

Determining Measurement Uncertainty Parameters for Calibration Processes

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-----ABSTRACT-----

The paper explains the concept of measurement uncertainty attributed to the calibration of measuring and testing instruments used for various industrial functions. All the steps to calculate measurement uncertainty during calibration are described in a way which is easy to understand. It also helps to develop reliable and standardized uncertainty estimates which in turn will provide assurance to the calibration process and reduce disagreements and confusion in scientific findings pertaining to quality of the result. The structured, step-by-step uncertainty analysis for calibration scenarios of instruments such as Micrometer and Pressure Gauge described herein will assist to address the important aspects of identifying measurement process uncertainties and using appropriate uncertainty estimates/models (in accordance with Guide to Uncertainty Measurement – GUM). This will also help to take valid managerial decisions by the measurement quality assurance team.

Keywords - Calibration scenarios, Measurement Uncertainty, Guide to Uncertainty Measurement (GUM), Quality of the result, Uncertainty estimates

Date of Submission: 19 October 2015



Date of Accepted: 01 November 2015

I. INTRODUCTION

It is now widely acknowledged that, when all of the known or suspected components of error have been evaluated and the appropriate corrections are made, there still remains an uncertainty about the correctness of the stated result during calibration, that is, a doubt about how well the result of the measurement represents the value of the quantity being measured. This situation regarding the incompleteness of measurement results, unqualified by uncertainty estimates is evident when two technicians in the same lab determine different measurement results, or different labs determine different results, or when disagreements arise between customer and supplier. In order to make the measurement results full proof, the task was to co-relate the doubt in the result of the measurement with the environmental, surroundings, physical factors causing the doubts. The result was mathematically presented so that a standardized conclusion could be made about the doubts that persist. Accreditation institutions want laboratories to present a standardized, well organized and reliable method of calibration comprising of measurement uncertainty analysis procedure.

The experts nominated by BIPM, IEC, ISO and OIML developed the Guide to Measurement uncertainty- GUM (1995) which gave the basis for estimating measurement uncertainty and the international comparison of the measurement results. But only knowing the basis of uncertainty was of less help, until and unless ISO/IEC 17025: (2000) Requirements for competence of testing and calibration laboratories was released. As the importance of measurement uncertainty has risen over couple of year, it has placed accrediting bodies and laboratories alike in a “catch-up” mode that has led to some hastily decisions and errors in estimating the uncertainty [1]. Different countries use different accreditation policies thereby stressing the need to provide

measurement uncertainty and also the process used to measure it. But all the policies explore the international consensus rules given by GUM and its revisions.

The most important aspect which needs to be determine for measurement uncertainty is whether the errors generated during calibration are Type A or Type B. Howard Castrup (2001) has explained the need to choose proper probability distribution for Type B errors which in turn will justify the quality of the errors. To report any measurement, a level of confidence is important and when we deal with probability distribution, then it becomes more predominant. Suzanne Castrup (2010) had distinguished various methods used to compute the confidence limits for the uncertainty analysis.

So, the aim is to give laboratories source of information regarding uncertainty so that they can enhance their calibration techniques and thereby improve the reliability of measurement/calibration results. Confusion related to the calculation, interpretation and analysis of uncertainties could be reduced.

II. OBJECTIVES

The estimation of a value for uncertainty of a measurement needs our analysis and understanding regarding the interaction of the individual, the equipment and the environment to determine the manner in which they contribute to the measurement error and the expected magnitude of their contributions.

In this context, objective of the project is to develop standardized parameters to calculate and interpret the measurement uncertainties which occur during calibration of testing and measuring instruments.

The paper in a whole attempts

- i. To provide a comprehensive resource for all technical personnel responsible for estimating and reporting measurement uncertainty.
- ii. Explanation of Measurement uncertainty in a way that can be readily understood and interpreted by others.
- iii. To report at a minimum, the measured value, the combined standard uncertainty, its estimate type (A, B or A/B) and degrees of freedom.
- iv. To develop an uncertainty budget model keeping in mind an associated confidence level and expanded uncertainty with associated coverage factor.

