

# Metric Projections to Identify Critical Points in Electric Power **Systems**

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ABSTRACT
The identification of weak nodes and branches involved have been analyzed with different technical of analysis as: sensitivities, modal and of the singular minimum value, applying the Jacobian matrix of load flows. We show up a metric projections application to identify weak nodes and branches with more participation in the electric
power system.
<i>Keywords:</i> Metric projections, Jacobian matrix, critical points, load flows, electric power system.

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#### **INTRODUCTION** I.

The identification of critical points in electric power systems is a problem of great interest, since voltage instability and even voltage collapse [1] - [3] can be reached if the relevant measures are not taken. Hence the importance of identifying critical points of the system in the face of contingencies or the growth of demand.

Knowing weak nodes and branches with greater participation can be taken actions to improve the support of reactive power. For this reason, the metric projections [4] are obtained, identifying different critical points in different scenarios, taking into account the minimum distances for each case with the cut-off values, determining the weak nodes and the branches of greater participation of the system without problems of voltage collapse.Traditionally the techniques known for the identification of weak nodes or branches are analysis of sensitivities, modal analysis and the least singular value, starting from the analysis of the Jacobian matrix [5] -[7]. The analysis of the distances in matrix form has been used to calculate the distances between several cities, locating the distances in matrix form and also the comparison between matrices has been made. Where the measurement of the distances is made based on the Euclidean norm [8].

It has also been used for the identification of leverage points in the state estimators of electric power systems [4] [9] [10]. It is observed that there is a relationship between the leverage points and the nodes and sensitive branches since these are generated by the electrical parameters and the topology of the electrical system [4], [11], [12]. And its structural characteristics are related to the parameters of the electrical systems (transmission lines, transformers). On the one hand the identification of atypical natures of suspect points is made and on the other hand the cut-off values of flat profiles are used as indicators of the limits in which our system is in acceptable voltage levels. In both cases in an n-dimensional system the distance of each point with respect to the total of the points is calculated. It is intended to identify the critical points of an electrical system in response to the demand for reactive power, the analysis considers the steady state study starting from the Jacobian matrix used in the solution of power flows.La capacidad máxima de transferencia de potencia es el límite de estabilidad y en forma tradicional para encontrar este punto se corren flujos de potencia incrementando la potencia reactiva y un punto antes de la no convergencia se considera como el límite de estabilidad.

The maximum power transfer capacity is the limit of stability and in traditional way to find this point power flows study are made increasing the reactive power and a point before the non-convergence is considered as the limit of stability.

#### II. METRIC SPACES

It is called metric space to the set of points of X that associates a real number between them, called distance (Wu, 1990). That is, if the points (x, z) of the set X associate d(x, z), called the distance from x to z, such that: d(x,z) > 0 if  $x \neq z$ d(x,x) = 0d(x,z) = d(z,x) $d(x,z) \le d(x,r) + d(r,z)$  for all  $r \in X$ 

Any function with these four properties is called metric.

#### III. EUCLIDEAN DISTANCE

The Euclidean distance of a vector  $X = [x_1, x_2, ..., x_n]^T$  is defined as follows:

$$\|\mathbf{X}\| = \left[\sum_{i=1}^{n} x_{i}^{2}\right]^{1/2}$$
(1)

This equation provides the length of the vector X. When the distance or the length between two vectors with pairs in space is desired, the above equation must be applied as a difference vector as follows:

#### where:

X and Z are pairs of vectors of order n.

$$\|\mathbf{X} - \mathbf{Z}\| = \left[\sum_{i=1}^{n} (\mathbf{x}_{i} - \mathbf{z}_{i})^{2}\right]^{1/2}$$
(2)

In some cases the Euclidean distance is replaced by a similar expression but eliminating the square root.

$$(\|\mathbf{X} - \mathbf{Z}\|)^2 = \left[\sum_{i=1}^{n} (\mathbf{x}_i - \mathbf{z}_i)^2\right]$$
(3)

#### **IV. METRIC PROJECTIONS**

For the case of a two-dimensional space this concept consists of calculating the distance between two points (x, z), adding up the absolute values which is a variation of the square of the Euclidean norm, where instead of adding the elements of differences squared the absolute values of the sums are added [4]. The distance of the absolute values of a vector  $X=[x_1,x_2,..,x_n]^T$  is given as follows:

$$d_j = \sum_{i=1}^n |x_i| \tag{4}$$

This equation provides the length of the vector X. When the distance or the length between two vectors with pairs in space is desired, the above equation must be applied as a difference vector as follows:

$$d_{ij} = \sum_{i=1}^{n} |x_i - z_i|$$
(5)

