

## A Novel Approach to Transient Stability Using Stochastic Energy Functions Suitable For Power System Risk Assessment

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### -----ABSTRACT-----

*It is very important to maintain supply reliability under the deregulated environment. The transient stability problem is one of the major concerns in studies of planning and operation of power systems. Although the equal-area criterion method is useful in determining the stability as a transient stability evaluation method, the method is only applicable to a one-machine system connected to an infinite bus or to a two machine system and the time domain simulation is the best available tool for allowing the use of detailed models and for providing reliable results. The main limitations of this approach involve a large computation time. This paper describes a method for estimating a normalized power system transient stability of a power system that is four machines, six bus system and three machines, nine bus systems. The critical clearing time is evaluated using corresponding energy function Therefore, the transient energy function (TEF) is constructed for large power system.*

**KEYWORDS** : Critical Clearing Time (CCT), Direct Stability Analysis (DSA), Structure preserved power system, Transient Energy Function (TEF).

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### I. INTRODUCTION

An interconnected power system consists of generating units run by prime-movers (including turbine-governor and excitation control systems) plus transmission lines, loads, transformers, static reactive compensators, and high-voltage direct-current lines. The size of the interconnection varies depending on the system but the technical problems are the same. At the planning level, the planner would invariably study the stability of the system for a set of disturbances ranging from a three-phase-to-ground fault (whose probability of occurrence is rare) to single-phase faults, which constitute about 70 percent of the disturbances. The planner desires to determine if a potential fault has an adequate margin of safety without the system losing synchronism. A system is said to be synchronously stable (i.e., retain synchronism) for a given fault if the system variables settle down to some steady-state values as time approaches infinity after the fault is removed. These simulation studies are called transient stability studies. Transient stability is one of the important items which should be investigated in power System planning and its operation. Present day transient stability analyses are mainly performed by simulations. This method is very reliable method, but it does not suit calculations of many cases because it takes much computing time. As a substitute, direct method was proposed, and many papers have been reported for this method. It has reached to some level for a simple model in which generators are represented by constant voltages behind transient reactance. Since the time of [1], there has been considerable progress made in the development of the appropriate tools necessary to address stochastic transient stability. There have been numerous recent advances in the application of Lyapunov stability methods to stochastic differential equation systems [4]–[6]. Furthermore, the past decade has seen significant advances in the development of numerical integration methods to simulate stochastic (ordinary) differential equations [7]. In this paper, these advances in stochastic Lyapunov stability methods and the numerical solution of systems of stochastic differential equations are merged to present a novel approach to developing a quantitative measure of stability that is suitable for power system risk assessment. Transient stability analysis programs are MATLAB, PSCAD, ETAP, etc... In these simulation programs, the behavior of a power system is evaluated to determine its stability and/or its operating limits, or eventually, in order to determine the need for additional facilities. Important decisions are made based only on the results of stability studies. It is therefore important to ensure that the results of stability studies are as timely and accurate as possible. Thus, it is important for a power system to remain in a state of operating equilibrium under normal operating conditions as well as during the presence of a disturbance.

The main purpose of this paper is to investigate the transient stability of four machine six bus power system and three machine nine bus power system with energy function method, when subjected to disturbances. The transient energy consists of two components: kinetic and potential energy. In the post-disturbance period, profiles of the kinetic energy (VKE), the potential energy (VPE) are obtained. These are used to develop a criterion for the degree of stress on a disturbed but stable machine, and to assess the extent of instability for an unstable machine.

## II. STRUCTURED PRESERVED STOCHASTIC TRANSIENT ENERGY FUNCTIONS

The concept of transient stability is based on whether, for a given disturbance, the trajectories of the system states during the disturbance remain in the domain of attraction of the post-disturbance equilibrium when the disturbance is removed. Transient instability in a power system is caused by a severe disturbance which creates a substantial imbalance between the input power supplied to the synchronous generators and their electrical outputs. Some of the severely disturbed generators may “swing” far enough from their equilibrium positions to lose synchronism. Such a severe disturbance may be due to a sudden and large change in load, generation, or network configuration. Since large disturbances may lead to nonlinear behavior, Lyapunov functions are well-suited to determine power system transient stability. Since true Lyapunov functions do not exist for lossy power systems, so-called “transient energy functions” are frequently used to assess the dynamic behavior of the system [8]. From a modeling point of view, the structure preserved model allows a more realistic representation of power system components including load behaviors and generator dynamic models.

To better understand how the structure preserved transient energy function will be developed and analyzed; a brief review of Lyapunov functions for stochastic differential equations is first presented.

