

## Proportional-Integral-Derivative Control (PID) Of A Continuous Stirred Tank Reactor (Cstr)

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### ABSTRACT

*Simulation is a very important tool especially now that computations speed of computer increase exponentially every day. With simulation, one will be exposed to the knowledge of various pieces of equipment. Simulation on mathematical model has several advantages over real experiment, this include; affordability, less time consuming, knowledge of non available equipments. This work deals on proportional integral derivative (PID) control of a continuous stirred tank reactor (CSTR) for virtual experiment. Non isothermal CSTR and different control modes were used in this work. Values were chosen at steady state and dynamic point. Results of this work showed the stability of the non isothermal continuous stirred tank reactor at different tuning point and disturbances. This work serves as a teaching aid for chemical engineering student.*

**KEYWORDS:** CSTR, PID, Simulation, MATLAB and Laboratory Experiment.

Date of Submission: 04 August 2015



Date of Accepted: 10 October 2015

### I. INTRODUCTION

Continuous stirred tank reactor CSTR also known as back mix is a workhorse at many chemical plants, frequently serving multi-purpose production objectives for fine and specialty chemicals. It can handle not only reaction, also solvent extraction, crystallization and distillation. It is used for continuous operation where production rate is greater than five million kg/hr, involves single product and established market. The successful installation of such a reactor depends, to a great degree, upon the proper design of its control system (Pablo et al, 2009). It is normally operated at steady state and also assumed to be perfectly mixed.

Temperature, concentration or the reaction rate inside the CSTR are not dependent on time, that is every variable is the same at every point inside the reactor, because the temperature and concentration are identical everywhere within the reaction vessel, they are same at the exit point as they are elsewhere in the tank. Thus the temperature and concentration in the exit stream are modeled as being the same as those inside the reactor (Fogler, 2008) and the rate of reaction is evaluated at the exit condition. The rate constant can be figured by using Arrhenius temperature dependence. As temperature increases so does the rate at which the reaction occurs. Numerical simulations for the closed loop system were performed by (Pablo et al, 2009). They implemented an ideal I/O linearising control, a standard sliding-mode and high order sliding mode control. They proposed a methodology in which the same value of the control gain was used. They also proposed another control where prior knowledge of the process was combined with the various existing modelling techniques leading to grey-box models (Madar, et al, 2005). Non linear control of CSTR for a reversible reaction using Neural Network as design tool was done by Dauda et al. Luyben (1990) used a proportional level control to manipulate the liquid leaving a CSTR as a function of the volume in the tank and a second controller manipulated the flow rate of cooling water to the jacket in direct proportion to the temperature in the reactor. He obtained a quite an oscillatory response on the results of feed composition and feed flow rate when he studied the responses of this reactor to 65% decrease in process gain and 20% increase in the reactor inlet concentration. With 10% increase in feed concentration, the system was stable. FORTRAN was used for the simulation.

Breedveld et al (2003) developed an alternative approach to modeling in chemical engineering using port based modeling approach which has an advantage of development of reusable models. The port based modeling approach implies the use of the basic thermodynamic axioms. So the intensive or extensive nature of thermodynamic variable was thoroughly used.

This led to write the entropy balance, to formulate the irreversible entropy production term and to rewrite the constitution relation in such a way the intensive variable are expressed as function of the extensive one. The intensive variable encountered in thermodynamic were the temperature, pressure and the chemical potentials of components of the system. The extensive ones were the entropy, the volume and the mole number of each space. This approach also led to rewriting of the kinetics rates in function of its conjugate variables, the forward and reverse affinities instead of concentration of reactants. Hoang et al (2008) also considered a thermodynamically nonlinear consistent model of a continuous stirred tank reactor to build the appropriate function for stabilization purpose. The aim was to stabilize the reactor around the unstable point. Fluid was assumed to be homogenous and this permitted the use of concavity property of the entropy function to build the function.