III. METHODOLOGY

3.1 Basic Concepts

A measurement is a process where the value of a quantity is estimated. The quantity put under calibration measurement is called as “measurand”. Errors occur in each and every measurement. Some are noticeable while some need to be deduced. Our lack of knowledge about the sign and magnitude of measurement error is called *measurement uncertainty*. A measurement uncertainty estimate is the characterization of what we know statistically about the measurement error. Therefore, a measurement result is only complete when accompanied by a statement of the uncertainty in that result. If all of the quantities on which the result of a measurement depends are varied, its uncertainty can be evaluated by statistical means. However, because this is rarely possible in practice due to limited time and resources, the uncertainty of a measurement result is usually evaluated using a mathematical model of the measurement and the law of propagation of uncertainty [2].

The general uncertainty analysis procedure consists of the following steps:

3.1.1 Define the Calibration Process

The first step in any uncertainty analysis is to identify the physical quantity that is measured. This quantity, sometimes referred to as the “**measurand**,”[2] may be a directly measured value or derived from the measurement of other quantities. At this initial stage of the analysis, it is important to describe the test setup, environmental conditions, technical information about the instruments, reference standards, or other equipment used and the entire procedure for obtaining the measurement(s).

3.1.2 Develop the Uncertainty Model

An uncertainty/error model is an algebraic expression that defines the total error in the value of a quantity in terms of all relevant measurement process or component errors. The error model for the quantity [3]

Q defined is :

$$E_q = C_a E_x + C_b E_y + C_c E_z$$

Where,

E_q = error in q

E_x = error in measure quantity x

E_y = error in measure quantity y and E_z = error in measure quantity z

C_a , C_b and C_c are sensitivity coefficients that determine the relative contribution of the errors in x, y and z to the total error in q. The sensitivity coefficients are defined below

$$c_a = \left(\frac{\partial q}{\partial x} \right), c_b = \left(\frac{\partial q}{\partial y} \right), c_c = \left(\frac{\partial q}{\partial z} \right)$$

Each partial derivative is called a sensitivity coefficient. It equals the partial derivative of the function $f(X_1, X_2, \dots, X_N)$ with respect to X_i , evaluated at $X_1=x_1, X_2=x_2, \dots, X_N=x_n$. It represents the sensitivity of y to changes in x_i , or the ratio of the change in y to a small change in x_i .

They are the essential conversion factors that allow one to convert the units of an input quantity into the units of the measurand.

• Example, If measurand is “Pressure” (measured in Pa) and if temperature (measured in degrees Celsius, °C) is an input quantity. So to convert the temperature into a resistance, multiply the temperature by some constant c with units of Pa/°C.

3.1.3 Identify Measurement Error/uncertainty Sources and Probability Distributions

The Errors that occur during measurement/calibration process are the main source of uncertainty analysis. Only after identification of these fundamental errors, the uncertainty estimates for the entire process could be developed.

The errors most often encountered in making measurements include, but are not limited to the following:

a. Uncertainty of the Reference (Master)

During calibration, the unit under test is compared with the master instrument. Error of the Master equipment itself is the reference attribute uncertainty. This excludes resolution error, random error, operator bias and other error sources that are not properties of the attribute.

b. Repeatability

Repeatability is differences in measured value from measurement to measurement during a measurement/calibration process.

c. Resolution Error

The smallest distinguishable value indicated in a measurement comprises the resolution of the measurement. For example, a Ammeter may indicate values to four, five or six significant digits.

d. Uncertainty due to Operator and his/her positioning

In reality, operator bias has a somewhat random character due to inconsistencies in human behaviour and response. It sometimes happens that two operators observing the same measurement result will systematically perceive or produce different measured values. Also, There are processes where 2 or more systematically people are required for taking the readings. This also results in operator bias uncertainty.

e. Environmental Factors Error

Errors can result from variations in environmental conditions, such as temperature, vibration or humidity etc. Additional errors are introduced when measurement results are corrected for environmental conditions. For example, when correcting a length measurement for thermal expansion, the error in the temperature measurement will introduce an error in the length correction.[4]

f. Errors during Computation of the result data

Data processing errors result during computation round-off, numerical interpolation of observed values, or the use of curve fit graphs and equations.

3.1.4 Estimate Standard Uncertainties

In order to eliminate the discrepancies which occur today while estimating uncertainties, a full proof and substantiate method is used to divide uncertainty in different types and then combine the same to make the concepts more understandable.