Consider the nonlinear stochastic system:

$$dx = f(x, t)dt + g(x, t)\Sigma(t)dW(t) \quad x(0) = x_0 \in \mathbb{R}^n \quad (1)$$

Whose solution can be written in the sense of :

$$x(t) = x_0 + \int_0^t f(x, s)ds + \int_0^t g(x, s)\Sigma(s)dW(s) \quad (2)$$

Where  $x(t) \in \mathbb{R}^n$  is the state  $W(t)$ ; is an  $m$ -dimensional standard Wiener process defined on the complete probability space  $(\mathcal{U}, \mathcal{F}, P, )$ ; the functions  $(f, g)$  are locally bounded and locally Lipschitz continuous in  $x \in \mathbb{R}^n$  with  $f(0, t) = 0, g(0, t) = 0$ , for all  $t \geq 0$ ; and the matrix  $\Sigma(t)$  is nonnegative-definite for each  $t \geq 0$ . These conditions ensure uniqueness and local existence of strong solutions to equation 1 [4], [9]. As with many nonlinear deterministic systems, Lyapunov functions can provide guidance regarding the stability of stochastic differential equation (SDE) systems. An SDE system is said to satisfy a stochastic Lyapunov condition at the origin if there exists a proper Lyapunov function  $V(x)$  defined in a neighborhood  $D$  of the origin in  $\mathbb{R}^n$  such that :

$$\mathcal{L}V(x) \leq 0 \quad (3)$$

for any  $x \in D \setminus \{0\}$  where the differential generator  $\mathcal{L}$  is given by :

$$\mathcal{L}V(x, t) = \frac{\partial V}{\partial x} f(x, t) + \frac{1}{2} \text{Tr} \{ \Sigma(t)^T g(x, t)^T \frac{\partial^2 V}{\partial x^2} g(x, t) \Sigma(t) \} \quad (4)$$

If (3) is satisfied, then the equilibrium solution  $x(t) \equiv 0$  of the stochastic differential equation 1 is considered to be *stable in probability* [10]. To accurately include the effects of the loads in the system, the so-called structure-preserved, center-of-inertia model of the power system is used, such that [2], [3].

$$\ddot{\theta}_i = \tilde{\omega}_i \quad (5)$$

$$M_i \ddot{\omega}_i = P_{M_i} - \sum_{j=1}^n B_{ij} V_i V_j \sin(\theta_i - \theta_j) - \frac{M_i}{M_T} P_{COI}$$

$$i = 1, \dots, m \quad (6)$$

$$0 = P_{d_i}^0 + D_i \dot{\theta}_i + \sum_{j=1}^n B_{ij} V_i V_j \sin(\theta_i - \theta_j) \quad (7)$$

$$0 = Q_{d_i}^0 + \sum_{j=1}^n B_{ij} V_i V_j \cos(\theta_i - \theta_j)$$

$$i = m + 1, \dots, n \quad (8)$$

Where,

$$\begin{aligned}\theta_i &= \delta_i - \delta_0 \\ \tilde{\omega}_i &= \omega_i - \omega_0\end{aligned}$$

And

$$\begin{aligned}\delta_0 &= \frac{1}{M_T} \sum_{i=1}^m M_i \delta_i ; \omega_0 = \frac{1}{M_T} \sum_{i=1}^m M_i \omega_i \\ M_T &= \sum_{i=1}^m M_i\end{aligned}$$

$$P_{COI} = \sum_{i=1}^m \{ P_{M_i} - \sum_{j=1}^n B_{ij} V_i V_j \sin(\theta_i - \theta_j) \} \quad (9)$$

Where,

$\delta_i$  generator rotor angle

$\theta_i$  COI bus angle

$\omega_i$  generator angular frequency

$\tilde{\omega}_i$  COI angular frequency

$M_i$  inertia constant

$P_{M_i}$  mechanical output

$V_i$  bus voltage

$B_{ij}$  ( $i, j$ )th entry of the reduced lossless admittance matrix

$D_i$  positive sensitivity coefficient representing the load frequency dependence

$m$  number of generators in the system

$n$  number of total buses in the system

$\omega_s$  synchronous speed in radians and  $P_{d_i}$  and  $Q_{d_i}$  are the load demands at each bus in the system.

and  $P_{d_i}$  and  $Q_{d_i}$  are the load demands at each bus in the system.

