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Heat and Mass Transfer in MHD Free Convection Flow over an Inclined Plate with Hall Current

Mohammad Shah Alam[1], Mohammad Ali, [1] Md. Delowar Hossain
[1], *Department of Mathematics*
Chittagong University of Engineering and Technology (CUET), Chittagong-4349, Bangladesh

--ABSTRACT--
In the present paper is an investigation of heat and mass transfer characteristics of MHD free convection of steady flow of an incompressible electrically conducting fluid over an inclined plate under the influence of an applied uniform magnetic field and the effects of Hall current are taken into account. Using suitable similarity transformations the governing equations of the problem are reduced to couple nonlinear ordinary differential equations and are solved numerically by Runge- Kutta fourth-fifth order method using symbolic software. The solution is found to be dependent on several governing parameters, including the magnetic field strength parameter, Schmid number, the inclination angle from the vertical direction, modified magnetic field and secondary magnetic parameter. The numerical results concerned with the velocity, secondary velocity, temperature and concentration profiles effects of various parameters on the flow fields are investigated and presented graphically.

KEY WORDS: *Boundary layer flow, Free convection, Hall current, Heat and Mass transfer, Inclined plate, MHD*

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I. INTRODUCTION

The study of boundary layer flow heat and mass transfer over an inclined plate has generated much interest from astrophysical, renewable energy systems and also hypersonic aerodynamics researchers for a number of decades. In recent years

MHD	Magnetohydrodynamics
c_p	Specific heat of with constant pressure
g	Gravitational acceleration
g_0	Secondary Velocity
f'	Velocity Profile
M^*	Modified magnetic field
M^{**}	Secondary magnetic parameter
M	Magnetic parameter
m	Hall parameter
ν	Kinematic viscosity
α	Thermal diffusivity
β	Thermal Expansion Coefficient
β^*	Coefficient of expansion with concentration
ρ	Density
σ	Fluid electrical conductivity
θ	Dimensionless temperature
u	Velocity component in x-direction
v	Velocity component in y-direction
w	Secondary Velocity
T	Temperature
D	Thermal molecular diffusivity
C	Concentration
C_∞	Concentration of the fluid outside the boundary layer

G_r	Grashof number
S_c	Schmidt number
N	Buoyancy ratio
B_0	Constant magnetic field intensity
T_w	Temperature at the Plate
T_∞	Temperature of the fluid outside the boundary layer
ψ	Stream function
γ	Inclination of the Plate
η	Similarity variable
ξ	Similarity variable
Subscripts	
W	Quantities at wall
∞	Quantities at the free stream

MHD flow problems have become in view of its significant applications in industrial manufacturing processes such as plasma studies, petroleum industries Magnetohydrodynamics power generator cooling of clear reactors, boundary layer control in aerodynamics. Many authors have studied the effects of magnetic field on mixed, natural and force convection heat and mass transfer problems. Indeed, MHD laminar boundary layer behavior over a stretching surface is a significant type of flow having considerable practical applications in chemical engineering, electrochemistry and polymer processing. This problem has also an important bearing on metallurgy where magnetohydrodynamic (MHD) techniques have recently been used. Rapits and Singh [27] studied the effects of uniform transverse magnetic field on the free convection flow of an electrically conducting fluid past an infinite vertical plate for the classes of impulsive and uniformly accelerated motions of the plate.

Hossain [26] studied the effects of viscous and Joule heating on the flow of viscous incompressible fluid past a semi-infinite plate in presence of a uniform transverse magnetic field. The combined effects of forced and natural convection heat transfer in the presence of a transverse magnetic field from vertical surfaces are also studied by many researchers.