Ruszkowski (2005) did same work using isothermal CSTR were he applied fundamental of irreversible thermodynamics in order to produce a Lyapunov function for stabilization purpose. Viel et al (1997) considered the design of feedback control laws that allow the global temperature stabilization, that were robust against kinetic uncertainties, and robust against control input saturations. They proposed a set of controllers that can guarantee the global temperature stabilization of CSTR in spite of strong uncertainties on the dependence of the kinetic function with respect to the temperature. This work could not give a robust stabilizing feedback controller in an exothermic case such that the input is nonnegative along the closed-loop trajectories. However, this problem has been recently addressed, and a solution has been proposed by (Viel et al, 2004). Emuoyibofarhe et al(2008) proposed a Kohkal network, which is a hybrid of the Counter-Propagation Neural Network (Kohonen Layer) and the Kalman Filter. This was done with the justifications that neural networks implementing both supervised and unsupervised learning algorithms are usually better than networks using only one learning algorithm and secondly, the outputs of Kohonen layer which implements unsupervised learning are entered as inputs into the Kalman's layer which uses a better iterative procedure to solve the linearsystem of equations. Ivan et al(2006) did a simulation study of a CSTR using a simple iteration (Runge Kutta) method. Coughanowr and koppel, tried to maintain the concentration of a reactant in two series isothermal CSTR at some desired value in spite of variation in inlet concentration, using an irreversible chemical reaction  $A \rightarrow B$  with liquid streams entering tank 1 at volumetric flow rate and concentration. These tanks were maintained at different temperature, the temperature in tank 2 was greater than that in thank 1. This was accomplished by adding a stream of pure A to tank 1 through a control valve. In a similar work by Mathworks (2009), the tanks level were assumed to stay constant because of the over flow nozzle and hence there was no level control involved.

## II. SIMULATION AND CONTROL

The reaction involved is an irreversible chemical reaction  $A \rightarrow B$ . A decomposes to form product B. This reaction led to the ordinary differential equations (1 – 4) which described the system. Nominal values were used for the steady state and dynamic analysis. The control system is to maintain the reactor temperature at 600K and holdup at 48m<sup>3</sup> which is the set point. This will be accomplished by manipulating the jacket flow rate  $F_j$  as described by equation (6) and the reactor outlet flow rate  $F$  as described by equation (7). A feedback controller with a proportional integral derivative control (PID) was used. The equations (1 – 4) show the ODEs which describe the system

- $dV/dt = F_o - F \dots\dots\dots(1)$

- $d(VCA)/dt = F_oCA_o - FCA - V_kCA \dots\dots\dots(2)$

- $d(VT)/dt = F_oT_o - FT - \lambda V_kCA / \rho C_p - UAH / \rho C_p(T - T_j) \dots(3)$

- $dT_j/dt = F_j(T_{j0} - T_j)/V_j + UAH / \rho_j V_j C_j(T - T_j) \dots(4)$

- $K = \alpha e^{-E/RT} \dots\dots\dots(5)$

The reaction rate per unit volume is described by Arrhenius rate law(5)

Equations (6) and (7) are the PID controller algorithm for jacket and reactor flow rate respectively.

$$F_j = F_{j_s} + k_c (\epsilon + 1/\gamma \int \epsilon dt + \gamma D d\epsilon / dt) \dots\dots\dots(6)$$

$$F = F_s + k_c v (\epsilon + 1/\gamma \int \epsilon v dt + \gamma D v d\epsilon v / dt) \dots\dots\dots(7)$$

Equations (8) to (14) are the MATLAB version of equations (1) to (7)

$$V_{DOT} = F_0 - F \dots\dots\dots (8)$$

$$V_{CDOT} = F_0 * CA_0 - F * CA - V * K * CA \dots\dots\dots (9)$$

$$V_{TDOT} = F_0 * T_0 - F * T + (30000 * V * K * CA - Q) / (0.75 * 50) \dots\dots\dots (10)$$

$$T_{JDOT} = F_J * (T_{J0} - T_J) / 3.85 + Q / 240 \dots\dots\dots (11)$$

$$K = 7.08e10 * \exp(-30000 / (1.99 * T)) \dots\dots\dots (12)$$

$$F_J = 0.8 + K_c * (E + (60 / \tau_{IUI}) * E_{INT} + (\tau_{AUD} / 60) * E_{DIFF}) \dots\dots\dots (13)$$

$$F = 40 + K_c * (E + (60 / \tau_{IUI}) * E_{INT} + (\tau_{AUD} / 60) * E_{DIFF}) \dots\dots\dots (14)$$

