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Stochastic Order Level Inventory Model with Inventory Returns and Special Sales

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ABSTRACT
This article deals with a single period inventory model with inventory returns and special sales. The demand is assumed to occur in a uniform pattern during a planning period say, t_p. Both non-deteriorating items and deteriorating items are considered for the discussion.
Keywords: *optimal quantity, disposable cost, cost function, order level system*

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I. INTRODUCTION

The problem of returning or of selling the inventory excess to optimal stock level is considered for an order level inventory system, when the demand is probabilistic in nature. The optimal stock level of an inventory system is smaller than the amount on-hand, and then the question of selling excess inventory arises in many business organizations. That is in any wholesale or retail business the demand of particular product decreases due to launch of new product which is cheaper or superior. This is so happened in the context of Pentium V computers. That means the sales of Pentium V. This type of context can be materialized when a new budget suggests the price increase or due to any other market fluctuations the stock-on-hand of the product becomes more than the optimal stock level. In such instances, the optimum amount to be returned or sold, if any should be determined by balancing the losses due to various costs involved in the system.

Naddor [1] has considered this problem for EOQ inventory system where as Dave [2] has extended for an order level inventory system. We now develop a single period inventory model with probabilistic demand for non- deteriorating and deteriorating items. Inventory models for deteriorating items have been considered by various researchers in the recent past, including Shah & jaiswal [3], Aggarwal [4], Nahamias [5 - 9].In all these models the convention problem of 'how much' 'and 'when to stock 'was discussed under varying assumptions of inventory systems.

Choosing appropriate hypothetical parameter values one can carry out numerical illustration. The applicability of the model requires specific parametric quantification with different distributions. Even with simple functions the cost function becomes complex in nature.In analysing such inventory systems, it is always challenging to estimate the demand distribution. Most of the researchers use parametric approaches for Estimation of the demand distribution.Usually a gamma distribution cf.Moomen l962[10], Burgin[11]; Das[l 2] a lognormal distribution.(Beckmann and Bobkoski [13]; Tadikamalla[14]) and other distributions. Since, with small data sets, it is difficult to reject a null hypothesis of any hypothesis of any of the above standard distribution; it is interesting to study a non-parametric approach. Strijhoch and Heuts [15] have used parametric and non-parametric approximation for the lead time demand distributions and demonstrated that the Kernal density (Non-Parametric approach) estimation approach has considerable advantage over the parametric density estimation in an (Q,s) inventory model with a cost criterion. A separate simulation study on these lines will be carried out highlighting the computational convenient over parametric approach. The pertinent results and analysis will be communicated elsewhere

In this study probabilistic inventory model with inventory returns for non-deteriorating items when the demand will be a random variable X, during the period t_p.

II.MODEL ASSUMPTIONS AND NOTATIONS:

We adopt the following notations in carrying the mathematical model.

(i) X: Random variable representing the demand during the period with known probability density function f(x).
(ii) The scheduling period is a prescribed constant say t_p
(iii) Replenishment rate is infinite, replenishment size is constant.
Lead time is zero. The fixed lot size q_p raises the inventory at the beginning of the scheduling period to the target level S.

(iv) Shortages if any are to be backlogged.
(v) The system starts with an amount of Q units out of which only P units are to be retained, after returning or selling. The problem is to determine the optimal value say P^0 of P.
(vi) The inventory holding cost C_1 per unit time, the shortage cost C_2 per unit time and the returning or selling cost C_4 per unit and known and constant during the planning period t_p.

III.MODEL WITH NON-DETERIROTATING ITEMS:

In this context the period T starts with initial inventory level of Q units and final inventory level of zero. Since (Q – P) units are returned (or) sold, the retained P units will be consumed by the time say t_1 ($t_1 \leq T$) due to demand. During the period (T - t_1) the optimal order level system will operate. Two typical situations may arise in the system depending on the relative values of demand x during a cycle and the quantity retained P units.