Duwairi and Al-Kablawi [7] formulated the MHD conjugative heat transfer problem from vertical surfaces embedded in saturated porous media. Chen [6] investigated the momentum. Heat and mass transfer characteristics of MHD natural convection flow over a permeable, inclined surface with variable wall temperature and concentration, taking into consideration the effects of Ohmic heating and viscous dissipation. Seddeek [28] analyzed the effect of variable viscosity and magnetic field on the flow and heat transfer past a continuously moving porous plate. Abdelkhalek [5] investigated the effects of mass transfer on steady two-dimensional laminar MHD mixed convection flow. Raghava Rao and Sekhar[12] considered the problem of steady 2-D , incompressible MHD flow past a circular cylinder with an applied magnetic field parallel to the main flow. Chowdhury and Islam [2] presented a theoretical analysis of a MHD free convection flow of a viso-elastic fluid adjacent to a vertical porous plate. Singh,P.K.[20] studied Heat and Mass Transfer in MHD Boundary Layer Flow past an Inclined Plate with Viscous Dissipation in Porous Medium. Hall effects on MHD boundary layer flow over a continuous semi- infinite flat plate moving with a uniform velocity in its own plane

in an incompressible viscous and electrically conducting fluid in the presence of a uniform transverse magnetic field were investigated by Watanabe and Pop [24]. Aboeldahab and Elbarbary [15] studied the Hall current effects on MHD free-convection flow past a semi-infinite vertical plate with mass transfer. The effect of Hall current on the steady magnetohydrodynamics flow of an electrically conducting, incompressible Burger's fluid between two parallel electrically insulating infinite planes was studied by Rana et al. [10]. Crane [21] first introduced the study of steady two-dimensional boundary layer flow caused by a stretching sheet whose velocity varies linearly with the distance from a fixed point in the sheet. The influence of a uniform transverse magnetic field on the motion of an electrically conducting fluid past a stretching sheet was investigated by Pavlov [25], Chakraborty and Gupta [19], Kumari et al. [22], Andersson [17], Andersson et al. [18] .The effect of chemical reaction on free-convective flow and mass transfer of a viscous, incompressible and electrically conducting fluid over a stretching sheet was investigated by Afify [16] in the presence of transverse magnetic field. Samad and Mohebujjaman [9] investigated the case along a vertical stretching sheet in presence of magnetic field and heat generation. Jhankal and Kumar [11] studied MHD Boundary Layer Flow Past a Stretching Plate with Heat Transfer Alam et.al [1] studied the combined effect of viscous dissipation and Joule heating on steady MHD free convective heat and mass transfer flow of a viscous incompressible fluid past a semi-infinite inclined radiate isothermal permeable moving surface in the presence of thermophoresis. The effect of thermal radiation on the forced or free convection flows are on the focus of research interest due to many applications especially involving high temperatures. Alam et al. [4] analyzed a two-dimensional steady MHD mixed convection and mass transfer flow over a semi-infinite porous inclined plate in the presence of thermal radiation with variable suction and thermophoresis.

Ganesan and Palani [3] studied the problem of unsteady natural convection flow of a viscous incompressible electrically conducting fluid past an inclined plate with variable heat and mass flux's. Orthan Aydm and Ahmet Kaya [8] studied MHD mixed convective heat transfer flow about an inclined plate. Since the study of heat and mass transfer is important in some cases, in the present paper we studied the Hall effects on the steady MHD free-convective flow and mass transfer over an inclined stretching sheet in the presence of a uniform magnetic field. The boundary layer equations are transformed by a similarity transformation into a system of coupled non-linear ordinary differential equations and which are solved numerically by Runge- Kutta fourth-fifth order method using symbolic software. Numerical calculations were performed for various values of the magnetic parameter, Hall parameter Schmid number, the inclination angle from the vertical direction, modified magnetic field and secondary magnetic parameter. The results are discussed from the physical point of view. Such a study is also applicable to the elongation to the bubbles and in bioengineering where the flexible surfaces of the biological conduits, cells and membranes in living systems are typically lined or surrounded with fluids which are electrically conducting (e.g., blood).

1.2 Mathematical formulation of the Problem

Consider a two dimensional steady laminar MHD viscous incompressible electrically conducting fluid along an inclined plate with an acute angle γ. X direction is taken along the leading edge of the inclined plate and y is normal to it and extends parallel to x-axis.

A magnetic field of strength B_0 **is** introduced to the normal to the direction to the flow. The uniform plate temperature T_w ($>T_\infty$), where T_∞ is the temperature of the fluid far away from the plate. Let u, v and w be the velocity components along the x and y axis and secondary velocity component along the z axis respectively in the boundary layer region. The sketch of the physical configuration and coordinate system are shown in Fig.1.