The table below shows the values used for the simulation

Nominal CSTR Parameter values

PARAMETER	VALUES
F	40(m <sup>3</sup> /h).
V	48(m <sup>3</sup> )
CA	0.245(kgmol/m <sup>3</sup> )
T	600K
T <sub>j</sub>	594K
F <sub>j</sub>	0.8(m <sup>3</sup> /h).
C <sub>ao</sub>	0.50(kgmol/m <sup>3</sup> )
V <sub>j</sub>	3.85(m <sup>3</sup> )
E	30000(kcal/kgmol).
U	150(kcal/K*h)
T <sub>o</sub>	530K
T <sub>jo</sub>	530K
C <sub>p</sub>	0.75(kcal/k).
ρ	50(kg/m <sup>3</sup> ).
K <sub>c</sub>	37.6
R	1.99[kcal/(kgmol*K)].
AH	250(m <sup>3</sup> )
λ	-30000(kcal/kgmol).
ρ <sub>j</sub>	62.3(kg/m <sup>3</sup> ).
C <sub>j</sub>	1.0(kcal/K)

Table 1.0 Steady state parameter values

### 2.1 Proportional-Integral-Derivative Control.

A feedback proportional-integral-derivative control was used for this work. Two variables temperature and holdup were the controlled variables. In the feedback control, the measured variable was fed back to the comparator which compared with the set point of 600k for temperature and 40m<sup>3</sup> for holdup. The difference between the measured variable and the set points generated errors which entered the controller (PID). The controller adjusted the final control element (control valves) in order to return the controlled variables to the set points. If the error signal is not equal to zero, the controller will make the appropriate changes in the system manipulated input and forces the output variable to return to its set point.

In this work three parameters process gain, derivative time and integral time were tuned (Seborg et al, 2002) using Zeigler-Nichols approach and which for the purpose of this work were called  $k_c$  and  $k_{cv}$ ,  $\tau_{AUD}$  and  $\tau_{AUDV}$  and  $\tau_{AUI}$  and  $\tau_{AUIV}$  respectively.

This work offers the advantage of studying the effects of other control modes on continuous stirred tank reactor. It also offers the advantage of studying the response of some variables to changes in the other variables.

A MATLAB version 7.0 was used in developing the process control module.

## 2.2 Proportional Integral Derivative Control Simulation

```
Kc=input('What is the value of Process gain:')
TAUI=input('What is the value of Integral time:')
TAUD=input('What is the value of Derivativetime:')
Kcv=input('What is the value of volume Process gain:')
TAUIV=input('What is the value of volume Integral time:')
TAUDV=input('What is the value of volume Derivativetime:')
T=600;CA=0.245;TJ=594.59;V=48;TIME=0;VC=V*CA;VT=V*T;
TJ0=530;F0=40;T0=530;CA0=0.5;F=45;DELTA=0.001;
DTIME=[];DCA=[];DT=[];DV=[];DF=[];DTJ=[];DFJ=[];
EINT=0;EDIFF=0;E=0;EINTV=0;EDIFFV=0;EV=0;
while TIME<4
    EDIFF=E;EDIFFV=EV;
    EV=(V-40);
    E=(T-600);
    FJ=0.8+Kc*(E+(60/TAUI)*EINT+(TAUD/60)*EDIFF);
    F=40+Kcv*(EV+(60/TAUIV)*EINTV+(TAUDV/60)*EDIFFV);
    DTIME=[DTIME TIME];DCA=[DCA CA];DT=[DT T];DV=[DV V];DF=[DF F];DTJ=[DTJ TJ];DFJ=[DFJ FJ];
    K=7.08e10*exp(-30000/(1.99*T));
    Q=150*250*(T-TJ);
    VDOT=F0-F;
    VCDOT=F0*CA0-F*CA-V*K*CA;
    VTDOT=F0*T0-F*T+(30000*V*K*CA-Q)/(0.75*50);
    TJDOT=FJ*(TJ0-TJ)/3.85+Q/240;
    V=V+VDOT*DELTA;
    VC=VC+VCDOT*DELTA;
    VT=VT+VTDOT*DELTA;
    TJ=TJ+TJDOT*DELTA;
    TIME=TIME+DELTA;
    CA=VC/V;
    T=VT/V;
```

```

EINT=EINT+E*DELTA;
EINTV=EINTV+EV*DELTA;
EDIFF=(E-EDIFF)/DELTA;
EDIFFV=(EV-EDIFFV)/DELTA;
end
solution=[DTIME' DCA' DT' DV' DF' DTJ' D