Case 1: $x \leq P$ i.e., demand size less than or equal to the quantity retained.

Let Q_1 (t) denotes inventory position at time t ($0 \leq t \leq t_1$). Then the differential equation governing the system during t, is given by

$$\frac{d}{dt} Q_1(t) = -X/t_1 , 0 \leq t \leq t_1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

The solution of the above equation using the boundary conditions

Q_1 (0) =P and Q_1 (t_1) = 0 gives

$$Q_1(t) = [P\text{-}x/t\,(e^t\text{-}1)]\, e^{-1}, \quad 0 \leq t \leq t_1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

To determine average number of units $I_{11}(x)$ carrying inventory per unit time, we assume that Q_1 (t) is approximately linear in t, so that

$$I_{11}(X) = \frac{1}{2}[Q_1(0) + Q_1(t_1)] = \frac{1}{2} P(1 + e^{-t_1}) - \frac{x}{2t_1}(1 - e^{t_1}) \ldots\ldots\ldots\ldots (3)$$

Case 2: x > P i.e., P < X < Q_1. Here we note that maximum quantity that can be returned (or) sold, if any is Q i.e. the optimal value of P must be less than or equal to Q. In this case, suppose that the system carries P units and these P units will be consumed during t_1 period and during (t_1 ,T), the order level inventory system will operate. Then it can be seen that the inventory level Q_1 (t) over (0, T) is given by

$$Q_1(t) = \left[\left\{P - \frac{x}{T}(e^t - 1)\right\} e^{-t}\right] \quad 0 \leq t \leq t_1 \ldots\ldots\ldots\ldots (4)$$

$$= \frac{x}{T}(t_1 - t)\; t_1 \leq t \leq T$$

Since Q_1 (0) = p and Q_1 (t_1) = 0, equation (4) gives

$$t_1 = \log\left(1 + \frac{PT}{x}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

Further since $t_1 < T$, we have

$x > Pt_1 / (e^{t_1} - 1) = P^*$ (say) …………… (6)

the average number of units per unit time during the cycle are

$$I_{12} = \frac{Pt_1}{2T} = \frac{P}{2T}\left[\log\left(1 + \frac{PT}{x}\right)\right] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

Here we use the series form of exponential and logarithms in the above equations (4.3.6) and (4.3.7) we get

$$I_{12} = \frac{P^2}{2x} \text{ and } P^* = P \ldots\ldots\ldots\ldots\ldots\ldots (8)$$

From equations (4.3.3) to (4.3.8) the expected average amount of inventory is

$$K(P) = \frac{1}{T}[C_4 \int_0^P (P - x) f(x)\, dx + C_1 \int_0^P I_{11}(x) f(x) dx + C_4 \int_P^0 (x - P) f(x)\, dx + C_1 \int_P^Q I_{12}(x) f(x) dx + (T - t_1)K(S) \ldots\ldots\ldots (9)$$

Where K(S) is the expected cost function as given by Naddor[l] . In order to reduce the complexity of the various expressions involved in the model we use series form of exponential terms and ignore the terms of second and higher order terms of P we get

$$K(P) = \frac{1}{T}[C_4 \int_0^P (P - x)\, f(x) dx + C_1\{\int_0^P \frac{P}{2}(2 - t_1) f(x) dx + \int_0^P x f(x) dx\} + C_1 \left\{\int_0^P \frac{P^2}{2x} f(x)\, dx\right\} \ldots\ldots\ldots (10)$$

The optimum P, $\frac{\partial K(P)}{\partial (P)} = 0$ yields

$$K(P^{\cdot}) = C_4\left[\int_0^P f(x)\,dx - \int_P^Q f(x)\,dx\right] + C_1\int_0^P f(x)\,dx - C_1\int_0^P \frac{PT}{x} f(x)\,dx + C_1\int_P^Q \frac{P}{x} f(x)\,dx - \frac{C_1 P}{2} f(P) - \frac{C_1 P}{2} Tf(P)$$
$$= 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (11)$$