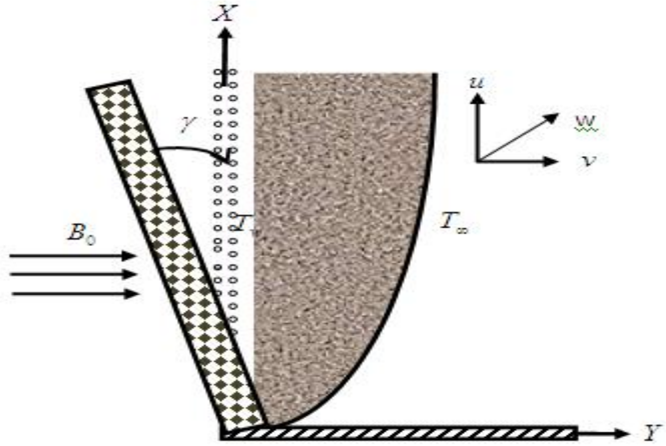

Fig.1 Physical configuration and coordinate system.

Under the above assumptions and usual boundary layer approximation, the dimensional governing equations of continuity, momentum, Concentration and energy under the influence of externally imposed magnetic field are:

Equation of continuity: $$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

Momentum equation:

$$u\,\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}+g\beta(T-T_\infty)cos\gamma+g\beta^*(C-C_\infty)cos\gamma-\frac{\sigma B_0^2}{\rho(1+m^2)}(u+mw) \tag{2}$$

$$u\,\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}=\nu\frac{\partial^2 w}{\partial y^2}+\frac{\sigma B_0^2}{\rho(1+m^2)}(mu-w) \tag{3}$$

Energy Equation: $u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{v}{c_p}\left(\frac{\partial u}{\partial y}\right)^2+\frac{\sigma B_0^2}{\rho c_p}u^2,\ \alpha=\frac{k}{\rho c_p}$ (4)

Concentration Equation: $u\frac{\partial c}{\partial x}+v\frac{\partial c}{\partial y}=D\frac{\partial^2 c}{\partial y^2}$ (5)

Boundary conditions are :

$$\left.\begin{aligned}&u=0,\ \ v=w=0,\ T=T_w, C=C_w \quad at \quad y=o\\ &u=0, w=0, \quad T=T_\infty, C=C_\infty \quad as\ y\to\infty\end{aligned}\right\} \tag{6}$$

To convert the governing equations into a set of similarity equations, we introduce the following transformation:

$$\left.\begin{aligned}&w=U_0 g_0(\xi,\eta),\ \ \eta=\frac{y}{x}\left(\frac{G_r}{4}\right)^{\frac{1}{4}}, \xi=\frac{g\beta x}{c_p}, \psi(\xi,\eta)=4v\left(\frac{G_r}{4}\right)^{\frac{1}{4}}f(\xi,\eta)\\ &T-T_\infty=\theta(\eta)(T_w-T_\infty), C-C_\infty=\varphi(\eta)(C_w-C_\infty)\end{aligned}\right\} \tag{7}$$

From the above transformations, the non-dimensional, nonlinear and coupled ordinary differential equations are obtained as

$$f'''+4f'^2-\frac{M}{1+m^2}f'+\theta\cos\gamma+N\varphi\cos\gamma-\frac{M^*m}{1+m^2}g_0=4\xi\left(f'\frac{\partial f'}{\partial\xi}-f''\frac{\partial f}{\partial\xi}\right) \tag{8}$$

$$g''_0+\frac{M^{**}m}{1+m^2}f'-\frac{2M}{1+m^2}g_0=4\xi\left(f'\frac{\partial g_0}{\partial\xi}-g'_0\frac{\partial f}{\partial\xi}\right) \tag{9}$$

$$\theta''=4\xi\left[f'\frac{\partial\theta}{\partial\xi}-\theta'\frac{\partial f}{\partial\xi}-f''^2-Mf'^2\right] \tag{10}$$

$$\varphi''=4S_c\xi\left[f'\frac{\partial\varphi}{\partial\xi}-\varphi'\frac{\partial f}{\partial\xi}\right] \tag{11}$$

