot d} \quad (18)$$

Total touched point capacitance by increasing pressure:

$$t = t_1 + t_2 \quad (19)$$

$$dC_t = \frac{\epsilon_d \cdot \epsilon_0 \cdot 2\pi r dr}{t} \quad (20)$$

The total Capacitance after touch point  $dC_{TM}$  :

$$dC_{TM} = dC_t + dC_{ut} \quad (21)$$

$$dC_{TM} = \frac{\epsilon_0 \cdot \sin \theta \cdot 2\pi r dr}{\theta \cdot d} + \frac{\epsilon_0 \cdot \epsilon_d \cdot 2\pi r dr}{t} \quad (22)$$

$$C_{TM} = \frac{\epsilon_0 \epsilon_d \cdot 2\pi}{t} \int_0^r r dr + \frac{2\pi \epsilon_0 \sin \theta}{\theta} \int_0^r \frac{r dr}{g - \frac{Pa^4}{64D} \left(1 - \frac{r^2}{a^2}\right)^2} \quad (23)$$

Where,  $r$ , is distance from the center of the diaphragm,  $\epsilon_d$  is the permittivity of the dielectric layer,  $t$  is the total value of dielectric thickness. Finally, by integrating  $C_{TM}$  the total capacitance of touched and after touch mode would be evaluated.

#### IV. Finite Elements Analysis

The above approximate formulas are derived without considering the influence of any fringe filed effects because of the gap of cavity depth is much less than the diaphragm thickness. FEA models were created under Coventor software for simulation. In the modeling it is important to understand the relationship between the applied electrostatic force and the mechanical restoring force. In a typical MEMS device, a voltage bias is applied between two conductors. This voltage bias causes charge migration that generates an attractive electrostatic force between the two conductors. This force will result in the mechanical deformation of the

structure. Deformation will reduce the distance between two conductors. It is this force relationship that leads to the phenomenon of pull-in voltage. CoSolv-EM is a couple solvers for Electro-Mechanics, the tools couple electrostatic and mechanical solvers. In an iterative process, the electrostatic results are input to mechanical solver, and the results are fed back until convergence is reached. The electrostatic load causes the device to deform. In general such deformation will lead to reorganization of all surface charges on the device. We are principally interested in that subset of MEMS devices in which this reorganization of charge is large enough to cause further deformation. Such devices considered to exhibit “coupled electro-mechanical behavior.”

We present some examples on Co Solve-EM, and it is used to evaluate capacitance of the sensors. The test model designed a 180  $\mu\text{m}$  radius circular plate with the different thicknesses, suspended over 0.5, 0.75, 1.0, 2  $\mu\text{m}$  cavity depth. A circular rim is been used to rigid link to inner circle and patched to the ground surface as of clamp. The devices are operating in the high pressure range (0.01 Mpa- 1.7 Mpa) and high-temperature (up to 425°C). The pressure is high enough to cause the plate to touch down on the bottom electrode. Figures 4 Shown the capacitance response of the pressure ranges within 0.01-1.7Mpa and the temperature ranges within 300°K to 700°K for the radius of 180  $\mu\text{m}$ .

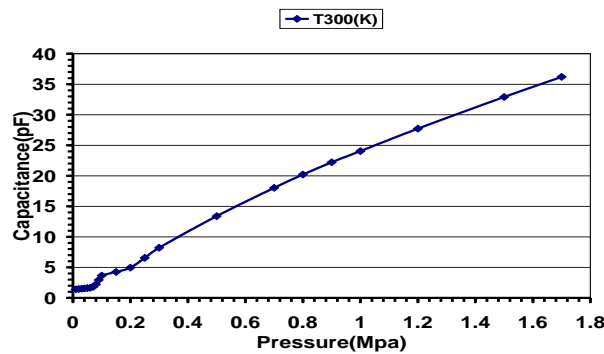


Figure 4: FEA simulation of the capacitive response using Co Solve at 300K temperatures.