The integrals involved in equation (10) and (1l) can be evaluated explicitly for some specific probability density functions f(x) only; for those functions we can obtain optimum value of P (say) P° by solving (11) and then minimum total expected cost of the entire system can be obtained by substituting P = P° in equation (10). This is what illustrated in the following example. Let the demand density function is given by

$$f(x) = c[0.002e^{-0.002x}], X > 0$$

= 0 otherwise. …………………………….. (12)

Where C I 0.002 is used to normalize the above function as density function
For this density function equation (11) becomes

$$C_4[q0.002e^{-0.002q} - 0.004Pe^{-0.002P} + e^{-0.002q} - 2e^{-0.002P} + 1] + \frac{C_1}{2}[1 - e^{-0.002P}(0.002P + 1)] +$$
$$C_1P[e^{-0.002P}.e^{-0.002Q}] + \frac{C_1P}{2}(0.002)^2e^{-0.002P} - \frac{C_1P^2}{2}(0.002)^2e^{-0.002P} = 0 .. (13)$$

The solution of the above equation gives optimal quantity to be retained say P0 of P. However, explicit solution in P for the above function is not possible. Hence, to find the optimal value of P say P°, we use Newton-Raphson iterative method for solving the equation (13).

IV. COMPUTATIONAL RESULTS:

We have endeavored to determine what effect varying in the parameter C_4 i.e., disposable cost or special sales over the changes of optimum cost. Such changes can take place due to uncertainties in any business context or they might be influenced by the decisions maker himself. To examine these implications of the model, consider a hypothetical system with the following parameter values. C_1 = 0.65; C_2 = 5.04; R = 2004; T = 0.5; and Q = 4800. All the parameters are expressed in consistent units per unit time. For different values of C_4, we have determined the optimal quantity to be retained and associated costs are summarized in the following table

C_4	t_1	P^0	$K(P^0)$	S	K(S)
0.20	0.104758	272.7947	4056.292	500	2882.321
0.22	0.115979	306.1074	5682.961	500	2882.321
0.24	0.126517	338.1883	7609.305	500	2882.321
0.26	0.136181	368.2986	9716.512	500	2882.321
0.28	0.144946	396.1952	11917.03	500	2882.321
0.30	0.152833	421.7822	14135.35	500	2882.321
0.32	0.159901	445.1115	16317.61	500	2882.321
0.34	0.166229	466.3219	18429.25	500	2882.321
0.36	0.171900	485.594	20450.61	500	2882.321
0.38	0.181582	519.0852	25473.23	500	2882.321

We note that, from equation (13) the optimum value of P^0 of P increases with an increase value of C_4. This is obvious since the higher the retaining or selling cost, the smaller should be the quantity to be returned. Thus the optimum value of P is sensitive with respective to the parameter C_4

V. LINEARITY ASSUMPTION AND ERROR ANALYSIS:

In deriving the model, it has been tactically assumed that the depletion of inventory Level I (t) of the system is linear in t, over the cycle time. In fact, I (t) is a non-linear function of t and the linearity assumption over estimates the average amount in inventory, in this case average amount to be retained and consequently the total cost of the system. This will induce an error in the cost function. However, if this error is too small in magnitude then the assumption of linearity is justified. We now present an error analysis and show that incurred error is significant. Note that, if the linearity function is not made then the expression for average amount to be retained in inventory per unit time during the cycle is given by

$$I_{11}^{e}(t) = \frac{1}{T_1}\int_0^T I_{11}(t)dt = \frac{P}{t_1}(1 - e^T) - x \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (14)$$

Hence the error due to linearity assumption is

$$eI_{11}(t_1) = I_{11}(t) - I_{11}^{e}(t) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (15)$$