Where the notation primes denote differentiation with respect to η and the parameters are defined as

$$\left.\begin{aligned}&G_r=\frac{g\beta x^3(T_w-T_\infty)}{v^2},\ M=\frac{2\sigma B_0^2 x^2}{\mu G_r^{1/2}}, M^*=\frac{\sigma U_0 B_0^2}{\rho g\beta(T_w-T_\infty)}\\ &M^{**}=\frac{\sigma B_0^2 x}{U_0\rho}, S_c=\frac{v}{D},\ \xi=\frac{g\beta x}{c_p}, N=\frac{\beta^*(C_w-C_\infty)}{\beta(T_w-T_\infty)}\end{aligned}\right\} \tag{12}$$

The transform boundary conditions:

$$\left.\begin{aligned}&f'(\xi,0)=0, f(\xi,0)=1, g_0(\xi,0)=0, \theta(\xi,0)=1, \varphi(\xi,0)=1, at\ \eta=0\\ &f'(\xi,\infty)=g_0(\xi,\infty)=\theta(\xi,\infty)=\varphi(\xi,\infty)=0 \quad \text{as } \eta\to\infty\end{aligned}\right\} \tag{13}$$

1.3 Results and Discussion:

The system of transformed ordinary differential equations (8)-(11) subject to the boundary conditions (13) is solved numerically by Runge- Kutta fourth-fifth order method using symbolic software. Representative velocity profiles for three typical values of M (5.0, 7.0, 9.0), m (1.0, 2.0, 10.0), N (1.0, 1.2, 1.4), $M^*(1.0, 1.2, 1.4)$ and $\gamma(30^0, 36^0, 45^0)$ respectively are presented in Fig. 2 - Fig.11. For $\xi=1, m=1.0, M^*=1.0, N=1.0, \gamma=30^0$, it is revealed from Fig. 2 - Fig. 7 that , the velocity is decreased by increasing the values of magnetic field strength, buoyancy ratio and angle of inclination but from Fig. 8 and Fig. 9 velocity is increased for increasing values of Modified magnetic field. It is interesting from Fig.10 and Fig.11, observed that the velocity is increased for Hall Effect $(0\le m<2.5)$ and then decreased. The effects of secondary modified magnetic and Hall parameters on the secondary velocity are shown in Fig. 12 - Fig.14.It is observed from these figures that there is no effect of those parameters on secondary velocity. The effects of various parameters on non-dimensional temperature and concentration are shown in Fig. 15 and Fig.16. It is observed from these figures that there is no effect of magnetic parameter M and Schmidt number, S_c on the temperature field and concentration.

II. CONCLUSIONS

From the study the following conclusions can be drawn:

[1] The velocity profiles for various of M, N and γ is decreased but velocity is increasing for M^*.
[2] The velocity profiles for the effect of m, in the certain interval is increased then decreased.
[3] There is no effect of various parameters on Secondary velocity, temperature and concentration.