A) Type A uncertainty

It is estimated on the basis of repeated observations. Applies to those situations where several independent observations have been made. It Involves Data Sampling and Analysis. Here, the uncertainty is equal to the standard deviation of the measurements taken. Ex: Repeatability may be estimated as the standard deviation of set of repeated measurements. This is one example of Type A uncertainty.

B) Type B uncertainty

It is obtained by previous measurement data, experience with or general knowledge of the behavior and properties of relevant materials and instruments, manufacturer's specifications, data provided in calibration and other certificates and uncertainties assigned to reference data taken from handbooks. Type B uncertainty can only be defined in terms of probability/ level of confidence. Any measurement will have some uncertainty and the quoted interval will be the range within which the true value lies at a certain level of confidence. Type B uncertainty evaluation involves estimating a bound, *a*, for the largest possible error in the estimate, *x_i*, then dividing the bound ‘a’ by an appropriate constant based on an assumed distribution for the error. The probability distribution for a type of measurement process error is a mathematical description of how likely an error or a range of errors is likely or unlikely to occur.

C) Type of Probability distribution considered for Type B uncertainty

i. Normal Distribution

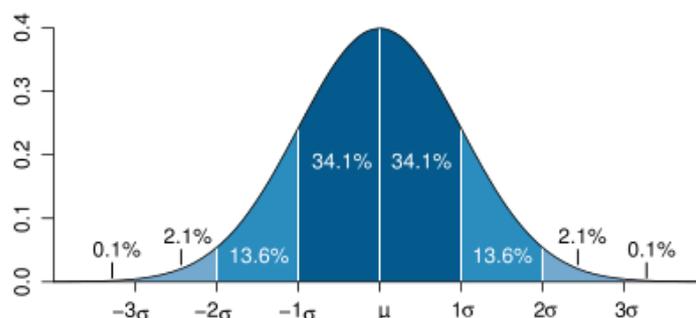


Fig. 1 : Normal distribution

It is used in some situations, the quoted uncertainty in an input or output quantity is stated along with level of confidence. Also, in the absence of any specific knowledge about the type of distribution, I have taken it to be normal distribution. It is also considered when the uncertainty in a calibration certificate is given as a confidence interval or in terms of standard deviation multiplied by coverage factor.

The percentage values of the covered area of the curve are the confidence level, which is an indication of Coverage factor K, as per table -1

Confidence level	67.27%	90%	95%	95.45%	99%	99.73%
Coverage factor K	1.000	1.645	1.96	2.000	2.576	3.000

Table 1: Coverage factor for various confidence level

ii. Rectangular Distribution

It is used where it is possible to estimate only the upper and lower limits of an input quantity (X) and there is no specific knowledge about the concentration of values of (X) within the interval.

The rectangular distribution can always be justified as it represents the worst case scenario. If the true value lies within $\pm a$ (Ex: $W=215.05 \pm 0.5\text{kg}$, Then $a=0.5$) of the estimated value, x_i , but nothing more is known than that, assume a **Rectangular Distribution**, and divide *a* by $\sqrt{3}$ to obtain $u(x_i)$.

Hence, $u(x_i) = a / \sqrt{3}$ Where $u(x_i)$ = uncertainty to be calculated of quantity x [4]

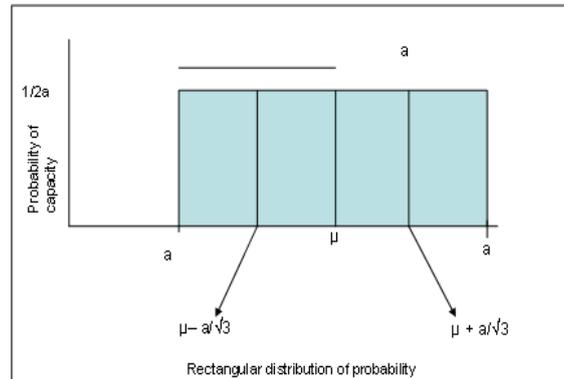


Fig. 2 : Rectangular Distribution

3.1.5 Combining Standard Uncertainties

$$u_x = \sqrt{u_1^2 + u_2^2 + 2\mu_1\mu_2\partial}$$

Equation for Standard combined uncertainty in x where

The correlation co-efficient $\partial = 0$ when the source of errors is independent of each other. This equation confides the law of propagation of uncertainty mentioned by GUM.