Here 1, 1(r) is given in (4.33) and the relative error in total cost REK (T) is given by the ratio

$$REC(t_1) = \frac{K(t_1) - K^E(t_1)}{K^E(t_1)} = \frac{C_1.e(t_1)}{K^E(t_1)} \ldots\ldots\ldots\ldots\ldots (16)$$

where $K(t_1) = C_1 \int_0^{\mu} I_{11}(x) f(x) dx$

$$K^E(t_1) = C_1 \frac{1}{T} \int_0^{\mu} [P(-1 - e^T) - x] f(x) dx$$
$$= C_1 [\int_0^{P} \frac{P(1-e^T)^0}{T}(x) f(x)\, dx - \int_0^{P} x\, f(x)\, dx]$$

Here, $K^E(t_1)$ can be retained for the given model by replacing $I_{11}(t)$ with $I_{11}^{e}(t)$ in equation (l0).By taking C_1 = Re 0.56 per unit per unit time and ranging $C_4 = 0.28$ (0.02) 0.38; different values of $K(t_1)$ (using equation 10), $K^E(T)$ are obtained. The corresponding relative error in total cost (REC) are obtained and summarized in the following table.

Table 1: Error Analysis

C_4	P	$K(t_1)$	$K^E(t_1)$	REK
0.28	396.1952	27.98553	15.86942	0.004228
0.30	421.7822	32.72707	18.7021	0.004086
0.32	445.1115	37.38957	21.51662	0.003989
0.34	466.3219	41.90605	24.26878	0.003922
0.36	485.594	46.23575	26.92975	0.003611
0.38	519.0852	54.26194	31.91789	0.003443

From the above table, it can be observed that the magnitude of the relative error in total cost is very small which shows that the linearity assumption is justifiable.

VI. MODEL WITH DETERIORATING ITEMS:

The probabilistic model is developed with the same assumptions as those given in section 2 except that a constant fraction θ(say) of the on-hand inventory deteriorates per unit time. Depending on the values of P and x of X, the following cases arise:

Case (i): When $x \leq P$

Let Q_1 (t) denote the inventory level of the system at time t ($0 \leq t \leq t_1$) when the demand x unit occurs during the time t_1, then during that period the demand rate is $\frac{x}{t_1}$ and hence differential equation governing the system is

$$\frac{d}{dt}Q_1(t) + Q_1(t) = \frac{-x}{t_1}, (0 \leq t \leq t_1) \ldots\ldots\ldots\ldots\ldots\ldots (17)$$

The solution of (17) gives

$$Q_1(t) = \left[P - \frac{x}{\theta t_1}\{\exp(\theta t) - 1\}\right] \exp(-\theta t)\ (0 \leq t \leq t_1), 0 < \theta < 1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (18)$$

Since $x \leq P$, we must have $Q_1(t_1) \geq 0$ (or) $x \leq P\theta t_1 / \{\exp(\theta t_1 - 1)\} = P^*$(say)

The number of unit, that deteriorate during the cycle is

$$D_1(x) = P - x - Q_1(t_1) = \left(1 - e^{-\theta t_1}\right)\left(P + \frac{x}{\theta t_1}\right) - x, X \leq P \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (19)$$

To determine average number of units $I_{11}(x)$ carried in inventory per unit time we assume that $Q_1(t)$ is approximately linear in t, so that

$$I_{11(x)} = \frac{1}{2}[Q_1(0) + Q_1(t_1)] = \frac{1}{2}P[1 + e^{-\theta t_1}] - \frac{x}{2\theta t_1}[1 - e^{-\theta t_1}], X \leq P \ldots\ldots\ldots\ldots (20)$$

Case (ii): When x>P:

In this case, suppose that the system that in inventory during $(0, t_1)$ an during $(t_1$,T) the optimal order level system will operate; then it can be seen that the optimal inventory level Q_1(t) over (0, T) is given by

$$Q_1(t) = \left[P - \frac{x}{\theta T}\left(e^{\theta t} - 1\right)\right] e^{-\theta t_1}, 0 \leq t \leq t_1 = [I_1(s) + I_2(s)](t - t_1), t_1 \leq t \leq t_1 \ldots\ldots\ldots\ldots\ldots (21)$$