The corresponding energy function is [18]

$$\begin{aligned}V(\tilde{\omega}_{gi}, \theta, V) &= \\ &\frac{1}{2} \sum_{i=1}^m M_i \tilde{\omega}_{gi}^2 - \sum_{i=1}^m P_{M_i} (\theta_i - \theta_i^s) + \sum_{i=1}^{n+m} P_{d_i} (\theta_i - \theta_i^s) \\ &- \frac{1}{2} \sum_{i=1}^{n+m} B_{ii} (V_i^2 - (V_i^s)^2) + \sum_{i=1}^{n+m} \frac{Q_{d_i}^s}{a(V_i^s)^a} (V_i^a - (V_i^s)^a) \\ &- \sum_{i=1}^{n+m-1} \sum_{j=i+1}^{n+m} B_{ij} (V_i V_j \cos(\theta_i - \theta_j) - V_i^s V_j^s \cos(\theta_i^s - \theta_j^s))\end{aligned} \quad (10)$$

Where is usually 2 and the superscript “s” indicates the stable equilibrium point. It is assumed that the power system frequency deviations can be represented by an appropriately scaled Wiener process  $W_i(\mathbf{t})$  (i.e., zero mean, finite covariance). Note that this is why the load variation is represented by wiener process as opposed to a Gaussian noise input ( $dW_i(\mathbf{t})$ ). Therefore a frequency dependent load gives rise to:

$$P_{d_i} = P_{d_i}^0 + D_i \ddot{\theta}_i \tag{11}$$

$$= P_{d_i}^0 + D_i (\omega_i - \omega_s) \tag{12}$$

$$= P_{d_i}^0 + D_i W_i(t) \tag{13}$$

$$= P_{d_i}^0 (1 + \alpha_{p_i} W_i(t)) \tag{14}$$

Where  $\alpha_{p_i} = D_i/P_{d_i}^0$ . Similarly (but less commonly)

$$Q_{d_i} = Q_{d_i}^0 (1 + \alpha_{q_i} W_i(t))$$

$$\begin{aligned} \ddot{\theta}_i &= \ddot{\omega}_i \\ M_i \ddot{\omega}_i &= P_{M_i} - \sum_{j=1}^n B_{ij} V_i V_j \sin(\theta_i - \theta_j) - \frac{M_i}{M_T} P_{COI} \\ i &= 1, \dots, m \end{aligned}$$

$$0 = P_{d_i}^0 (1 + \alpha_{p_i} W_i(t)) + \sum_{j=1}^n B_{ij} V_i V_j \sin(\theta_i - \theta_j) \tag{15}$$

$$0 = Q_{d_i}^0 (1 + \alpha_{q_i} W_i(t)) + \sum_{j=1}^n B_{ij} V_i V_j \cos(\theta_i - \theta_j) \tag{16}$$

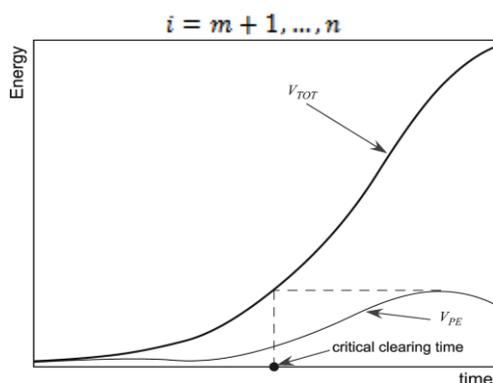


Figure 1. Total energy  $V_{TOT}$  versus the potential energy  $V_{PE}$

Critical clearing time is the time at which the total energy equals the maximum potential energy.

Now compute the energy function given in equation 10 and if the energy is less than the critical energy then the system is stable else the system is not stable.

### III. SIMULATION AND RESULTS

#### Four machine six bus system

The validity of the proposed method is shown by Simulation studies. For the simulation studies, we use four machine six bus systems shown in Fig. 2.

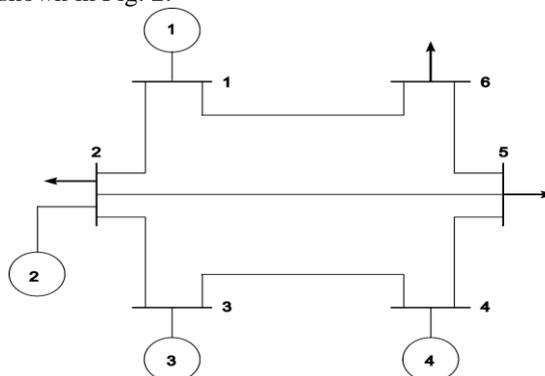


Figure 2. Four machine six bus system

TABLE I: Line data for 6 bus system

Transmission Line Data					
From Bus	To Bus	R (p.u)	X (p.u)	B (p.u)	Transformer Tap Setting Value (p.u)
1	2	0.05	0.2	0	1
2	3	0.10	0.5	0	1
3	4	0.20	0.8	0	1
4	5	0.10	0.3	0	1
5	6	0.20	0.4	0	1
6	1	0.10	0.15	0	1
2	5	0.20	0.5	0	1