3.1.6 Final Step – Expanded Uncertainty

The Expanded uncertainty is intended to produce an interval about the result that has a high probability of containing the (true) value of the measurand. Expanded uncertainties provide intervals that consider a larger fraction of the *measurand* value distribution, compared to that of the combined uncertainty. The combined standard uncertainty is in the form of one standard deviation and therefore may not provide sufficient confidence [Laboratory Accreditation Bureau (2001)]. For this reason the expanded uncertainty U is calculated by multiplying the standard uncertainty by a coverage factor k as follows:

$$U = k \cdot u(y)$$

Where $u(y)$ = Combined Standard Uncertainty

IV. EXPERIMENTS AND CALCULATIONS

4.1 Uncertainty Analysis for Micrometer Calibration

The micrometer that was subjected to calibration had the following characteristics:

Range of Measurement: 0-25mm [5]

Resolution: 0.01mm

Type of measurement Display = Analog

The gage block served as the “master” was made of a special grade of steel, which is capable of being hardened, and which will retain a high degree of dimensional stability.

4.1.1 Data collection and Uncertainty Analysis

i. Uncertainty of Master: Calibration is done by comparing the result of the measurand (device under calibration) with the master. So, uncertainty of the master itself needs to be taken into consideration. Here the master is gage blocks. The calibration certificate for the gage block indicates that it has an expanded uncertainty of $0.1\mu\text{m}$. Since the standard uncertainty is not determined statistically, there are no degrees of freedom. For the same reason it is a Type B uncertainty. It will follow Normal distribution as mentioned earlier.

Therefore, Standard Uncertainty = $0.1/2 = 0.05 \mu\text{m}$

ii. Uncertainty due to Repeatability: 5 repeated measurements of the gage block were taken with the micrometer and the standard deviation (standard error) of these values was determined to be $0\mu\text{m}$. Also since it is statistically determined it is a Type A uncertainty. The measurements produce a standard deviation of mean which is calculated as $S_x = S_d / \sqrt{5}$ [6]

iii. Uncertainty of Resolution: In an analogue instrument the effect of resolution is determined by the practical ability to read the position of a pointer on a scale [7]. The micrometer reads to the nearest $10\mu\text{m}$. The expanded uncertainty considered here is one half of the resolution, $5\mu\text{m}$. This is a Type B uncertainty with no degrees of freedom since it is not determined statistically. The distribution is “Rectangular” since the actual reading that is rounded off to determine the displayed value is equal to the displayed value $\pm 5\mu\text{m}$. The divisor for a “Rectangular” distribution is $\sqrt{3}$ as discussed earlier. The standard uncertainty is $5/\sqrt{3} = 2.887\mu\text{m}$.

iv. Uncertainty due to environmental temperature control: The laboratory temperature is controlled to $\pm 1^\circ\text{C}$ however the thermometer is not exact and we need to determine the uncertainty associated with it. The laboratory temperature range is $20^\circ\text{C} \pm 2^\circ\text{C}$. The following basic equation to calculate the resulting uncertainty associated with the inaccuracy of the thermometer is used.

$$\Delta L = L * \Delta T * \alpha$$

Where, ΔL = error in length

L = Nominal length of the gauge block

$\Delta\alpha$ = change in the coefficient of thermal expansion between micrometer and gauge block

As, both micrometer spindle and gauge block are made of steel, change in thermal expansion between is supposed to be **0**. Therefore, Contribution to uncertainty is also zero.

v. Uncertainty due to temperature differential: If the temperature of the micrometer and the gage block are not the same they will experience unequal thermal expansion or contraction and this will produce an error in the measurement. Laboratory procedures must address the need for both items to be at the same temperature. To reduce this error, the instruments for test were placed in the controlled environment of the laboratory a minimum of 24 hours prior to measurement so that thermal equilibrium may be attained. Small fluctuations are still possible as the temperature control system of the laboratory makes slight adjustments to maintain the stated temperature of $20^\circ\text{C} \pm 2^\circ\text{C}$. we can reasonably expect that under laboratory conditions the temperature differential does not exceed 0.5°C . Uncertainty Contribution for such type of difference between block and micrometer is also calculated by the same equation for error in length which is:

$$\Delta L = L * \Delta T * \alpha - (0.025 * 11.5 \text{ act as a sensitivity coefficient})$$

$$\text{Uncertainty contribution} = 0.025\text{m} * 0.5^\circ\text{C} * 11.5\mu\text{m}/\text{m}^\circ\text{C} = 0.14375\mu\text{m}$$