### V. Comparison of Analytical and Simulation Results On Capacitance

Discrepancy has been noticed in comparison of the results, that is because of complete material properties, and parasitic capacitances has been considered in software for simulation, which in theoretical has not been considered, and all parasitic capacitances has been neglected. As shows in figure 5(a), the result of capacitance before touch based on applied pressure of 0.01\_0.07MPa with discrepancy of approximately 0.15pF, and figure 5(b) shows the capacitance after touch based on applied pressure of 0.07\_1.7MPa with discrepancy of approximately of 0.02pF.

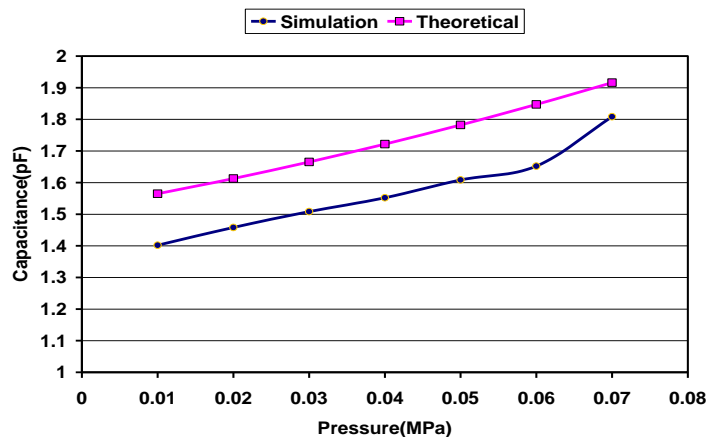


Figure 5 (a): the comparison of capacitance before touch.

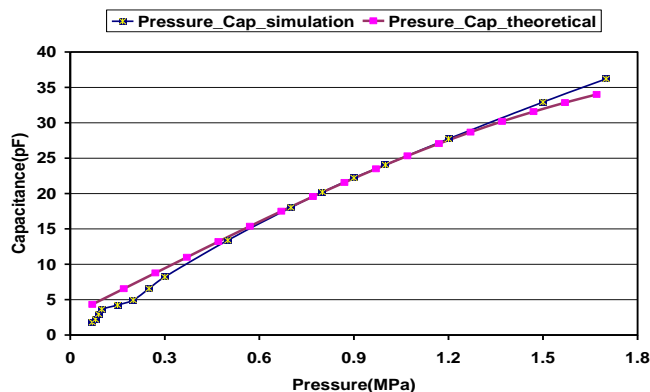


Figure 5 (b): the comparison of capacitance after touch.

### VI. Characteristic Of Operation At Pressure And Temperature

Pressure sensor analysis with considering the temperature effect using FEA simulation has the ability to solve problems related to temperature variation. Users can set temperature related boundary conditions, and the solver can compute temperatures at a surface or through a volume. The simulation on temperature and pressure were performed on the pressure sensors to determine the impact of the temperature effects and to determine the temperature and pressure operating range of the sensors. The simulation results shown that for the pressure sensor temperature compensation techniques will be necessary to correct the pressure data to be as linear as possible. Figure 6, shown the characteristic of pressure vs. capacitance at various temperatures, the changes caused by the parameters is as a result of thermal expansion at elevated temperature. The increase in initial capacitance and sensitivity at higher temperatures are due to the thermal expansion mismatched between the material properties, as noticed in Figure 6, at a certain pressure the moving electrode defects at its maximum point and will contact to the fixed electrode. After touch down the change in capacitance with pressure is quasi-linear, the changes happen based on increasing the area of the contact point rather than the gap depth. Beyond a certain pressure, it will saturates and the sensitivity of the sensor decreases, the problem can be solve by adjusting the design parameters such as diaphragm diameter, diaphragm thickness and the gap depth, dielectric thickness.

