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A numerical Technique Finite Volume Method for Solving Diffusion 2D Problem

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ABSTRACT

In this paper, a numerical finite volume technique was used to solve transient partial differential equations for heat transfer in two dimensions with the boundary condition of mixed Dirichlet (constant, not constant) in a rectangular field. We explained the procedures step by step, for the digital solution we used our Fortran code and a line by line TDMA solver for algebraic equations. Finally, the numerical results are compared with the exact solution.

Keywords – *Conduction, Dirichlet boundary condition, finite volume method, heat transfer, TDMA.*

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I. INTRODUCTION

The partial derivative equations are very often used in science and engineering. Many partial differential equations can't be solved analytically in a closed form solution. There are several and different techniques for the construction of numerical methods of the PDE solution [1]. As the mathematical modeling has become an integral part of the analysis of engineering problems, a variety of techniques for digital network have been developed [2].

The most popular numerical techniques are the finite difference (FD), finite elements (FE) and the finite volume method (FVM) which was originally developed as a special formulation of finite difference. Each of these methods has its own advantages and disadvantages depending on the problem to solve. The technique of finite volume is one of the most versatile and flexible technique for solving problems in fluid dynamics [2].

The present paper deals with the description of the method of finite volume (FVM) to solve differential equations. The comparison is made between the analytical solution (AS), and the solution obtained by implementing the finite volume method. This paper is organized as follows:

• Section 2: contains the description of the finite volume method (FVM), with the help of TDMA solver (Tri-diagonal matrix algorithm).

• Section 3: contains diffusion problem, the analytical solution (AS) and the finite volume method (FVM).

• Section 4: in this section the numerical solutions obtained by this technique are compared with the exact solution.

• Section 5: contains the conclusion of this paper.

II. FINITE VOLUME METHOD

The finite volume method represents and evaluates the partial differential equations in the form of algebraic equations [3]. Values are calculated at a discrete nodes of the control volume for geometry. "Finite volume" refers to the small volume surrounding each point on a mesh nodes. In the finite volume method, volume integrals of a partial differential equation that contains the divergence term are converted to the full surface, using the divergence theorem. These terms are then evaluated as a flow at the surfaces of each finite volume. Because the inflow into a given volume is identical to the outflow at the adjacent volume, these methods are conservative. The method is used in many fluid dynamics calculation software. It is always preferred to use the governing equation under its conservative form in the finite volume approach for solving any problems that ensures the preservation of all properties in each cell / volume control. Now we will discuss the steps involving FVM to solve the differential equation [3], and for more details we can consult [4].

2. 1 Grid generation

The first step of the finite volume method is the generation of the grid by dividing the area into small discrete control volumes. The limits of the control volumes are positioned halfway between adjacent nodes. Thus, each node is surrounded by a control volume or cell [5]. It is common practice to set up the control volumes close to the edge of the field so that the physical boundaries coincide with boundaries between control volumes. A general nodal point "P" is defined by its neighbors, in a two-dimensional geometry, nodes on the west and east, north and south are defined as follow; W, E, N and S, respectively. The four faces of the volume control are defined at their sides by 'w', 'e', 'n', and 's', respectively. The distances of these points are given in Fig 1.