It is a Type B uncertainty therefore there are no degrees of freedom. Again it is unlikely that the worst case as calculated will always happen so consider this as a “Rectangular” distribution (we know the range of fluctuation) for which the divisor is $\sqrt{3}$. The standard uncertainty is $0.143/\sqrt{3} = 0.083\mu\text{m}$

vi. Uncertainty of CTE (Coefficient of Thermal Expansion): The micrometer and the gage block are made of steel and the commonly stated value for CTE is of $11.5\mu\text{m}/\text{m}^\circ\text{C}$ length per degree Celsius. Although this value is satisfactory for most engineering calculations, it is not an exact value and when considering precise measurements it is important to consider the uncertainty associated with the value for CTE. It is reasonable to expect that this value might vary by as much as 10% of length per degree Celsius. To calculate the resulting uncertainty associated with the CTE

Equation again used is the same i.e.

$$\Delta L = L * \Delta T * \alpha - (0.025 * 2 \text{ act as sensitive coefficients})$$

$$\text{Uncertainty Analysis} = 0.025 * 2 * 1.150 = \text{Expanded Uncertainty}$$

$$\text{Standard Uncertainty} = 0.025 * 2 * 1.150 / \sqrt{3} = 0.033\mu\text{m}$$

1. Purpose : **Determine Uncertainty and Prepare Uncertainty Budget**

2. Device Under Calibration **External MicrometerM187** Length= 0.025m
 Range **0.25 mm**
 L.C. **0.01 mm**

3. Standards / Equipment Used for Calibration:

Sr. No	Standard / Equipment	Range	LC	Uncertainty in	Accuracy
1	SOMET Slip Gauge set	2.5 to 25 mm	Grade : 0	0.1 μm	0.2 μm

4. **Type A**

Measured reading on digital pressure gague						Average	Standard deviation
Reading 1	Reading 2	Reading 3	Reading 4	Reading 5			
2.500	2.50	2.50	2.50	2.50	2.50	2.5	0.00
5.100	5.10	5.10	5.10	5.10	5.10	5.1	0.00
10.300	10.30	10.30	10.30	10.30	10.30	10.3	0.00
15.000	15.00	15.00	15.00	15.00	15.00	15	0.00
20.200	20.20	20.20	20.20	20.20	20.20	20.2	0.00
25.000	25.00	25.00	25.00	25.00	25.00	25	0.00

max. standard deviation 0.00 μm

5. Standard deviation of mean $S/\sqrt{5}$ **0.00 μm**

6. Degree of Freedom n-1 4

Fig. 3 : Type A Uncertainty Estimation For External Micrometer

TYPE B CONTRIBUTION

Source of Uncertainty	Distribution	Calculation	Value
UB1 Accuracy of Carbide Slip Gauge Set Value (width of distribution) 0.20 μm	Rectangular (Value / $\sqrt{3}$)	Calculation = 0.200 /1.7321 μm 0.115 μm	0.115 μm
UB2 Uncertainty of Measurement in Slip Gauge set reported in Calibration certificate Value (width of distribution) 0.1 μm	Normal (Value / 2)	Calculation = 0.100 /2 μm 0.050 μm	0.050 μm
UB3 Uncertainty due to resolution of Unit under calibration (50% of L.C. 10 μm) Value (width of distribution) 5 μm	Rectangular (Value / $\sqrt{3}$)	Calculation = 5.000 /1.7321 μm 2.887 μm	2.887 μm
UB4 Uncertainty due to environmental temperature control (20°C +/- 2°C) Value (width of distribution) 2 °C	Rectangular (Value / $\sqrt{3}$)	Calculation = 2.000 /1.7321 °C 1.155 °C Sen. Co-eff.= 0.025 m x (11.5-11.5) $\mu\text{m}/\text{m}\cdot\text{C}$ 0.000 $\mu\text{m}/\text{C}$	0.000 μm
UB5 Uncertainty due to difference in temp. between UUC & STD. after stabilization Value (width of distribution) 0.5 °C	Rectangular (Value / $\sqrt{3}$)	Calculation = 0.500 /1.7321 °C 0.289 °C Sen. Co-eff.= 0.025 m x 11.5 $\mu\text{m}/\text{m}\cdot\text{C}$ 0.288 $\mu\text{m}/\text{C}$	0.083 μm
UB6 Uncertainty due to thermal expansion co-efficient of UUC (10% of 11.5) $\mu\text{m}/\text{m}\cdot\text{C}$ Value (width of distribution) 1.15 $\mu\text{m}/\text{m}\cdot\text{C}$	Rectangular (Value / $\sqrt{3}$)	Calculation = 1.150 /1.7321 $\mu\text{m}/\text{m}\cdot\text{C}$ 0.664 $\mu\text{m}/\text{m}\cdot\text{C}$ Sen. Co-eff.= 0.025 m x 2°C 0.050 m·C	0.033 μm
UB7 Uncertainty due to thermal expansion co-eff. of STD. (10% of 11.5) $\mu\text{m}/\text{m}\cdot\text{C}$ Value (width of distribution) 1.15 $\mu\text{m}/\text{m}\cdot\text{C}$	Rectangular (Value / $\sqrt{3}$)	Calculation = 1.150 /1.7321 $\mu\text{m}/\text{m}\cdot\text{C}$ 0.664 $\mu\text{m}/\text{m}\cdot\text{C}$ Sen. Co-eff.= 0.025 m x 2°C 0.050 m·C	0.033 μm