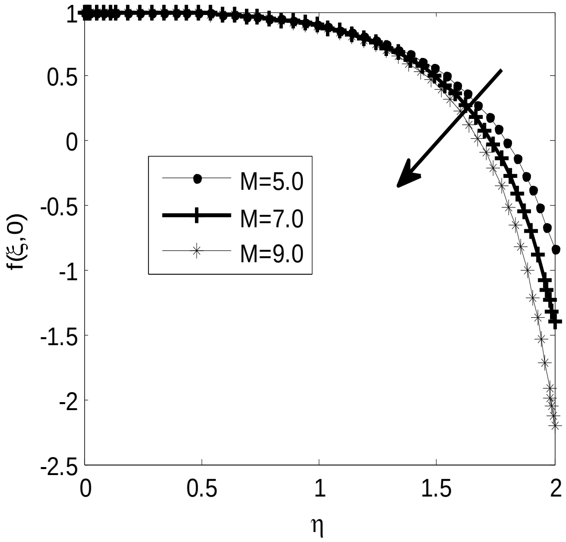

Fig.2: Velocity profile for various values of M

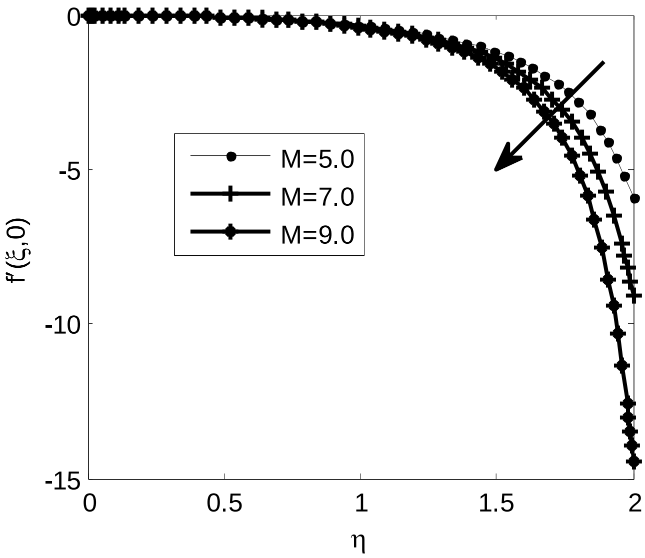

Fig.3: Velocity profile for various values of M

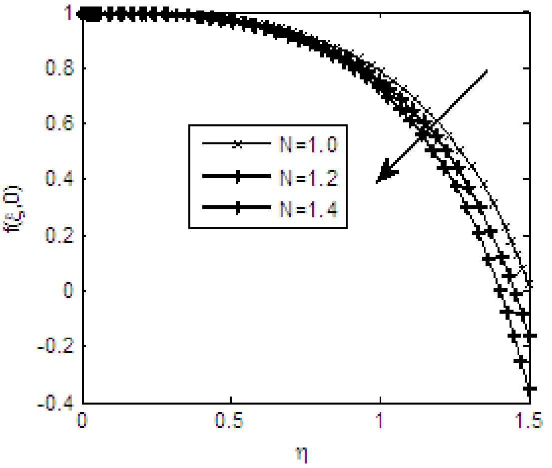

Fig.4: Velocity profile for various values of N

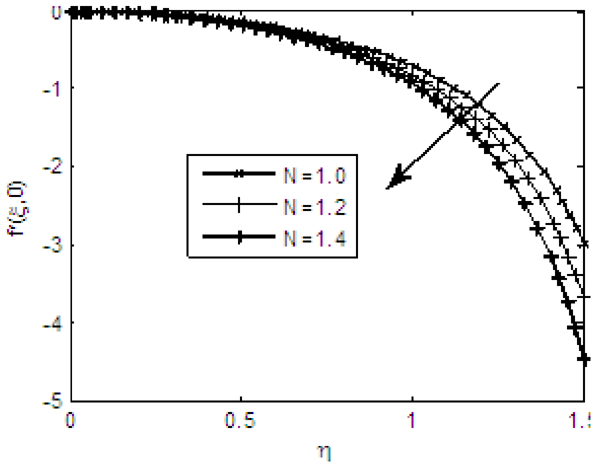

Fig.5: Velocity profile for various values of N

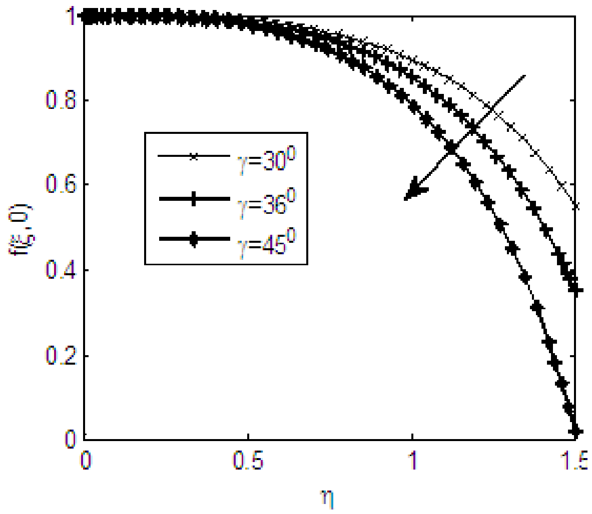

Fig.6: Velocity profile for various values of γ

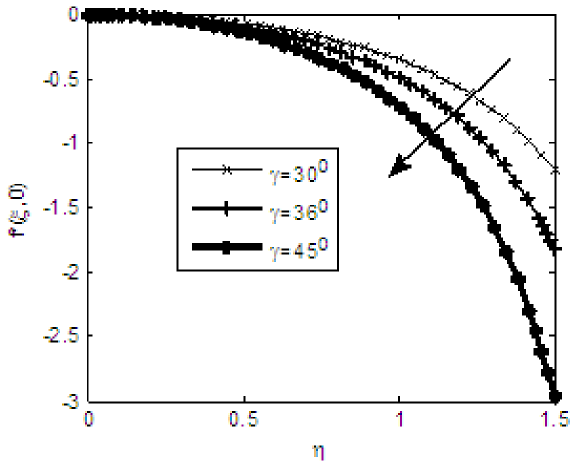

