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An Experimental Design Method for Solving Constrained Optimization Problem

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ABSTRACT
This paper presents a new version of super convergent line series for solving optimization problems. It has proven to be more precise because it converges very fast. This method does not constitute the non-negativity of the constraint equations. Unlike other general purpose solution methods of solving constrained optimization problems it guarantees global optimal solution. Useful information about the direction of search $\underline{d}$, *the step-length* δ *and the optimal point search* $\left(\underline{\bar{x}}^*\right)$ *were provided by this method.*

***KEY WORDS**: Optimal Design, Optimality Criteria, Stopping Rule, Regression Modeling, Convergence Test and Feasibility Checking.*

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I. INTRODUCTION:

OPTIMIZATION PROBLEM

An optimization problem is a well known problem in operations research. Optimization problems as defined by Inyama (2007) are problems that seek to maximize or minimize a given quantity called the objective function which depends on a finite number of input variables. These input variables may be independent or related through one or more constraints. The linear constraint in optimization problems guarantees a convex solution space. The constraints of linear optimization problem can be either equality constraints or inequality constraints. Equality constraints problem can be represented as:

Max (min) $Z = f(x)$ … (1)

Subject to : $g_i(x) = \alpha_i$ … (2)

While inequality constraints can be represented as

Max (Min) $Z = f(x)$ … (3)

Subject to: $g_i(x) \geq$ or $\leq \alpha_i$ … (4)

Many real life problems can be stated as constrained numerical optimization problems.
There are many constrained optimization problems with simply given postulated solutions, and there are different solution algorithms in solving different kinds of linearly constrained optimization problems.

This proposed algorithm introduces a new method of solving linearly constrained optimization problems that is so effective were other classical methods are not.

II. OPTIMAL DESIGN OF EXPERIMENTS

Optimal designs are class of experimental designs that are optimal with respect to some statistical criterion.In design of experiments for estimating statistical models, optimal designs allow parameters to be estimated without bias and with a minimum variance. A non-optimal design requires a greater number of experimental runs to estimate the parameters with the same precision as an optimal design.
The optimality of a design depends on the statistical model and is assessed with respect to a statistical criterion, which is related to the variance matrix of the estimator. Optimal designs are called optimum designs.

III. OPTIMALITY CERTERIA

Onukogu (1997) cited some optimality criteria of a design of experiment.

- **G-optimality criteria**: This criterion minimizes the maximum variance of the estimate of the surface.
- **D-optimality criteria**: This criterion ensures that the determinant of the information matrix is maximized.

- **D_s-optimality criteria**: This criterion is appropriate when an experimenter's interest is on estimating a subject of S parameters very precisely.
- **A-optimality criteria**: The average variance of the parameter estimates is minimized.
- **E-optimality criteria**: In this type of criterion, the variance of the least well-estimated contrast is minimized subject to the constraint.

IV. CONDITIONS FOR CONVERGENCE OF A LINE SERIES

This condition follows kolmogorov's criterion for convergence.

A sufficient condition for a series of independent random variables $\{\underline{x}_j\}$ with finite variance $\{\delta^2{}_j\}$ to be convergent with probability one is for,

$$\sum \delta_j^2 / j^2 < \infty \qquad \ldots \qquad (5)$$

At the jth step, let the sequence be defined by the equation.

$$\underline{x}_j = \underline{x}_{j-1} - \rho_{j-1}\underline{d}_{j-1} \qquad \ldots \qquad (6)$$

Let X_j be an N x n design matrix and X_j, be spanned by the vector

$$\{x_{1j}, x_{2j}, \ldots x_{nj}\} \qquad \ldots \qquad (7)$$

Then the column space of the matrix at $\underline{X}_{j+1}$ is spanned by the vector

$$\{x_{1j} - \rho_j d_{1j}, x_{2j} - \rho_j d_{2j}, \ldots, x_{nj} - \rho_j d_{nj}\} \qquad \ldots \qquad (8)$$

Hence the design matrices at j_{+1} step are:

$$X_{j+1} = X_j - \rho_j \underline{1}^1 \underline{d}_j \qquad \ldots \qquad (9)$$

$\underline{1}^1 = (1,1,\ldots,1)^1$ is an N- component vector of ones.

And the information matrix at j + 1 step is

$$X_{j+1}^1 X_{j+1} = X_j^1 X_j + N_j \rho_j^2 d_j \hat{d}_j \underline{1}^1 X_j = 0 \qquad \ldots \qquad (10)$$

See Onukogu (1997)

V. SIGNIFICANCE OF STUDY

This paper is important as its outcome will add to knowledge on the methods of solving constrained optimization problems. This method does not require partitioning or segmentation unlike the initial method of super convergent line series. This work is a contribution to the field of operations research. It will be relevant to all using quadratic optimization method to solve certain problems as facing real life. It will be useful to researcher as it will help to prove other arguments as concerns constrained optimization problems.

Finally, it will help to guide researchers who want to carry out similar study on how to go about their work.

VI. SCOPE OF STUDY

This work shows how a line search algorithm is used to solve constrained optimization problems.