Combined Standard Uncertainty (Uc) = 2.8909833 μm

Coverage Factor K = 2 at 95% Confidence Level

Expanded Uncertainty (Uc x K) \pm 5.7819665 μm

Reporting the Result 25mm \pm 5.78 μm

Fig. 4: Type B and Expanded Uncertainty for Calibration of External Micrometer.

By mentioning the contribution of each factor on the uncertainty analysis, a clear view is developed so that proper care could be taken in future. In short, it shows the maximum impact of the factors contributing uncertainty [8]

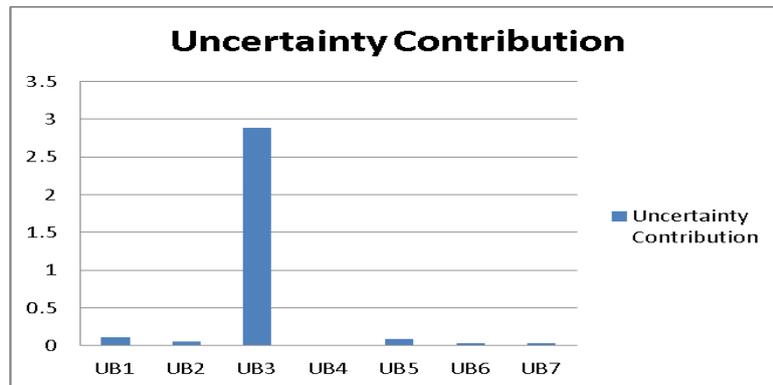


Fig. 5: Uncertainty contribution for calibration of External Micrometer

4.2 Uncertainty Analysis for Pressure gauge Calibration

Important Note:

- i. Before Calibration, A zero Reading at the Unit under Calibration while the hydraulic is opened to an atmosphere after exercise shall be observed
- ii. The number of the pressure calibration points of the UUC can be determined according to the requirement in the DKD R-6-1: 2002. [9]

Important formulae

To specifically find the uncertainty contributed by change in gravity, temperature and change in altitude, following are the formulae which were found to be useful when taken into consideration.

$$u_{gravity} = (p / g) \sqrt{\frac{1}{3} a_g^2} \quad u_{temp} = -(\alpha + \beta) * p * \sqrt{\frac{1}{3} a_t^2}$$

$$u_{\Delta h} = (\Delta p \cdot g) \sqrt{\frac{1}{3} a_h^2}$$

The detailed calibration readings and uncertainty calculation is showed below:

i. Uncertainty Contributed by Accuracy of DWT (Dead Weight Tester)

Width of Distribution = 0.05% * P = 0.05 % * 60.5 (0.05% from the certificate) = 0.03025 bar

Probability Distribution = Rectangular

Uncertainty U(xi) = Width / $\sqrt{3}$ = 0.03025 / $\sqrt{3}$ = 0.0175 bar

Sensitivity coefficient (Ci) = 1 . Therefore, Uncertainty contribution, Ub1 = U(xi) * Ci = 0.0175 bar

i. Uncertainty of measurement in DWT (Dead Weight Tester)