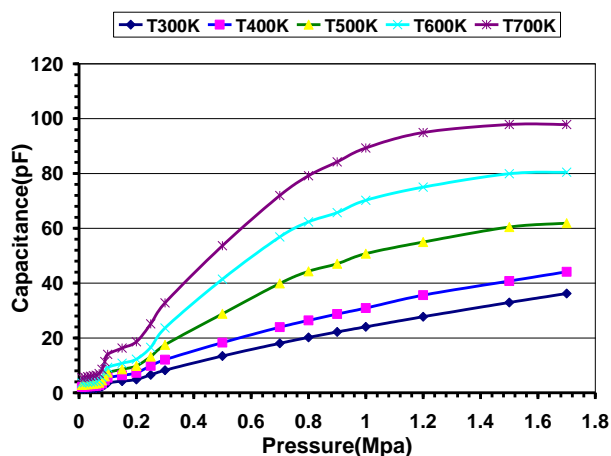


Figure 6: FEA simulation of the capacitive response of a pressure sensor at Various temperatures

There will be four modes of operation by increasing the pressure such as 1: normal mode, it happens due to applied low pressure, figure 7 shows pressure vs. capacitance before touch point(0.01-0.07 Mpa), as it has been noticed the sensor has very slow-slop capacitive response and the sensitivity at room temperature is approximately 58 fF/Psa. 2: transition mode, as shown in figure 8, in this mode the moving electrode start touching the fixed electrode just above the touch point from 0.07-0.1 Mpa, the sensor experience large sensitivity over a short range of pressure with the sensitivity approximately 61 fF/Psa. 3- Third mode, as shown in figure 8, from 0.1-0.25 Mpa is defined as touched area, with increasing the pressure, touched area will

increase, it will become more linear, the value of capacitance will increase, sensitivity decreases suddenly, and the sensor presents sensitivity approximately 12.7 fF/Psa.

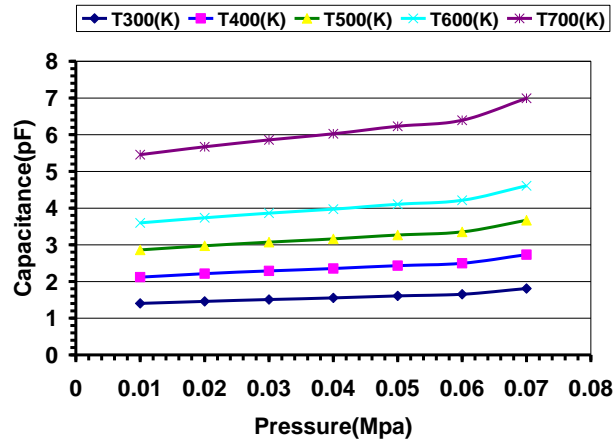


Figure 7: FEA simulation of the capacitive response before touch-mode at Various temperatures

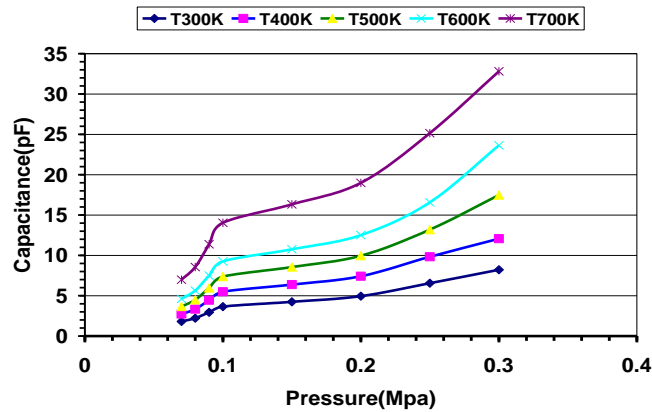


Figure 8: FEA simulation of the capacitive response after touch-mode at Various Temperatures

4- As shown in figure 9, within the pressure ranges 0.25-1.7 Mpa, the shape of the sensing curve at different temperature does not change, the sensor presents sensitivity approximately 19.98 fF/Psa, but it proves that the operation beyond that point is insensitive to temperature, the non-linearity becomes more noticeable and capacitance at higher temperature will start decreasing.