TABLE II: Bus data for 6 bus system

Bus Data							
Bus No	Bus Type	V  (p.u)	$\theta$ (degree)	$P_G$ (MW)	$Q_G$ (MVAR)	$P_L$ (MW)	$Q_L$ (MVAR)
1	1	1	0	33.2	0	0	0
2	2	1	0	10	0	20	10
3	2	1	0	30	0	0	0
4	2	1	0	20	0	0	0
5	3	1	0	0	0	40	15
6	3	1	0	0	0	30	10

TABLE III: Generator data for 6 bus system

Generator Data			
Generator bus #	Transient reactance	Inertia constant	Generation
1	0.004	100.0	33.2
2	1.0	1.5	10.0
3	0.5	3.0	30.0
4	0.4	2.0	20.0

The above TABLE I, II and III gives the line, bus and generator data for three machine six bus system respectively.

TABLE IV: Energy variations of a 6 bus system

Time (sec)	Total Energy (p.u)	Potential Energy(p.u)
0.0000	1.2443	1.2188
0.0000	1.2443	1.2188
0.0001	1.2443	1.2188
0.0003	1.2443	1.2188
0.0014	1.2443	1.2188
0.0064	1.2454	1.2195
0.0114	1.2480	1.2211
0.0164	1.2519	1.2236
0.0214	1.2572	1.2270
0.0264	1.2639	1.2313
0.0314	1.2720	1.2365
0.0364	1.2815	1.2426
0.0414	1.2924	1.2495
0.0464	1.3047	1.2574
0.0500	1.3143	1.2636
0.0500	1.3143	1.2636
0.0517	1.3189	1.2665
0.0599	1.3418	1.2812
0.0910	1.4245	1.3341
0.1161	1.4856	1.3730
0.1322	1.5212	1.3956
0.1448	1.5468	1.4117
0.1574	1.5701	1.4265
0.1730	1.5958	1.4426

0.1921	1.6218	1.4588
0.2132	1.6431	1.4719
0.2381	1.6573	1.4803
0.2683	1.6581	1.4800
0.2982	1.6407	1.4681
0.3243	1.6110	1.4486
0.3396	1.5874	1.4333
0.3550	1.5595	1.4154
0.3740	1.5196	1.3898
0.3987	1.4594	1.3515
0.4276	1.3794	1.3010
0.4541	1.2996	1.2509
0.4784	1.2230	1.2030

**Critical Clearing Time = 0.1161 sec**

From the TABLE IV, we see that the system is stable with  $t_{cr}=0.1161$  sec and system becomes unstable after this  $t_{cr}$ . Therefore the critical clearing time 0.1161 sec. is proved when critical energy ( $V_{cr}$ ) is equal to the total energy, therefore critical clearing time at this point is 0.1161sec.

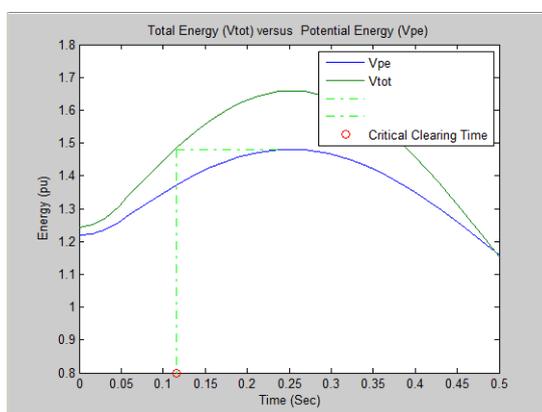


Figure 3. Total energy ( $V_{TOT}$ ) and potential energy ( $V_{PE}$ ) versus time for 6 bus system.

Critical clearing time is the time at which the total energy equals the maximum potential energy. From the Fig. 3 the critical clearing time is determined as 0.1161 sec.