Where $I_1(s)$ and $I_2(s)$ can be had from Shah and Jaiswal [3] since $Q_1(t_1)$ = 0, (4.3.21) gives

$$t_1 = \frac{1}{\theta}\log\left(1 + \frac{PT\theta}{x}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (22)$$

Further, since t_1 < T, we also have in this case

$$x > \theta PT / \{e^{\theta T} - 1\} = P$$

The no. of units that deteriorate during the cycle are

$$D_2(x) = P - \frac{x}{\theta T}\log\left(1 + \frac{PT\theta}{x}\right), x > P \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (23)$$

The average amount in inventory per unit time is

$$I_{12}(P) = \frac{Pt_1}{2T} = \frac{P}{2T\theta}\log\left(1+\frac{PT\theta}{x}\right), x > P \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (24)$$

From (19) and (24), we have

$$D(P) = \int_0^P D_1(x)f(x)dx + \int_0^Q D_2(x)f(x)\,dx$$
$$= \int_0^P Pdt_1(1-\frac{\theta t_1}{2} - x)f(x)dx - \int_P^Q \frac{P^2T\theta}{x}f(x)dx \ldots\ldots\ldots\ldots\ldots\ldots (25)$$

From (4.3.20), (4. 3.24) and (4.325), the expected total cost of the system is given by

$$k(P) = \frac{c_4}{T}\int_0^P (P-x)f(x)\,dx + \frac{c_1}{T}\left[\int_0^P I_1(P)f(x)\,dx + \int_P^Q I_{12}(P)\,f(x)dx\right] + \frac{c_4}{T}\int_P^Q (x-P)f(x)\,dx$$
$$+\frac{c}{T}\left[\int_0^P D\,(P)\,f(X) + \int_P^Q D_2(x)\,f(x)dx\right] + (T-t_1)K(S)\ \ldots\ldots (26)$$

Where K(S) is the expected cost in the order level system and is given by

$$K(S) = ((\theta + c_1 - \frac{1}{2}c_1\theta T)\int_0^{s(1-\frac{1}{2}\theta T)} \left(s-\frac{1}{2}x\right)f(x)dx + \frac{1}{2}s^2(c\theta + c_1 + c_2\theta T)\int_{s(1-\frac{1}{2}\theta T)}^{\infty} \frac{f(x)}{x}dx$$
$$+\frac{c_2}{2}\int_{s(1-\frac{1}{2}\theta T)}^{\infty} \frac{(x-s)^2}{2x}f(x)\,dx - \frac{s^3\theta T}{4}(c_1+2c_1)\int_{s(1-\frac{1}{2}\theta T)}^{\infty} \frac{f(x)}{x^2}dx \ldots.\ldots..(27)$$

Optimum value of S^0 of S, in the solution of the following equation

$$\left(c\theta - \frac{1}{2}c_1\theta T + c_1 + c_2\right)F\left[S\left(1-\frac{1}{2}\theta T\right)\right] + (c\theta + c_2\theta T + c_1 + c_2)\,S\int_{s(1-\frac{1}{2}\theta T)}^{\infty}\frac{f(x)}{x}dx - \frac{3}{4}S^2\theta T(c_1 +$$
$$2c_2)\int_{s(1-\frac{1}{2}\theta T)}^{\infty}\frac{f(x)}{x^2}dx - c_2 = 0 \ldots\ldots\ldots..(28)$$

For the derivations of K(S) in (27) and S in (28) see Shah & Jaiswal [3].The optimum value P^0 of P is the solution of the following equation.

