I. INTRODUCTION

An optimality criterion is a criterion which summarizes how good a design is and it is maximized or minimized by an optimal design. To estimate the treatment effects with a certain number of restrictions on allotment of treatments to the experimental units, and also to estimate how much information would have been lost by using a smaller number of restrictions, the optimality criteria can be used. Optimality criteria can be used to select best among the available designs belongs to a particular class of experimental design satisfying the conditions, and easy to analyze and are generally optimal only for a specific statistical model.

Kiefer (1959) developed a useful criterion for finding optimum designs based on the Information matrix. The information matrix is proportional to the inverse of the variance-covariance matrix of the least squares estimates of the linear parameters of the model. Some of the alphabetical optimality criteria are: A-, C-, D-, E-, G- optimality criteria.

DEFINITION 1.1: Let D be the class of designs, X be any design belongs to D, is said to be optimal if

\[ \phi \{ \text{Var}(X) \} \leq \phi \{ \text{Var}(X^*) \} \quad \forall X, X^* \in D \]  

where \( \phi \) is a criterion function of information matrix used for estimating the parameters in the model.

DEFINITION 1.2: A design X \( \in D \) is said to be A-optimum in the class of designs D if

\[ \text{Trace} \{ (XX')^{-1} \} \leq \text{Min} \{ \text{Trace} \{ (X'X')^{-1} \} \} \quad \forall X, X^* \in D \]

DEFINITION 1.3: A design X \( \in D \) is said to be C-optimum in the class of designs D, if

\[ \lambda_i / \lambda_n \leq \lambda_i / \lambda_n \quad \forall X, X^* \in D \]

where \( \lambda_i \) and \( \lambda_n \) are the largest and the smallest eigen values of \( XX' \).

DEFINITION 1.4: A design X \( \in D \) is said to be D-optimum in the class of designs D, if

\[ \{ (XX')^{-1} \} \leq \{ \text{Inf} (XX')^{-1} \} \quad \forall X, X^* \in D \]

where \( \{ (XX')^{-1} \} \) indicates the Determinant of \( (XX')^{-1} \).matrix.

DEFINITION 1.5: A design X \( \in D \) is said to be E-optimum in the class of designs D, if

\[ \lambda_{\max} \{ (XX')^{-1} \} \leq \lambda_{\max} \{ XX^{-1} \} \quad \forall X, X^* \in D \]

DEFINITION 1.6: A design X \( \in D \) is said to be G-optimum in the class of designs D if

\[ \text{Min} \{ \text{Var} (\hat{Y}(X)) \} \geq \text{Min} \{ \text{Var} (\hat{Y}(X^*)) \} \quad \forall X, X^* \in D \]

2. INFORMATION MATRICES FOR VARIOUS DESIGNS

In this section an attempt is made to obtain information matrices and optimality values for the experimental designs, Completely Randomized Design, Randomized Block Design, Latin Square Design and Balanced Incomplete Block Design and are presented below with suitable examples.

2.1 COMPLETELY RANDOMIZED DESIGN: The statistical model of Completely Randomized Design \( Y_{ij} = \mu + \alpha_i + e_{ij} \) can be expressed in a general linear model as \( \mathbf{Y} = \mathbf{Xb} + \mathbf{g} \). Where \( \mathbf{Y} = [Y_{11}, Y_{12}, ..., Y_{1n}] [Y_{21}, Y_{22}, ..., Y_{2n}] [Y_{m1}, Y_{m2}, ..., Y_{mn}] \), vector of responses, \( \mathbf{b} = [\mu, \alpha_1, ..., \alpha_m] \) where \( \mu \) is the mean, \( \alpha_i \) is effect due to treatment.
(with ‘k’ treatments), \( \mathbf{e} = [e_{11}, e_{12}, \ldots e_{1n1}, \ldots, e_{i1}, \ldots e_{ijn}, e_{i2}, \ldots, e_{ik1}, \ldots e_{ikn}]' \) is the vector of random errors, follows \( N(0, \sigma^2) \) and \( X \) is the design matrix where

\[
X_{n \times (k+1)} = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
1 & 0 & 1 & 0 & 0 & 0 \\
1 & 0 & 0 & 1 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
1 & 0 & 0 & 0 & 1 & 0 \\
1 & 0 & 0 & 0 & 0 & 1 \\
\end{bmatrix}
\]

\[
X'X = \begin{bmatrix}
N & n_1 & n_2 & \ldots & n_j \\
n_1 & 1 & 0 & 0 & 0 \\
n_2 & 0 & 1 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
n_j & 0 & 0 & 0 & n_{k'} \\
\end{bmatrix}_{(k+1) 	imes (k+1)}
\]

Note: Augment the conditions \( \Sigma \alpha_i = 0 \) in \( X \) under \( H_0 \) to estimate the parameters in the model.