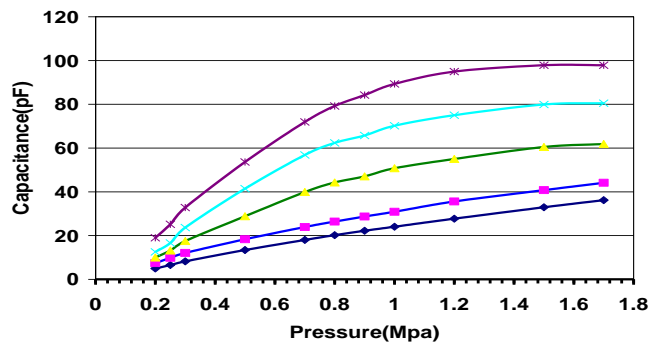


Figure 9: FEA Simulation of the Capacitive Response of the Sensor's Non-Linearity at various temperatures



## VII. Conclusion

The Paper Presented The theoretical and FEA approach to find a solution for evaluating the capacitance variation value with variation of pressure and temperature for before and after touch point. The theoretical equations for evaluation of capacitance value for pressure sensor have been derived. The FEA results shows that the shape of the sensing curve at higher pressure with the variation of temperature does not change noticeably, it proves that beyond this pressure range, the sensor is insensitive to temperature variations and still there will be changes on capacitance value and the non-linearity becomes more noticeable. So far there were no formulas in any literatures to find the values of capacitance at and after touch point. The suggested methodologies are reliable and novel approach.

## References

- [1] M. Mehregany, C. A. Zorman, N. Rajan, and C. H. Wu, "Silicon carbide MEMS for harsh environments," Proc. IEEE, vol. 86, pp. 1594-1610, Aug. 1998.
- [2] M. Suster, W. H. Ko., and D. J. Young, "Optically-Powered wireless transmitter for high temperature MEMS sensing and communication," in proc. Int. conf. solid-state Sensors and Actuators, pp.1703-1706, 2003.
- [3] M. A. Fonseca, J. M. English, M. Von Arx, and M. G. Allen, "Wireless Micromachined ceramic pressure Sensor for high-temperature applications," J. Microelectromech. Syst. pp. 337-343, Mar. 2002
- [4] Wen H. Ko., Qiang Wang, "Touch mode capacitive Pressure sensors," Sensors and Actuators, 75(1999), pp.242-251.
- [5] Hezarjaribi Y., Hamidon M. N., Keshmiri S. H., Bahadorimehr A. R. "Capacitive pressure sensors Based On MEMS, operating in harsh environments" ICSE2008 IEEE International Conference, pp.184-187(2008).
- [6] S. Timoshenko, "Theory of Plates and Shells," McGraw- Hill, New York, 1940, p.343.
- [7] S. Timoshenko, Nonlinear Problems in Bending of Circular Plates, Theory of plates and shells, The Maple Press, 1959, p.308.
- [8] M. Z. sheikh, Dr. S. F. Kodad, Dr. B.C. Jinaga, " Modeling and Simulation of MEMS Characteristics: A numerical integration approach", JATIT, pp.415-418(2008).
- [9] D. Peters, St. Bechtold, R. Laur, "Optimized Behavioral Modes of a Pressure Sensors Including the Touch Mode effect", MSM 99, pp.237-240(1999).
- [10] Guangqing Meng, Wen H. Ko, "Modeling of circular diaphragm and spreadsheet solution programming for touch mode capacitive sensors, "sensors and actuators75 (1999), pp. 45-52.
- [11] Sung-Pil Chang, Jeong-Bong Lee, and Mark Allen, "An 8X8 Robust Capacitive Pressure Sensor Array", Micro- Electromechanical Systems, MEMS, 1998.
- [12] F H Tay, Xu Jun, Y C Liang, V. J. Logeeswaran, and Tao Yufeng, "the effect of non-parallel plates in differential capacitive micro accelerometer", J. Microtech. Microeng, 9(1999), 283-293.