$$\frac{\partial}{\partial P}K(P) = 0$$
$$\Rightarrow K'(P) = \frac{C_4}{T}\left[\int_0^P f(x)\,dx + \frac{PT\theta}{2}f(P)\right] + c_1\left[\int_0^P \left\{1 + \frac{P^2T^2\theta^2}{x^2} - \frac{-PT\theta}{x}\right\}f(x)\,dx + \left\{P + \frac{P^2T\theta}{P}\left(\frac{PT\theta}{P}-1\right)\right\}f(P)\right.$$
$$+\frac{c_1}{2}[\int_P^Q \frac{T\theta}{2}f(X)\,dx - \left(P - \frac{PT\theta}{2}\right)fP)] + \frac{c_4}{T}[-\int_P^Q f(x)\,dx] + \frac{c^P}{T}\int_0^2 [\frac{2PT\theta}{x}\left(1-\frac{T\theta}{2}\right)$$
$$-\frac{T\theta}{2}]f(x)dx + [PT\theta\left(1-\frac{T\theta}{2P}\right) - \frac{PT\theta}{2}]f(P) - \int_P^Q \frac{2PT\theta}{x}f(X)dx + PT\theta f(P). \ldots\ldots\ldots (29)$$

In practice, for obtaining the optimal value of P° of P, we are supposed to evaluate various integrals of $\frac{\partial K(P)}{\partial P} = 0$. It is not possible to obtain explicit expression for some of the integrals. To overcome this difficulty we have used the expansion form of the exponentials and logarithms in the cost function (26) by ignoring the second and higher order levels of6 under the assumption that0 ≤ T. The integrals involved in (29) can be evaluated only for some specific functions f(x) only, for such functions we can determine optimum quantity to be retained say P^0 of P using equation (29) and the corresponding minimum cost can be obtained by substituting P° in (28). This is illustrated with the following example. Let the demand density function be

$$f(x) = \frac{1}{2}x^2 \exp(-x), x \geq 0 = 0 \; otherwise$$

For this demand density, equation (29) becomes

$$K(P) = K_1(P) + K_2(P) + K_3(P) + K_4(P) + K_5(P) + K_6(P) + (T - t_1)K(S) \ldots\ldots..(30)$$

Where

$$K_1(P) = \frac{C_4}{2T}[P\{-P^{x^2}e^{-P^x} - 2P^x e^{-P^x} + 2(1 - e^{-P^x})\} + P^{x^3}e^{-P^x} + 3P^{x^2}e^{-P^x} - 6\{-P^x e^{-P^x}4)1 + e^{-P^x})\}] \ldots\ldots\ldots..(31)$$

$$K_2(P) = c_1 P\left\{(1 - e^{-P^x}) - P^x e^{-P} - \frac{Px^2 e^{-P}}{2}\right\} + \frac{P^2(T\theta)^2}{2}(e^{-P^x} - 1) + \frac{P^2 T\theta}{2}\{e^{-P^x}(1 + P^x) - 1\}\frac{P^2 T\theta}{2}\{e^{-P^x}(1 + P^x) - 1\} \ldots \ldots.(32)$$

$$K_3(P) = \frac{c_1}{2}[-Q^3 e^{-P^x} - P^x e^{-P^x} - 3\{-q^2 e^{-P^x} + P^{x^2}e^{-P^x} + 2\{-qe^{-q} + P^x e^{-P^x} - e^{-q} + e^{-P}\}] \ldots..(33)$$

$$K_4(P) = \frac{c_4}{2}\left[Q^3 e^{-q} - P^{x^3}e^{-P^x} - 3Q^2 e^{-q} - 3P^{x^2} - e^{-P^x} + 6\{-Qe^{-q} + P^x e^{px^2} + (e^{-q} - e^{-P^x})\} - P\left\{-q^2 e^{-q} + P^{x^2}e^{-P^x} + 2\{(-qe^{-q} + P^X e^{-P^x}) + (-e^{-q} + e^{-P})\}\right\}\right] \ldots \ldots.(34)$$