Width of Distribution = 0.006% * P = 0.006 % * 60.5 = 0.00363 bar

Probability Distribution = Normal Uncertainty U(xi) = Width / 2 = 0.00363/2 = 0.00182

Sensitivity coefficient (Ci) = 1

Therefore, Uncertainty contribution, $U_{b2} = U(x_i) * C_i = 0.00182 \text{ bar}$

ii. **Uncertainty due to change in gravity**

Acceleration due to gravity with worst case scenario = 9.7865244 ± 0.00097 – (0.01% of the ‘g’)

Width of Distribution = 0.00097 ms^{-2}

Probability Distribution = Rectangular

Uncertainty $U(x_i) = \text{Width} / \sqrt{3} = 0.00097 / \sqrt{3} = 0.00056 \text{ ms}^{-2}$

Sensitivity coefficient (Ci) = $p/g = 60.5/9.78652 = 6.18197 \text{ bar/ms}^{-2}$ Therefore, Uncertainty contribution,

$U_{b3} = U(x_i) * C_i = 0.00346 \text{ bar}$

iii. **Uncertainty due to thermal expansion of coefficient**

Width of Distribution = ± 2

Probability Distribution = Rectangular

Uncertainty $U(x_i) = \text{Width} / \sqrt{3} = 2 / \sqrt{3} = 1.155 \text{ k}$

Sensitivity coefficient (Ci) = $-(\alpha+\beta)*p = 0.000011 * 2 * 60.5 \text{ bar/k}$ Therefore, Uncertainty contribution,

$U_{b4} = U(x_i) * C_i = -0.00077 \text{ bar}$

v. **Uncertainty due to Hysteresis**

Width of Distribution = 0 (Max hysteresis-fluctuation)

Probability Distribution = Rectangular

Uncertainty $U(x_i) = \text{Width} / 2\sqrt{3} = 0 / \sqrt{3} = 0$

Sensitivity coefficient (Ci) = 1 . Therefore, Uncertainty contribution, $U_{b5} = U(x_i) * C_i = 0$

vi. **Uncertainty due to zero offset of Pressure gauge**

Width of Distribution = 0 bar

Probability Distribution = Rectangular

Uncertainty $U(x_i) = \text{Width} / \sqrt{3} = 0 / \sqrt{3} = 0 \text{ bar}$

Sensitivity coefficient (Ci) = 1 . Therefore, uncertainty contribution, $U_{b6} = U(x_i) * C_i = 0 \text{ bar}$

vii. **Expanded Uncertainty**

Expanded Uncertainty = k (Coverage factor) * Standard uncertainty

Standard uncertainty [10] = $\sqrt{u_{b1}^2 + u_{b2}^2 + u_{b3}^2 + u_{b4}^2 + u_{b5}^2 + u_{b6}^2} = 0.018 \text{ kg/cm}^2$

Now, $k = 2$ for 95% confidence level . Therefore, Expanded Uncertainty = $2 * 0.018 = 0.04 \text{ kg/cm}^2$

Evaluation of uncertainty during calibration of Pressure Gauge

1. Purpose: Uncertainty budget for Pressure Gauge Calibration

2. Device Under Calibration Analogue Pressure gauge
 Range 0 to 60 kg/cm² Full Scale = 60 kg/cm²
 Least 1 kg/cm²
 Count

3. Standards / Equipment Used for Calibration:

S. No	Standard / Equipment	Range	Uncertainty in measurement(%)	Accuracy (%)
1	Dead weight Pressure Gauge	20 TO 700 kg/cm ²	0.006	0.05

4. **Type A (all value in kg/cm²) Type A taken from repeatability of measurement at max. error point**

Which is enclosed this.