### Three machine nine bus system

The validity of the proposed method is shown by Simulation studies. For the simulation studies, we use three machine nine bus systems shown in Fig. 4

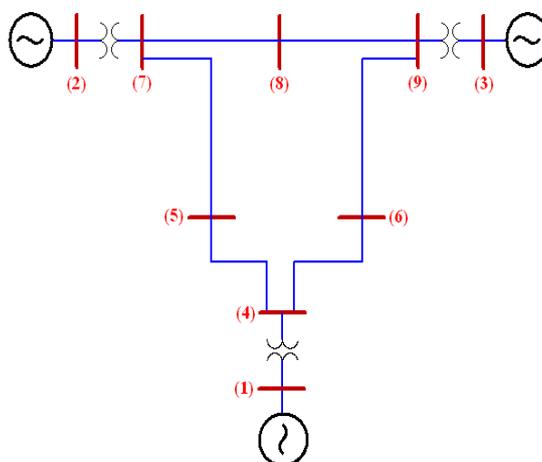


Figure 4. Three machine nine bus system

TABLE V: Line data for 9 bus system

No	Type	V  (p.u)	$\theta$ (deg)	$P_G$ (MW)	$Q_G$ (MVAR)	$P_L$ (MW)	$Q_L$ (MVAR)	$Q_{MIN}$ (MVAR)	$Q_{MAX}$ (MVAR)
1	1	1.04	0	0	0	0	0	0	0
2	2	1.025	0	163	0	0	0	-500	500
3	2	1.025	0	85	0	0	0	-500	500
4	3	1	0	0	0	0	0	-500	500
5	3	1	0	0	0	125	50	-500	500
6	3	1	0	0	0	90	30	-500	500
7	3	1	0	0	0	0	0	-500	500
8	3	1	0	0	0	100	35	-500	500
9	3	1	0	0	0	0	0	0	

TABLE VI: Bus data for 9 bus system

Branch no	From bus	To bus	R(p.u)	X(p.u)	B/2(p.u)	Tx. Tap
1	2	7	0.0	0.063	0.0	1
2	1	4	0.0	0.058	0.0	1
3	3	9	0.0	0.059	0.0	1
4	4	6	0.017	0.092	0.079	0
5	4	5	0.01	0.085	0.088	0
6	5	7	0.032	0.161	0.153	0
7	6	9	0.039	0.17	0.179	0
8	9	8	0.012	0.101	0.105	0
9	8	7	0.009	0.072	0.0745	0

TABLE VII: Energy variations of 9 bus system

Time (sec)	Total Energy (p.u)	Potential Energy (p.u)
0.1039	3.0790	2.1101
0.1325	3.5906	2.4743
0.1616	4.1355	2.8703
0.1899	4.6691	3.2682
0.2116	5.0724	3.5768
0.2334	5.4620	3.8822
0.2602	5.9128	4.2461
0.2908	6.3797	4.6369
0.3252	6.8293	5.0301
0.3661	7.2424	5.4119
0.4161	7.5504	5.7223
0.4732	7.6252	5.8331
0.5242	7.4418	5.6948
0.5695	7.0885	5.3860
0.6104	6.6281	4.9712
0.6382	6.2465	4.6268
0.6575	5.9531	4.3638
0.6768	5.6394	4.0854
0.6944	5.3386	3.8220

**Critical Clearing Time = 0.2602 sec**

From the TABLE VII, we see that the system is stable with  $t_{cr}=0.2602$  sec and system becomes unstable after this  $t_{cr}$ . Therefore the critical clearing time 0.2602 seconds is proved when critical energy ( $V_{cr}$ ) is equal to the total energy, therefore critical clearing time at this point is 0.2602 sec.

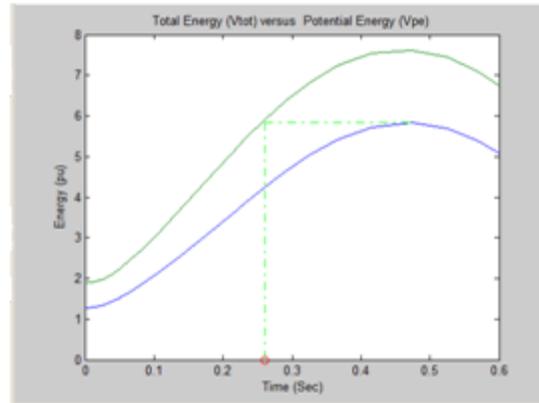


Figure 5. Total energy ( $V_{TOT}$ ) and potential energy ( $V_{PE}$ ) versus time for 9 bus system.

Critical clearing time is the time at which the total energy equals the maximum potential energy. From the fig. 5 the critical clearing time is determined as 0.2602 sec.

#### IV. CONCLUSION

This paper develops an approach to analyze the impact of random load and generation variations on the transient stability of a structure preserved power system. The well-known energy function method for power system transient stability is used as a basis to explore the stochastic power system stability through a stochastic Lyapunov stability analysis. The stability of the system is analyzed based on the Critical Clearing Time of the system. It is concluded that this type (direct method) of analysis provides fast computing time compared to other type of analysis. Further work may include exploring the impact of non-Gaussian distributions on critical clearing times. An additional area of study would include modeling the stochastic behavior of generation scheduling.

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