```

### III. RESULTS AND DISCUSSION

The graphs below show some of the results of the simulation

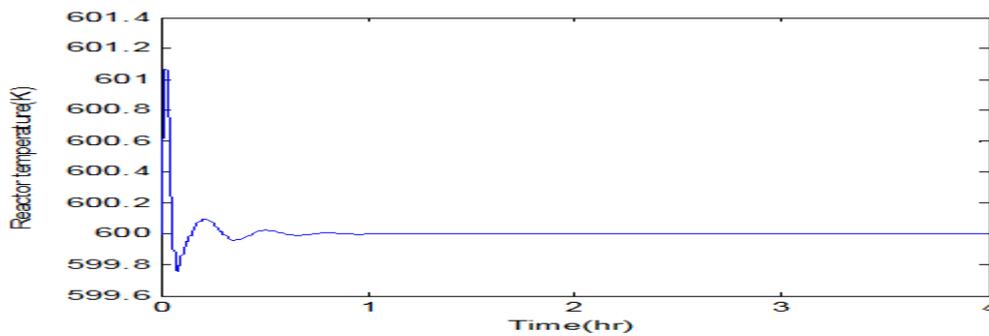


Figure 1: temperature – time trajectory for PID control

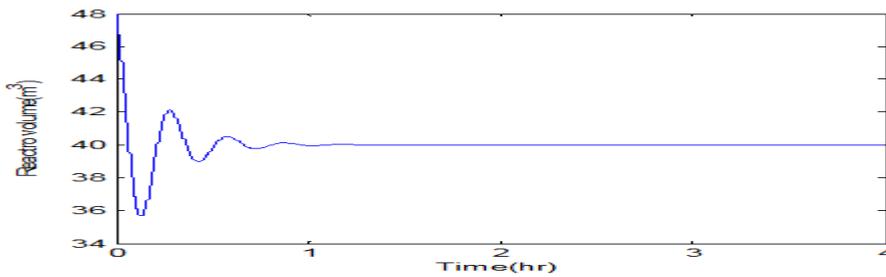


Figure 2: Volume-time trajectory for PID control

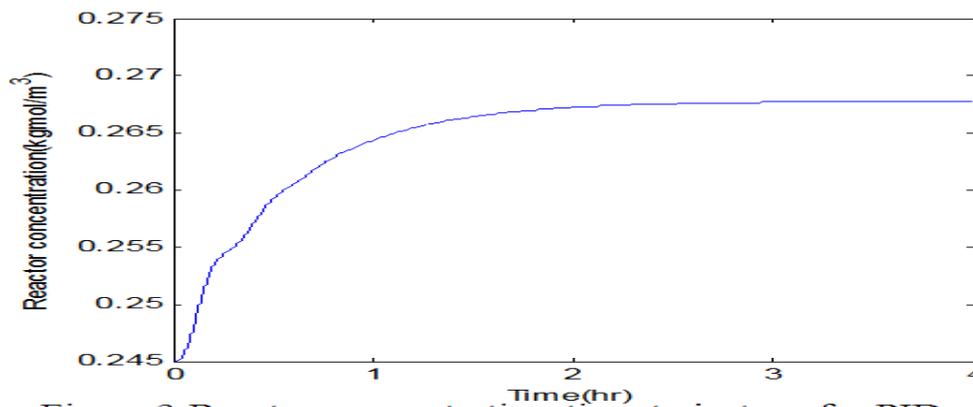


Figure 3: Reactor concentration-time trajectory for PID control

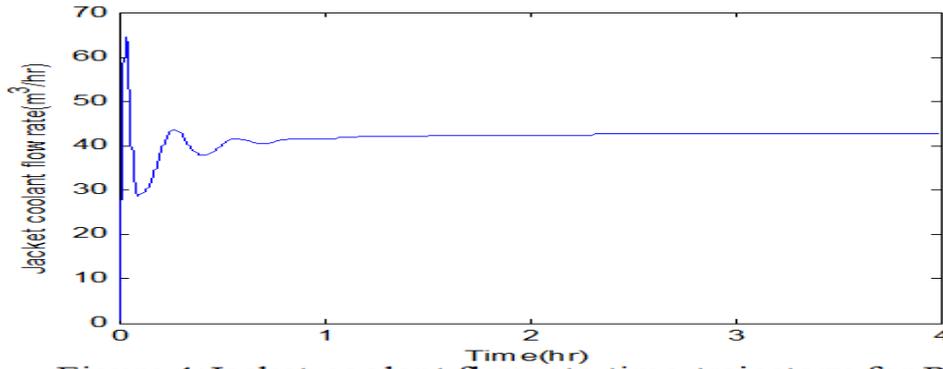


Figure 4: Jacket coolant flowrate-time trajectory for PID control

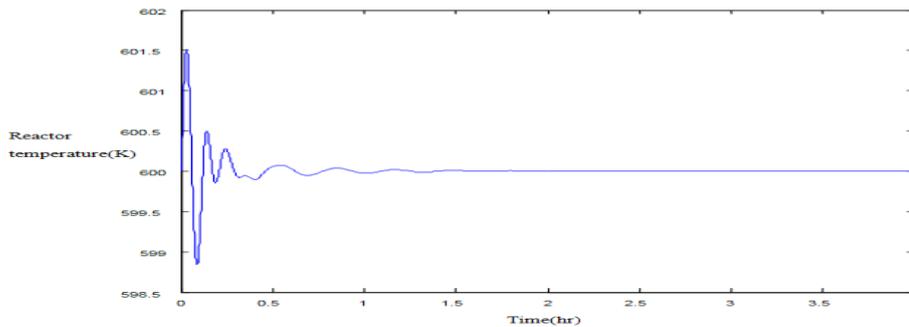


Figure 5: Reactor temperature - time trajectory for PID control with 50% reduction in process gain and control parameters