Measured reading on digital pressure gauge				Average	Standard deviation
Cycle 1		Cycle 2			
0.0	0.0	0.0	0.0	0.00	0.000
10.50	10.50	10.50	10.50	10.50	0.000
20.50	20.50	20.50	20.50	20.50	0.000
30.50	30.50	30.50	30.50	30.50	0.000
40.50	40.50	40.50	40.50	40.50	0.000
50.50	50.50	50.50	50.50	50.50	0.000
60.50	60.50	60.50	60.50	60.50	0.000

Hysteresis (kg/cm ²)	
Cycle 1	Cycle 2
0.0	0.0
0.0	0.0
0.0	0.0
0.0	0.0
0.0	0.0
0.0	0.0
0.0	0.0

Maximum Standard Deviation 0.00 kg/cm²
 Maximum Hysteresis 0.00 kg/cm²

5. Standard deviation of mean $S / \sqrt{4}$ **0.0000** kg/cm² Gravity(m/s²) 9.786524 ± 0.00097 n-1 3

Fig. 5: Type A uncertainty for Pressure Gauge Calibration

Fig. 6: Type B & Expanded Uncertainty for Pressure Gauge Calibration

Type B distribution		Degree of Freedom	Calculation	Uncertainty
UB1	Accuracy of Dead Weight Tester Value (Width of Distribution) $60.5 \times 0.05\% = 0.03025$	Rectangular (Value / $\sqrt{3}$)	Calculation = $0.03025 / 1.7321$ 0.02 kg/cm^2	0.0175 kg/cm^2
UB2	Uncertainty of Measurement in DWT reported in Calibration certificate Value (Width of Distribution) $60.5 \times 0.006\% = 0.00363$	Normal (Value / 2)	Calculation = $0.00363 / 2$ 0.001815 kg/cm^2	0.001815 kg/cm^2
UB3	Uncertainty of gravitational force "g" applied on DWT (0.01% change-worst case) Value (Width of Distribution) ± 0.00097	Rectangular (Value / $\sqrt{3}$)	Calculation = $0.00097 / 1.7321$ 0.00056 Sensitivity Coeff. = $P/g = 60.5/9.788 = 6.2$ Uncertainty = $0.0056 \times 6.2 = 0.00347209$	0.00347 kg/cm^2
UB4	Uncertainty due to temperature variation Temp. Variation (+/- °C): 2 Value (Width of Distribution) 2	Rectangular (Value / $\sqrt{3}$)	Calculation = $2 / 1.7321$ 1.155 Sensitivity Coeff. = $-(\alpha + \beta) \times P = -0.001331$ Uncertainty = $1.155 \times (-0.001331) = -0.00154$	-0.0015 kg/cm^2
UB5	Uncertainty due to Hysteresis of UUC Value (Width of Distribution) 0	Rectangular (Value / $\sqrt{3}$)	Calculation = $0 / 1.7321$ 0.00	0.0000 kg/cm^2
UB6	Uncertainty due to Zero offset Value (Width of Distribution) 0	Rectangular (Value / $\sqrt{3}$)	Calculation = $0 / 1.7321$ 0.00	0.0000 kg/cm^2
Calculations to find Sensitivity co-efficients		Formula	Value	
1. Uncertainty due to temp. correction		$C1 = -(\alpha + \beta) \times P$	-0.001	$\alpha = \beta = 0.000011 \text{ /K}$
2. Uncertainty due to change in Acceleration due to gravity		$C2 = P/g$	6.182	
		Combined Standard Uncertainty (Uc)	0.018 kg/cm ²	
		Coverage Factor k	2 at 95% Confidence Level	
		Expanded Uncertainty (Uc x k)	± 0.04 kg/cm ²	

V. RESULTS AND DISCUSSIONS

The uncertainties for Micrometer and Pressure Gauge Calibrations are $5.78\mu\text{m}$ and 0.04 kg/cm^2 . These numbers represent the effect of environment, temperature, physical conditions/positions and errors in instruments on the calibration process. It also shows a realistic approach towards the measurement process and thereby provides valuable information to laboratories and customers about the quality and reliability of the measuring and testing instruments. The method mentioned above is applicable to various measuring and testing instruments such as Vernier caliper, dial gauge, Lux meter, Measuring Tape, Thermocouple etc. It is imperative for all the laboratories to provide a certain level of uncertainty which occur during calibration. The explained methodology therefore helps in easing the task of reporting uncertainty.

VI. CONCLUSIONS

Industries and governments spend billions of dollars to acquire, install and maintain measurement and test equipments. There is a need for measurement quality assurance program which is already undertaken by many multinational companies. Uncertainty analysis helps to provide support to the measurement quality assurance program by justifying it as cost benefit. It gives a base where the calibration process of the instruments could be compared anywhere round the world, thus, making it easy for the technical experts to ensure reliable and accurate products to the industries. It also puts light on the main factor that causes the measurements during calibration process to vary and so the lab assistants could control those factors.

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