#### V. CUT-OFF VALUE

(6)

The cut-off value is intended to identify the points of the projections that are outside the range of the values. It is proposed to determine the cut-off value with the minimum distance in the plane or space as:

$$vc = min\left(d_{ij}\left(X_i, Z_j\right)\right)$$

i = 1, ..., k - 1 j = i + 1, ..., kk = number of points

#### VI. IDENTIFICATION OF CRITICAL POINTS

To identify critical points, we start from the information obtained from the elements  $\partial Q / \partial V$  of the Jacobian matrix of power flows. With these elements of the Jacobian the metric projections of all the points of the system are obtained, the calculations of the metrics include the voltage and the angle of all the nodes of the system. The critical points are those whose metric projection is below the previously established cut-off value. The cut-off value is determined by considering a voltage of 0.90 and an angle of zero degrees at all nodes in the system. This allows us to identify the closest points to the cut-off value, which relates the points that approach or deviate from the cuts of the voltage profiles. These profiles are a range where the system is considered to be within operating limits.

The cut-off value is compared with the metric projections of the Jacobian matrix according to a given load operation scenario. Considering that the critical points are the values that are below the cut-off value, in this case we consider that our system is stable without any problems of voltage.

This was done for a 5-node test system [12], in which different scenarios were generated by performing the variation of reactive power in one or several nodes by comparing the cut-off values against those obtained with the voltage profiles.

#### VII. RESULTS

For the 5-node test system, the cut-off value (vc = 35.93) for a voltage of 0.90 was determined from the elements  $\partial Q / \partial V$  of the Jacobian matrix. Subsequently, the reactive power was varied in nodes 3 (case 5-3) and 5 (case 5-5) in individual form and also the variation of the reactive power in three nodes simultaneously, nodes 3, 4 and 5 was considered (case 5-345).

Thus, Fig. 1, 2 and 3 show V vs Q were obtained in which the variation of the voltage with respect to the reactive power is observed; the Fig. 4, 5 and 6 show D vs Q in which the distances with respect to the cut-off value are appreciated.

Metric projections are considered acceptable if they are above the cut-off value of 35.93. Tables 1, 2 and 3 show the projections below the cut-off value, allowing the identification of the weaker node and the branch with the highest participation.







Table I.					
5-NOE	DE TEST	<b>SYSTEM</b>		Case 5-3	
Reactiv	Reactive power variation				
Metric	Metrics considering the elements $\partial Q/\partial V$ of the Jabobian				
Cut-of	f value =	= 35.93			
Node	Q	D	Branch	V	ANG
1	-200	68.3821	1-2	1.06000	0.00000
2	-200	44.4433	2-5	1.11671	-3.95924
3	-200	68.2390	1-3	1.18936	-7.37301
4	-200	54.3877	4-5	1.17044	-7.33704
5	-200	68.0778	2-5	1.11546	-7.23431
1	250	57.9063	1-2	1.06	0.00000
2	250	35.7590	2-5	0.91127	-1.4725
3	250	45.8376	4-5	0.70503	-1.31275
4	250	36.2550	4-5	0.73815	-2.75166
5	250	55.4321	2-5	0.82525	-5.59059
1	280	51.7028	1-3	1.06000	0.00000
2	280	33.2649	2-5	0.85341	-1.35561
3	280	39.4188	4-5	0.57156	-0.39994
4	280	31.1590	4-5	0.61787	-2.58843
5	280	47.1555	4-5	0.74244	-6.26112