Fig.7: Velocity profile for various values of γ

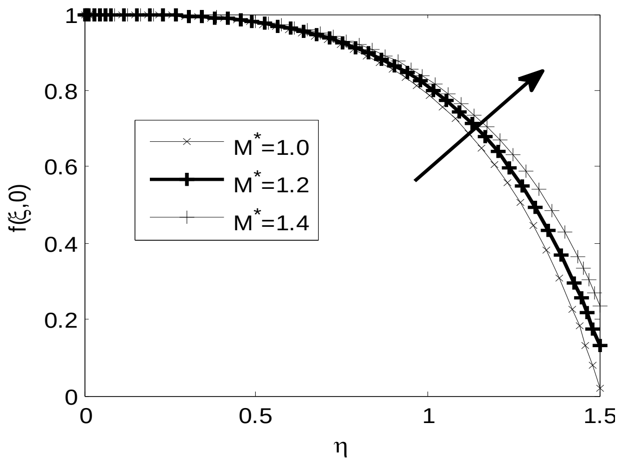

Fig.8: Velocity profile for various values of M^*

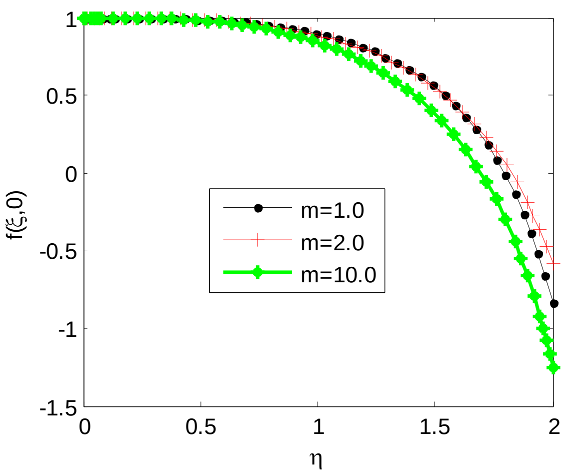

Fig.10: Velocity profile for various values of m

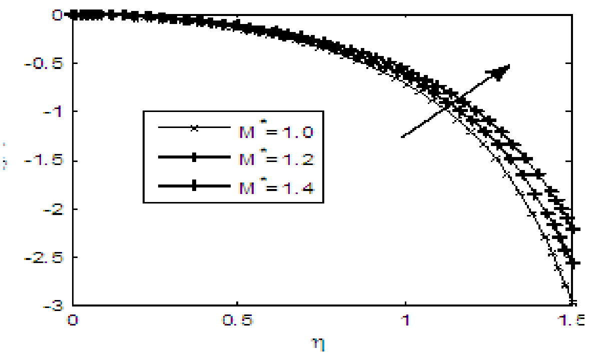

Fig.9: Velocity profile for various values of M^*

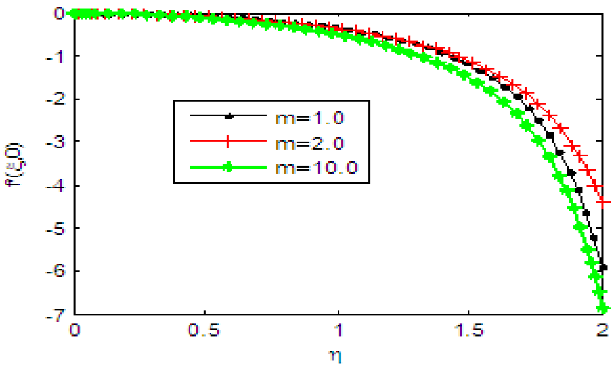

Fig.11: Velocity profile for various values of m

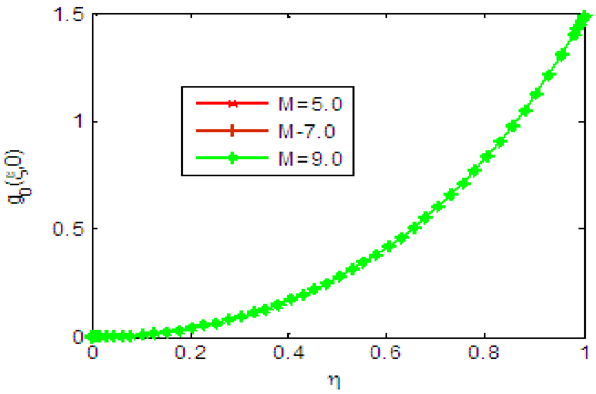