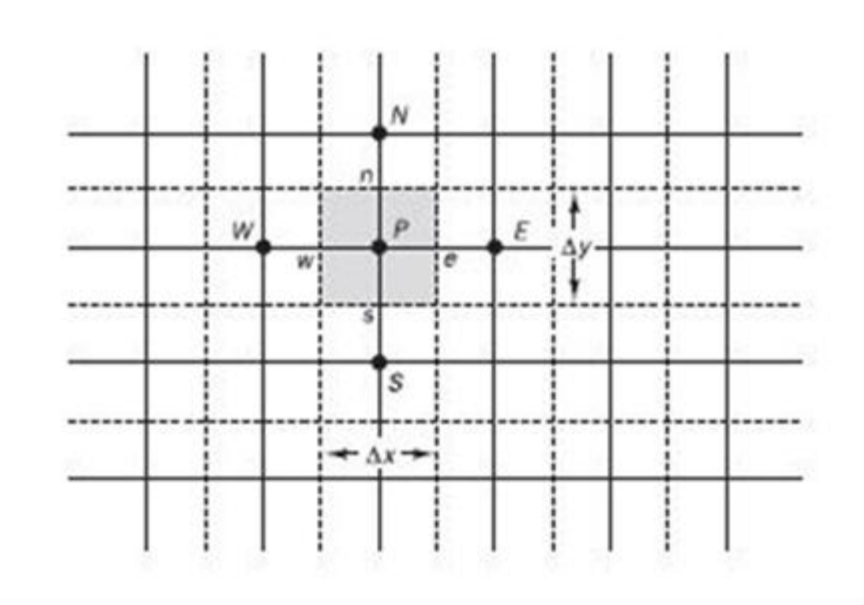

Fig 1- A part of the two-dimensional grid

2. 2 Discretization

The most important characteristics of the method of finite volume are the integration of the governing equation on a volume control to obtain a discretized equation to its nodal points P.

$$a_p T_p = \sum a_{np} T_{np} + S_u \qquad (1)$$

where Σ indicates summation over all neighboring nodes (np), are the coefficients of the neighbor nodes, , is the value of the property T at the neighboring nodes; and is the linearized source term.

In all cases, the coefficients surrounding the point P satisfy the following relationship:

$$a_p = \sum a_{np} - S_p \qquad (2)$$

2. 3 Solution

After discretization on each volume control, we find a system of algebraic equation. Which fills a sparse matrix that can be easily solved TDMA algorithm (Tri-Diagonal Matrix Algorithm). The algorithm of tri-diagonal matrix (TDMA), also known under the name of Thomas algorithm, is a simplified form of Gaussian elimination which can be used to solve the system of equations with three diagonal.

TDMA is based on Gaussian elimination procedure and consist of two parts - a forward elimination phase and a backward substitution phase. TDMA is in fact a direct method, but can be applied iteratively in a line by line fashion, to solve multidimensional problems and is widely used in CFD programs as shown in [4].Consider the equation system 1 for j = 1, - - -, n and we use equation (1) the general form of the TDMA solver which is given by:

$$-a_S T_S + a_p T_p - a_N T_N = a_W T_W + a_N T_N + S_u \qquad (3)$$

To solve the above TDMA system along North-South lines, the discrete equation (3) is reorganized in the form:

$$-\beta_j T_{j-1} + D_j T_j - \alpha_{j+1} T_{j+1} = C_j \qquad (4)$$

Where $T_j = A_j T_{j+1} + C_j^{'}$ (5)

$$A_j = \frac{\alpha_j}{\left(D_j - \beta_j A_{j-1}\right)} \qquad C'_j = \frac{\left(\beta_j C'_{j-1} + C_j\right)}{\left(D_j - \beta_j A_{j-1}\right)}$$

III. PROBLEM FORMULATION

We consider solving two-dimensional steady heat conduction problems in rectangular plate made of uniform material thus the thickness of the plate is about D = 0.01 m and the thermal conductivity of material is 1W/mC0. The boundary conditions are as follow Fig. 2:

- East boundary: fixed temperature T0 = 100 °C.
- West boundary: fixed temperature T0 = 100 °C.
- South boundary: fixed temperature T0 = 100 °C.
- North boundary: variable temperature T = f(x).

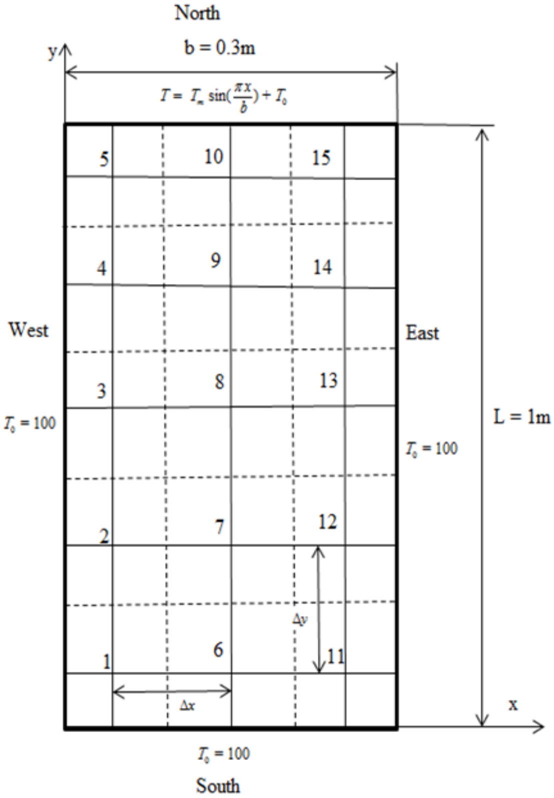

Fig 2 - Solution region with mixed boundary condition

The mathematical formulation of this problem is given by

$$\frac{\partial}{\partial X}\left(k\frac{\partial T}{\partial X}\right) + \frac{\partial}{\partial Y}\left(k\frac{\partial T}{\partial Y}\right) = 0 \tag{6}$$