#### Table II.

5-NOE	5-NODE TEST SYSTEM Case 5-5				
Reactiv	Reactive power variation				
Metric	Metrics considering the elements $\partial Q/\partial V$ of the Jabobian				
Cut-of	Cut-off value = 35.93				
Node	Q	D	Branch	V	ANG
1	-191	68.7864	1-2	1.06000	0.00000
2	-191	47.2542	2-5	1.12404	-4.20808
3	-191	66.2682	1-3	1.11023	-6.22002
4	-191	56.6331	4-5	1.12141	-6.70613
5	-191	68.7937	2-5	1.23472	-9.19908
1	150	57.5148	2-5	1.06	0.00000
2	150	34.8334	2-5	0.956	-1.73211
3	150	49.0978	4-5	0.92101	-4.28258
4	150	38.3527	4-5	0.90659	-4.50057
5	150	57.8742	2-5	0.76362	-3.31094
1	190	49.3931	2-5	1.06000	0.00000
2	190	29.5030	2-5	0.88223	-1.40709
3	190	39.5891	4-5	0.83719	-4.42204
4	190	30.4291	4-5	0.8118	-4.64479
5	190	52.9708	2-5	0.56074	-2.14986

### Table III.

5-NODE TEST SYSTEM Caso 5-345					
Reactive power variation					
Metric	Metrics considering the elements $\partial Q/\partial V$ of the Jabobian				
Cut-of	Cut-off value = 35.93				
Node	Q	D	Branch	V	ANG
1	-200	74.5952	1-2	1.06000	0.00000
2	-200	50.9094	1-2	1.23630	-6.3243
3	-200	70.6774	1-3	1.36380	-10.0282
4	-200	66.9613	4-5	1.38260	-10.487
5	-200	75.5099	2-5	1.39020	-11.1369
1	100	52.4348	2-5	1.06000	0.00000
2	100	31.9747	2-5	0.87234	-0.93534
3	100	38.2127	4-5	0.71816	-2.00604
4	100	31.0686	4-5	0.69670	-2.17763
5	100	50.4077	4-5	0.67251	-3.57437
1	109	47.2392	2-5	1.06000	0.00000
2	109	28.7067	2-5	0.81340	-0.72569
3	109	30.612	4-5	0.61800	-1.69764
4	109	25.2694	4-5	0.58940	-1.86073
5	109	41.8144	4-5	0.55300	-3.72888

### VIII. ANALYSIS OF RESULTS

According to Fig. 4, 5 and 6, and to tables 1, 2 and 3 it is observed for the test system:

- 1. Case 5-3: reactive power is varied in node 3 from-200MVAR to 280MVAR. At -200MVAR all metric projections are above the value, the lowest metric at this point is presented at node 2 with a value of 44.4433. It is observed that with a power of 250 MVAR the node 2 has a distance of 35.759 exceeding the cut-off value. In the 280MVAR the cut value is exceeded by nodes 2 and 4 with metrics of 33.2649 and 31.159 respectively, the branch 4-5 has the highest participation and node 4 with the lowest metric. Node 3 presents the lowest voltage of the system with a value of 0.57156 P.U.
- 2. Case 5-5: the reactive power is varied from -191 MVAR to 190MVAR. Metric projections at 150 MVAR start to exceed the cut-off value, the only one being node 2 whose metric of 34.8334 is below the cut-off value, presenting a voltage of 1.00363 P.U. At 190MVAR, metrics of nodes 2 and 4 exceed the cut-off value, presenting values of 29.503 and 30.4291 respectively. Branch 2-5 is the one with the greatest participation. The lowest voltage of 0.56074 P.U. belongs to node 5.
- 3. Case 5-345: the reactive power goes from -200MVAR to 109MVAR. At 100 MVAR, for the first time, metrics exceeded the cut-off value, with nodes 2 and 4 being involved, with a metric of 31.9797 for node 2 and 31.0686 for node 4. At 109 MVAR metrics at nodes 2, 3 and 4 are below the cut-off value with values of 28.7067, 30.612 and 25.2694 respectively, with branch 4-5 being the most participatory. The lowest nodal voltage corresponds to node 5 with a value of 0.553 P.U.

#### IX. CONCLUSIONS

In the analyzed system it is observed that there is a relation between the metric projections and the nodal voltages, since as the voltage decreases in the nodes the metric projections also decrease. If the cut-off value is not exceeded, the system is considered to be stable and in this range there are no voltage problems, however, if the reactive power continues to increase, it cannot be guaranteed that there are no problems in the system solution of convergence.

With the method used it is shown that it is able to identify in a fast and reliable way the weak nodes as well as the involved branches with greater participation, since the algorithm to calculate the metric projections requires little computational effort.

The computational requirements are minimal and the algorithm has the flexibility to be implemented in any power system.

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