$$K_5(P) = \frac{c}{T}\left[-\frac{P^2 T\theta}{2}e^{-P^x}(P^x + 1) + (PT\theta)^2(e^{-Px} - 1) + \frac{p^{x^2}e^{-P^x}}{4} + \frac{1}{2}P^2 e^{-px} + \frac{e^{-px}}{4}\right] \ldots \ldots.(35)$$

$$K_6(P) = \frac{cP^2 q}{2}[e^{-q}(1 + q) - e^{-px}(1 + P^x)] \ldots\ldots\ldots(36)$$

$$K(S) = \left(c\theta - \frac{1}{2}c_1\theta T + c_1 + c_2\right)\left(S - \frac{3}{4}\right) + \frac{1}{8}[4(c\theta + c_1 + c_2)(S + 3) - c_1 S\theta T(S + 6) - 2c_2\theta T(2S - 3)]\exp(-S) \ldots..(37)$$

and the optimum order level of S^0 ofs is a solution of

$$\left(c\theta - \frac{1}{2}c_1\theta T + c_1 + c_2\right) - \frac{1}{8}(c_4 c\theta T + c_1 + c_2)(S + 2) - c_1\theta T(S^2 + 4S - 6) - 2c_2\theta S(2S - 5)]\exp(-S) = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(38)$$

For derivation of K(S) and S1 see Shah and Jaiswal [3]. The optimum quantity to be retained P° of P is the solution of

$$K'(P_1) + K'(P_2) + K'(P_3) + K'(P_4) = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(39)$$

$$K'(P_1) = \frac{C_4}{2T}[\{-P^{x^2}e^{-P^x} - 2P^x e^{P^x} + 2(-e^{P^x} + 1)\} + \frac{C_4}{2T}\left\{\frac{PT\theta}{2}P^{x^2}e^{-P^x}\right\} + \frac{C_4}{2T}[q^2 e^{-q} - P^x e^{-P^x} - 2\{e^{-P^x}(1 + P^x) - (q + 1)e^{-q}\}] \ldots\ldots\ldots\ldots\ldots.(40)$$

$$K'(P_2) = \frac{C_1}{2}[\{-P^{x^2}e^{-P^x} - 2P^x e^{P^x} + 2(1 - e^{P^x})\} + 3P^2(\theta T)^2 - PT\theta - 2P^x e^{-P^x} + 2(1 - e^{-P^x}) + \{P(T\theta)^2 - PT\theta\}P^x e^{-P^x}] \ldots\ldots\ldots\ldots\ldots.(41)$$

$$K'(P_3) = \frac{C_1}{2}\left[\frac{T\theta}{4}\{-q^2 - e^{-q} + P^x e^{-P^x} + 2\{e^{-P^x}(1 + P^x) - e^{-q}(q + 1)\}\} - \frac{p^{x^3}e^{-P^x}}{2}\left(P - \frac{PT\theta}{2}\right)\right] \ldots(42)$$

$$K'(P_4) = \frac{c}{T}[PT\theta\{1 - (1 + P^x)e^{-P^x}\} + \frac{P(T\theta)^2}{2}(1 - e^{-P^x}) - \frac{T\theta}{4}\{- P^{x^3}e^{-P^x} - 2P^x e^{-P^x} + 2(1 - e^{-P^x}) + \frac{1}{2}P^{x^3}e^{-P^x}\left[\frac{PT\theta}{2} - \frac{(T\theta)^2}{2}\right]$$

$$PT\theta\{(q + 1)e^{-q} - e^{-P^x}(1 + P^x)\} + \frac{1}{2}PT\theta(P^{x^3}e^{-P^x})] \dots \dots .. (43)$$

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VII. DISCUSSION

In this article we have attempted to derive an analytical solution to the optimum quantity to be retained in the context of probabilistic demand. Both non-deteriorating and deteriorating cases were considered. However, a closed form solution could not be obtained. To handle this type of situation one can use a non-parametric approach in estimating demand distribution. In all these cases the Kernel density estimation approach proposed by Strijbosch and Heuts [15], can be used. This suggestion will enhance the further scope of this research problem.

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