The exact temperature field is

$$T(X,Y) = T_0(X,Y) + \sum_{n=1}^{\infty} C_n \sin\left(\frac{n\pi X}{b}\right)\sinh\left(\frac{n\pi Y}{b}\right) \tag{7}$$

Where C_n is given by:

$$C_n = \frac{(b/2)}{\sin\left(\frac{n\pi L}{b}\right)} \int_{X=0}^{X=b} \left[f(X) - T_0(X,Y)\right] \sin\left(\frac{n\pi X}{b}\right) dX \tag{8}$$

For n > 1, the reader may consult [6]; the final solution of equation (6) is therefore

$$T(x,y) = T_m \frac{\sinh(\pi y/b)}{\sinh(\pi L/b)} \sin(\frac{\pi x}{b}) + T_o(x,y) \quad (9)$$

The error of temperature field is evaluated by:

$$Error(x,y) = ABS(T_{Num} - T_{Exact}) \quad (10)$$

The grid size , $\Delta x = 0.1m$, $\Delta y = 0.2\ m$

THE FINITE VOLUME DISCRETIZATION:

Equations the general form of discretized for problems is given by equation (3).

$$a_p T_P = a_W T_W + a_E T_E + a_S T_S + a_N T_N + S_u \quad (11)$$

And we use the coefficients surrounding the point P, equation (2).

$$a_p = a_W + a_E + a_S + a_N - S_p \quad (12)$$

The area $A_w = A_e = D\ \Delta y$, $A_n = A_s = D\ \Delta x$

Where $a_W = \frac{k}{\Delta x} A_w$ $a_E = \frac{k}{\Delta x} A_e$ $a_S = \frac{k}{\Delta y} A_s$ $a_N = \frac{k}{\Delta y} A_n$

At interior points 7, 8 and 9

Where $S_u = 0$, $S_p = 0$

At a boundary node the discretized equation takes the form

For Fixed value T $S_u = \frac{2\ k\ A}{\Delta} T$ and $S_p = -\frac{2\ k\ A}{\Delta}$

IV. RESULTS AND DISCUSSION

All numerical calculations were carried out with volume control method and analytical solution using computing FORTRAN code.

The coefficients and the source term of the discretization equation for all nodes are summarized in Table 1.

Node	a_N	a_S	a_W	a_E	a_P	S_u
1	0.005	0	0	0.02	0.075	5
2	0.005	0.005	0	0.02	0.07	4
3	0.005	0.005	0	0.02	0.07	4
4	0.005	0.005	0	0.02	0.07	4
5	0.0	0.005	0	0.02	0.075	5.099
6	0.005	0.	0.02	0.02	0.055	1
7	0.005	0.005	0.02	0.02	0.05	0.
8	0.005	0.005	0.02	0.02	0.05	0.
9	0.005	0.005	0.02	0.02	0.05	0.
10	0.0	0.005	0.02	0.02	0.055	1.2
11	0.005	0.	0.02	0.	0.075	5
12	0.005	0.005	0.02	0.	0.07	4
13	0.005	0.005	0.02	0.	0.07	4
14	0.005	0.005	0.02	0.	0.07	4
15	0.	0.005	0.02	0.	0.075	5.099

Table 1- The coefficients and source term for all nodes

The numerical solution of the discretized equations system is calculated using TDMA as shown in Table 2.