2.2 RANDOMIZED BLOCK DESIGN: The general linear model for a Randomized Block Design is \( \mathbf{Y} = \mathbf{X}\mathbf{\beta} + \mathbf{e} \) Where \( \mathbf{Y} = [y_{11}, y_{1j}, \ldots, y_{1n1}, \ldots, y_{i1}, \ldots y_{ijn}, y_{i2}, \ldots, y_{ik1}, \ldots y_{ikn}]' \) vector of responses, \( \mathbf{e} = [\mu, \alpha_1, \ldots, \alpha_k, \beta_1, \ldots, \beta_n]' \) where \( \mu \) is the mean, \( \alpha_i \) is effect due to \( i \)th treatment, \( \beta_j \) is effect due to \( j \)th block (with ‘k’ treatments and ‘n’ blocks), \( \mathbf{e} = [e_{11}, e_{12}, \ldots, e_{1n1}, \ldots, e_{i1}, \ldots e_{ijn}, e_{i2}, \ldots, e_{ik1}, \ldots e_{ikn}]' \) is the vector of random errors and follows \( N(0, \sigma^2) \) and \( X \) is the design matrix where

\[
X = \begin{bmatrix}
1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\
1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 \\
1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \\
1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \\
\end{bmatrix}
\]

\[
(X'X)^{-1} = \begin{bmatrix}
\frac{-1}{k+n+kn} & \frac{1}{k+n+kn} & \frac{1+k}{(1+k)^2} & \frac{1+n}{k+n+kn} \\
\frac{1+k}{k+n+kn} & \frac{1}{k+n+kn} & \frac{1}{n+k+n+kn} & \frac{1}{k+n+kn} \\
\frac{1+k}{k+n+kn} & \frac{1+k}{k+n+kn} & \frac{1}{k+n+kn} & \frac{1+n}{k+n+kn} \\
\frac{1+n}{k+n+kn} & \frac{1+n}{k+n+kn} & \frac{1+n}{k+n+kn} & \frac{1+n}{k+n+kn} \\
\end{bmatrix}
\]

Note: Augment the conditions \( \Sigma \alpha_i = 0, \Sigma \beta_j = 0 \) in \( X \) under \( H_0 \) to estimate the parameters in the model.

2.3 LATIN SQUARE DESIGN: The general linear model for a Latin Square Design is \( \mathbf{Y} = \mathbf{X}\mathbf{\beta} + \mathbf{e} \), where \( \mathbf{Y} = [y_{111}, \ldots, y_{1jk}, \ldots, y_{1y}, \ldots, y_{i1j}, \ldots, y_{ijk}, \ldots, y_{iy}, \ldots, y_{i1y}, \ldots, y_{i1y}, \ldots, y_{iky}, \ldots, y_{iky}, \ldots, y_{iky}, \ldots, y_{iky}]' \) is the vector of observations where \( y_{ijk} \) is the observation belongs to \( i^\text{th} \) row, \( j^\text{th} \) column and \( k^\text{th} \) treatment (design has ‘v’ rows, ‘v’ columns and ‘v’ treatments), \( \mathbf{e} = [\mu, \alpha_1, \ldots, \alpha_v, \beta_1, \ldots, \beta_v, \gamma_1, \ldots, \gamma_v]' \) where \( \mu \) is the mean, \( \alpha_i, \beta_j, \gamma_k \) are the \( i^\text{th} \) row,
Consider an experimental design conducted in two way blocked design, the responses are locking, RBD and LSD respectively. \( \mathbf{r} = [\varepsilon_{111} \ldots \varepsilon_{1jk} \ldots \varepsilon_{11v}] \) is the vector of random errors follows \( \mathcal{N}(0, \sigma^2) \) and \( X \) is the design matrix where

\[
X = \begin{bmatrix}
1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\
1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
\end{bmatrix}
\]

Note: Augment the conditions \( \Sigma \alpha_i = 0, \Sigma \beta_j = 0, \Sigma \gamma_k = 0 \) in \( X \) under \( H_0 \) to estimate the parameters in the model.