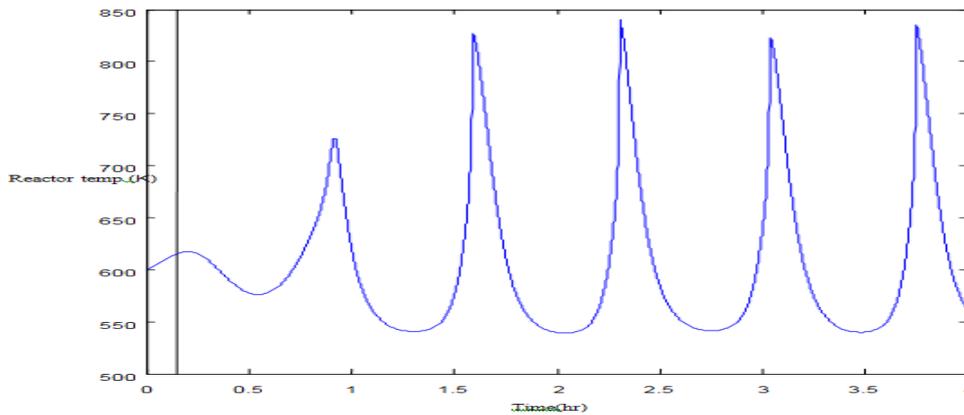


Figure 6 :Temperature time trajectory for PID control with low process gains

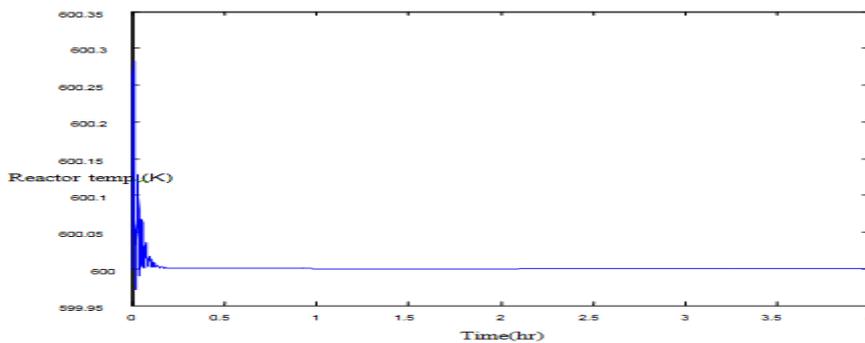


Fig 7 Reactor temperature-time trajectory for PID control with high process gain

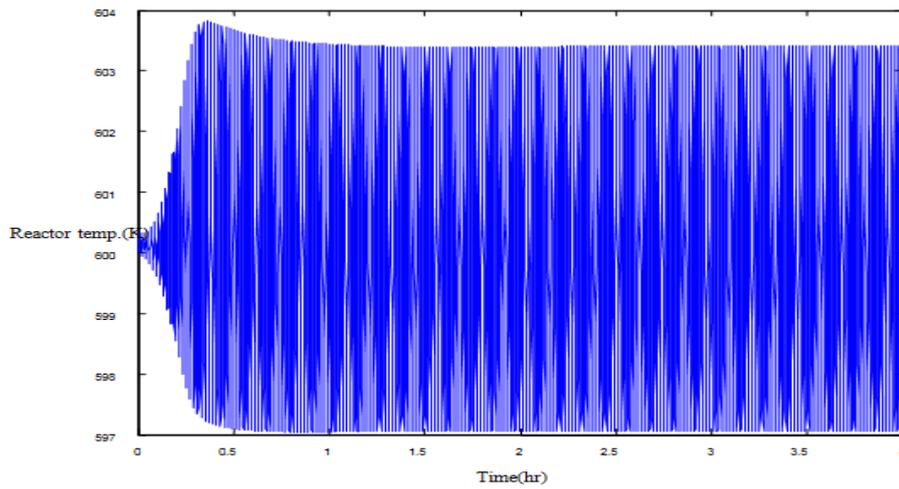


Figure 8: Reactor temperature - time trajectory for PID control with very high TAUD

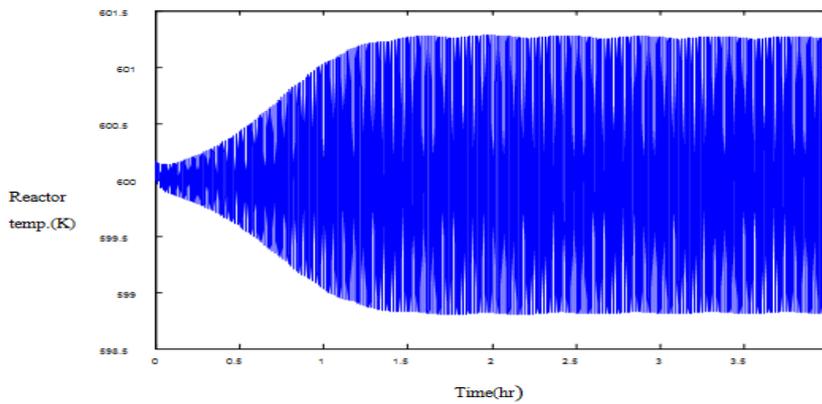


Figure 9: Reactor temperature - time trajectory for PID control with very high process gains

Parameters	$K_c$	TAUI	TAUD	$K_{cv}$	TAUIV	TAUDV
Values	37.6	1.82	0.453	10	1.5	0.35

Table 2: Values for the control analysis

Parameters	Very High gain	High gain	High TAUI	Low TAUI	High TAUD
$K_c$	376	200	37.6	37.6	37.6
TAUI	1.82	1.82	200	0.1	1.82
TAUD	0.453	0.453	0.45	0.45	100
$K_{Cv}$	200	100	10	10	10
TAUIV	1.5	1.5	100	0.1	1.5
TAUDV	0.18	0.18	0.18	0.18	50

Table 3: Values for the process gain and controller parameters

### 3.1 Discussion

In the figures above, there are different responses for different input parameters under different control modes. With proportional –integral- derivative control mode, reactor temperature, volume and flow rate had good responses; they were able to maintained offset and stability after a short period. Concentration and jacket coolant temperature could not reach their steady state value. Under proportional –integral control mode these variables had similar responses to that of PID control.

A further study on these responses determines which of the controller will be more suitable for a particular loop. Although concentration is not a controlled variable, but with the response under proportional control mode, it indicates that it is more suitable for concentration control. Also volume and reactor flow rate had better responses with proportional only control. This indicates that such control action is good for these loops.