Node	β_j	D_j	α_j	C_j	A_j	$C_j^{'}$
1	0	0.075	0.005	5.	0,07	66,67
2	0.005	0.07	0.005	4.	0,07	62,20
3	0.005	0.07	0.005	4.	0,07	61,90
4	0.005	0.07	0.005	4.	0,07	61,88
5	0.005	0.075	0.0	5.099	0,00	72,46
6	0.	0.055	0.005	2.4226	0,09	44,05
7	0.005	0.05	0.005	1.3397	0,10	31,48
8	0.005	0.05	0.005	1.3343	0,10	30,14
9	0.005	0.05	0.005	1.3417	0,10	30,15
10	0.005	0.055	0.0	2.6494	0,00	51,38
11	0.	0.075	0.005	5.944	0,07	79,25
12	0.005	0.07	0.005	4.6977	0,07	73,12
13	0.005	0.07	0.005	4.6742	0,07	72,37
14	0.005	0.07	0.005	4.7068	0,07	72,78
15	0.005	0.075	0.	6.1276	0,00	86,97

Table 2- The Numerical coefficients TDMA after first iteration

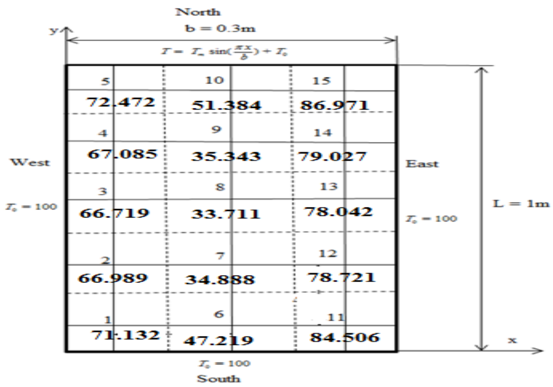

Fig 3- The Numerical solution after first iteration

Finally, the solutions obtained by numerical finite-volume techniques, were compared with an exact solution, to verify the accuracy of the results on calculating errors, as shown in Table 3.

Node	FVM	Exact	Error
1	100.002	100.000	0,002
2	100.014	100.006	0,008
3	100.086	100.053	0,033
4	100.502	100.432	0,07
5	102.928	103.509	0,581
6	100.004	100.001	0,003
7	100.029	100.013	0,016
8	100.172	100.106	0,066
9	101.005	100.864	0,141
10	105.857	107.018	1,161
11	100.002	100.000	0,002
12	100.014	100.006	0,008
13	100.086	100.053	0,033
14	100.502	100.432	0,07
15	102.928	103.506	0,578

Table 3- A Comparison between Numerical and Exact Solutions

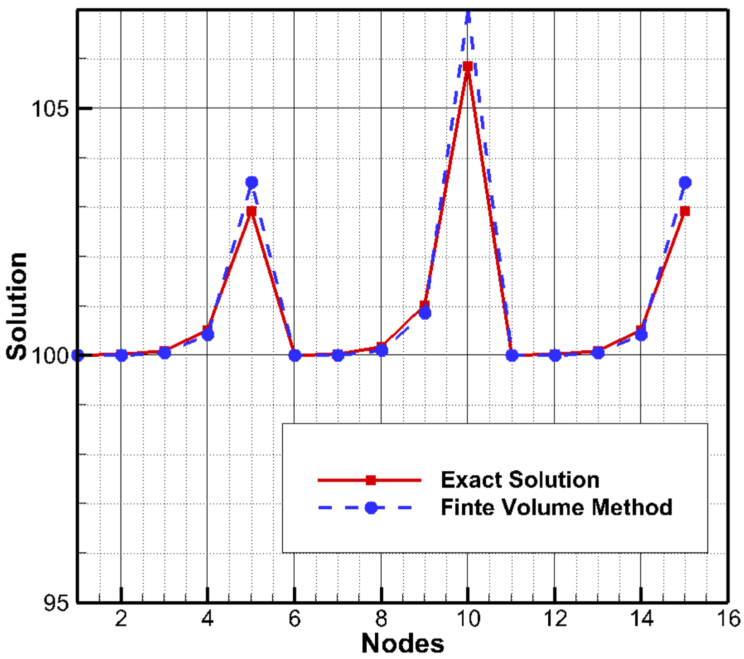

Fig 4- A comparisons between Finite Volume Numerical Solution with Exact Solution

V. CONCLUSION

In this paper finite volume numerical grid technique for steady state heat flow problems was studied and numerical solution of the two dimensional heat flow equation with Dirichlet boundary conditions was obtained. We have used TDMA solver for solving algebraic equations and the results obtained by this technique are all in good agreement with the exact solutions under study.

Furthermore, this technique is effective, reliable, precise and easier to implement in programming in FORTRAN, compared to other costly techniques.

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