**EXAMPLE 2.1:** Consider an experimental design conducted in two way blocked design, the responses are presented below.

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.10 (C)</td>
<td>5.95 (F)</td>
<td>1.75 (A)</td>
<td>6.40 (E)</td>
<td>3.85 (B)</td>
<td>5.30 (D)</td>
<td>4.80 (B)</td>
<td>3.30 (C)</td>
</tr>
<tr>
<td>3.00 (A)</td>
<td>2.95 (B)</td>
<td>6.70 (E)</td>
<td>5.95 (D)</td>
<td>7.75 (F)</td>
<td>7.10 (C)</td>
<td>5.80 (D)</td>
<td>6.55 (C)</td>
</tr>
<tr>
<td>6.40 (E)</td>
<td>5.80 (D)</td>
<td>6.60 (D)</td>
<td>4.60 (B)</td>
<td>7.00 (E)</td>
<td>5.00 (A)</td>
<td>4.85 (C)</td>
<td>6.65 (E)</td>
</tr>
<tr>
<td>5.20 (F)</td>
<td>4.85 (C)</td>
<td>6.65 (E)</td>
<td>9.30 (F)</td>
<td>4.95 (A)</td>
<td>8.40 (F)</td>
<td>6.65 (E)</td>
<td>9.30 (F)</td>
</tr>
</tbody>
</table>

The optimality values are evaluated in case of CRD, RBD and LSD and are presented in Table 2.2, by assuming the design as CRD by ignoring the blocking, RBD by ignoring the columns as blocks and LSD by consider the two way blocking.

<table>
<thead>
<tr>
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<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>CRD</td>
<td>0.02040</td>
<td>7.8729</td>
<td>0.0029</td>
<td>0.0204</td>
</tr>
<tr>
<td>RBD</td>
<td>1.6875</td>
<td>-</td>
<td>0.0156</td>
<td>1.6875</td>
</tr>
<tr>
<td>LSD</td>
<td>2.5185</td>
<td>-</td>
<td>0.0123</td>
<td>2.5185</td>
</tr>
</tbody>
</table>

Table 2.2

**2.4 BALANCED INCOMPLETE BLOCK DESIGN:** The General Linear Model for a Balanced Incomplete Block Design is \( \mathbf{Y} = \mathbf{X} \mathbf{\beta} + \mathbf{\varepsilon} \). It can be expressed as:
Information matrices and optimality values for various block designs are presented as follows:

\[
\left[
\begin{array}{cccccc}
0 & 0 & 0 & 0 & \cdots & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & 0 & \cdots & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 \\
\end{array}
\right]
\]

Then its \( X'X \) is

\[
X'X = \begin{bmatrix}
bk & rJ_{1,v} & kJ_{1,b} \\
rJ_{1,v} & rI_v & N \\
kJ_{1,b} & N' & kI_b
\end{bmatrix}
\]

The various optimality values for the different parameters of BIBD are evaluated and presented in the following Table 2.3.

<table>
<thead>
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<th>No</th>
<th>V</th>
<th>B</th>
<th>r</th>
<th>k</th>
<th>( \lambda )</th>
<th>E=( \lambda \nu/rk )</th>
<th>A-opt</th>
<th>D-opt</th>
<th>E-opt</th>
<th>G-opt</th>
<th>I-opt</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>6</td>
<td>3</td>
<td>2</td>
<td>1</td>
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<td>1.6660</td>
<td>0.020800</td>
<td>6</td>
<td>0.416</td>
<td>1.66</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>4</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>0.888</td>
<td>3.1100</td>
<td>0.111100</td>
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<td>0.77</td>
<td>3.11</td>
</tr>
<tr>
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<td>9</td>
<td>12</td>
<td>4</td>
<td>3</td>
<td>1</td>
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<td>2.7500</td>
<td>0.000012</td>
<td>12</td>
<td>0.3055</td>
<td>2.75</td>
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<td>8</td>
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<td>1</td>
<td>0.560</td>
<td>1.2050</td>
<td>0.000001</td>
<td>16</td>
<td>0.1338</td>
<td>1.205</td>
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Information matrices and optimality values For various block ...

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Table 2.3

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REFERENCES