Some of the parameter that were not controlled responded well to certain control modes, it can be deduced that those controlled modes are good control modes for those parameter.

Furthermore, a 50% reduction in control parameters and the process gain, shows that the system maintained stability with more transient time but with 50% increase in these parameters, it also maintained stability but with less transient time. If process gain is lower than the controller parameters the system becomes unstable with more overshoots and undershoots. High process gain introduces a slight excessive response but with derivative action, stability is maintained.

High derivative (TAUD) action causes an excessive response. Excessive response can also be caused by a very high process gain. If the integral time is very high or very low it causes instability in the system response. Low process gain also causes instability in a system. Stability is improved when TAUD is smaller than TAUI and KC higher than TAUI and TAUD. A step change in some of the input parameter can change the system's stability depending on the control mode

## IV. CONCLUSION

In this work, a process control module for CSTR was developed for a teaching aid. It also studied the responses of CSTR under different control mode, step changes, low and high control parameters. This work studied good tuning of the control parameters; process gain and process modes for various variables. The stability of the controller system has also been proved to be consistent at various level of disturbance of  $T_0$ ,  $F_0$  and  $T_j$ . Effects of high and low process gain, integral time and derivative time has also been proved through this simulation. The set of the reactor temperature was maintained above the inlet temperature by each of the control modes otherwise no reaction will occur and non isothermal CSTR will not be ideal for such a reaction. Reaction in a non isothermal reaction is an exothermic reaction with jacket cooling water reducing the heat of reaction. A simulation manual was also developed for this work. Users of this work is expected to have a basic knowledge of MATLAB.

## REFERENCE

- [1] Breedveld .P.,Couenne .F.,Jallut . C., Maschke .B., Taya .M. (2003.Feb)'Modelling of Continuous Stirred Tank Reactor using bond graph formalism' *4th Mathmod Vienna Fourth International Symposium on Mathematical Modelling*. Feb 5th- 7th, 2003.
- [2] Brain R. H., Ronald L. L. and Jonathan M. R (2001). '*A guide to MATLAB for beginners and experienced users*'. pp Xiii. Cambridge university press New York.
- [3] IDauda O.A., ITinuade J. A., 2Duncan A. (2006). '*Neural Network control of CSTR for Reversible Reaction Using Reverence Model Approach*'. 1Ladoke Akintola University of Technology Ogbomosho. 2Federal University Of Technology Minna.
- [4] Emuoyibofarhe O.J. and Reju. A. S. (2008). 'An optimum solution for a process control problem (continuous stirred tank reactor) using a hybrid neural network'.*Journal of Theoretical and Applied Information Technology*.vol 4,no 10,pp 906-915.
- [5] Fogler .H.S.(2008). *Elements of Chemical Reaction Engineering*, 4th Edition. pp.12 – 13 New Delhi: Prentice Hall of India
- [6] Hoang H. Couenne .F., Jallut C., Le Gorrec .Y.,(2008,July) '.The international Federation of Automatic Control' *proceedings of the 17th world congress Seonl Korea* July 6-11 ,2008.
- [7] Ivan Zelinka, Jiri Vojtesek and Zuzana Oplatkova 'simulation study of the cstr reactor for control Purposes' *Proceedings 20th European Conference on Modelling and Simulation*
- [8] Madar Janos, Janos Abonyi and Ferenc Szeifert. 'Feedback linearizing control using hybrid neural networks identified by sensitivity approach'.*Engineering Applications of Artificial Intelligence*. Vol. 18, No. 3, April 2005, pp. 343-351.
- [9] Pablo A., Lopez P., Lopez R. A., 2009, 'Dynamic non linear feedback for temperature control for a continuous stirred tank reactor with complex behaviour' *journal of applied research and technology*, vol 7, no. 2, pp202-217.
- [10] Seborg D.E., T. F. Edgar, and D. A. Mellichamp,*Process Dynamics and Control 2nd ed.*(New York: Wiley, 2002).
- [11] Suja Mani .M.R. and Thyagarajan .T.(2009) 'Artificial Neural Networks Based Modeling and Control of Continuous Stirred Tank Reactor' *international journal on simulation and model* vol 8,no.3. pp.1726 – 4529.
- [12] The Mathworks United Kingdom Control System Toolbox. 2009 '*Internal Model Control Design for CSTR*'.