Fig.12: Velocity profile for various values of M

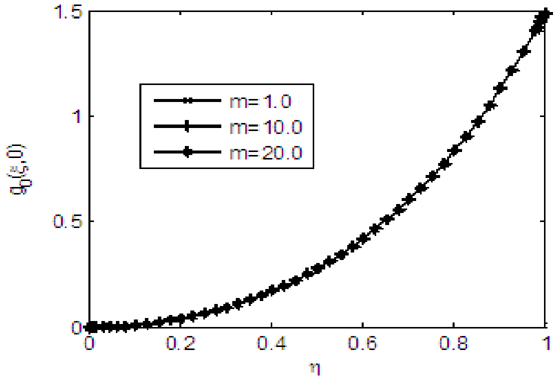

Fig.13: Velocity profile for various values of m

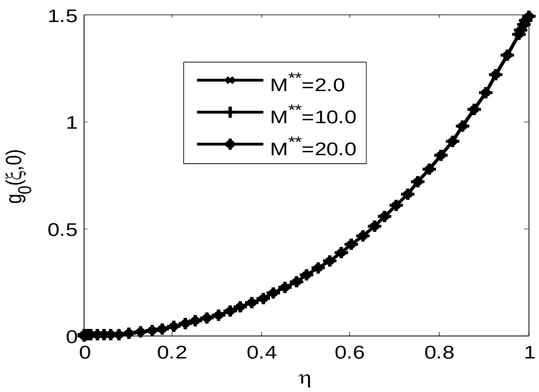

Fig.14: Velocity profile for various values of M**

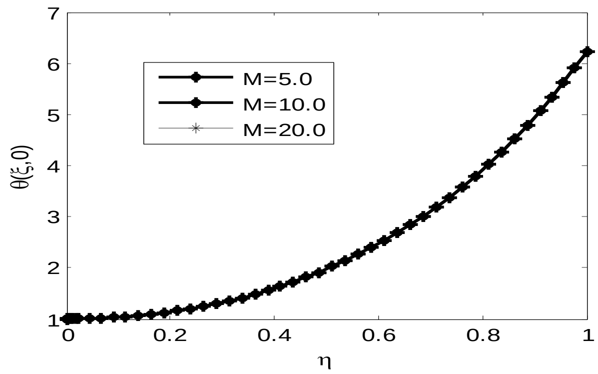

Fig.15: Temperature profile for various values of M

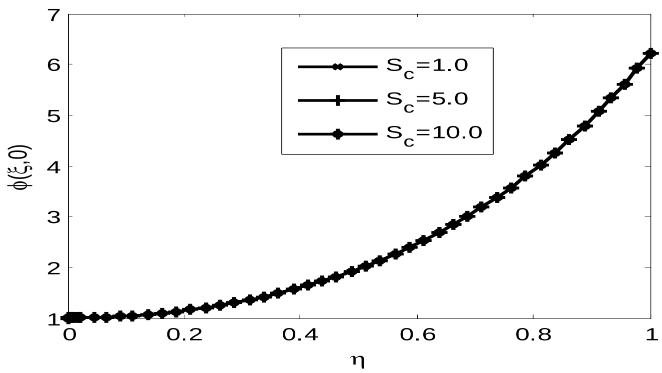

Fig.16: Concentration profile for various values of S_c

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