Using a polynomial objective function $f(\overline{x})$ of degree m, where m ≥ 2, subject to linear constraints,

$$\tilde{x} = \{C_k = c_{ik}x_1 + c_{2k}x_2 + \ldots + c_{mk}x_n \geq = \leq b_k; k = 1,2,\ldots, K\} \qquad \ldots \qquad (11)$$

the use of first order linear regression introduces an element of bias;

Thus from the 1st order partial derivative, we have

$$\frac{\partial f(x)}{\delta \underline{x}} = \underline{a}_1 + w\underline{a}_2 \qquad \ldots \qquad (12)$$

which is the first order model = $\underline{a}_1 + w\underline{a}_2$, … (13)

where $\underline{a}_1 = \begin{pmatrix} a_{01} \\ a_{02} \\ \vdots \\ a_{0n} \end{pmatrix}$, $\underline{a}_2 = \begin{pmatrix} a_{B0} \\ a_{B1} \\ \vdots \\ a_{Bn} \end{pmatrix}$

and $w\underline{a}_2$ is the biasing element/part

A matrix X was generated from the function f(X), the convex combination and the direction becomes

$$\underline{d} = H\underline{a}_1 + (I - H)W\underline{a}_2 \text{ and } \underline{d} = \begin{pmatrix} d_0 \\ d_1 \\ \vdots \\ d_n \end{pmatrix} \quad \ldots \quad (14)$$

The iterate was evaluated as

$$\underline{x}^* = \underline{\bar{x}} - \underline{\rho}\underline{d} \quad \ldots \quad (15)$$

where $\underline{\bar{x}}$ is the vector of the starting point,

$\underline{\rho}$ is the step length

$\underline{d}$ is the direction

This paper is to develop a new version of super convergent line series for solving constrained optimization problems with a necessity to partition the feasible region, and to show that the new method is globally convergent.

Literature Review

There are many optimization problems with simply given postulates known solutions. There are also different algorithms for solving different kinds of linearly constrained optimization problem.

One simple method for solving optimization problem is the gradient descent method. Fletcher (1981) explained this method as a first order optimization algorithm to find a local minimum of a function. He also showed that using Gradient descent method, steps taken are proportional to the negative of the Gradient or of the approximate Gradient of the function at the current point if instead of one takes steps proportional to the gradient one approaches the local maximum of that function. The procedure is known as Gradient ascent or steepest decent or method of steepest descent. This method yields zigzag phenomena in solving practical problems. The algorithm converges to an optimal solution very slowly or even fails to converge when solving large scale minimization problem.

Karmarkar (1984) developed another method of solving optimization problem known as interior point or Newton Barrier method. This method solves inequality constraint problem. It is characterized by preserving strict constraint feasibility at all times, by using a barrier term which is infinite on the constraint boundaries. This can be advantageous if the objective function is not defined when the constraints are violated. The sequence of minimizer is also feasible. The two most important cases are the inverse barrier function

$$\varnothing (X, r) = f(X) + r \sum, \{C_i (X)\}^{-1} \quad \ldots \quad (16)$$

and the logarithmic barrier function

$$\varnothing (X, r) = f(X) - r \sum \log [C_i (X)] \quad \ldots \quad (17)$$

The coefficient r is used to control the barrier function iteration. In this case sequence $\{r^{(k)}\} \rightarrow o$ is chosen which ensures that the barrier term becomes more and more negligible except close to the boundary.

Also X $(r^{(k)})$ is defined as $\varnothing$ (X, $r^{(k)}$). The barriers method has a difficulty of locating the minimizer due to ill-conditioning and large gradients. It is undefined for infeasible points and the simple expedient of setting it to infinity can make the line search inefficient. Also the singularity makes conventional quadratic or cubic interpolation in the line search work less efficiently. For this reason, Fletcher and McMann (1969) recommended special purpose line searches.

Another difficulty is that an initial interior feasible point is required and this in itself is a non-trivial problem involving the strict solution of a set of inequalities. In view of these difficulties and the general inefficiency of sequential techniques, barrier functions currently attract little interest. The calculation of the direction is the most time consuming step of the interior point method.

Another method of solving quadratic optimization problem is developed by Phil-Wolf and Frank in (1956) known as Frank and Phil-Wolf algorithm. This algorithm is also known as convex combination algorithm.

Fletcher (1981) explained that each step of the algorithm linearizes the objective function and a step is taken in a direction that reduces the objective function while maintaining feasibility.Fang and Li (1999) designed an efficient solution procedure for solving an optimization problem with one linear objective function and fuzzy relation equation constraints. A more general case of the problem, an optimization model with one linear objective function and finitely many constraints of fuzzy relation inequalities was presented in the work.

Compared with the known methods, the proposed algorithm shrinks the searching region and hence obtains an optimal solution fast. For some special cases the proposed algorithm reaches an optimal solution very fast since there is only one minimum solution in the shrunk searching region and hence obtains an optimal solution fast.

At the end of the work, two numerical examples were given to illustrate the differences between the proposed algorithm and the known ones. The work studied the new linear objective function optimization with respect to the relational inequality in the constraints. The fuzzy inequality enabled them to attain the optimal points that are better solution than those results from the resolution of the similar problem with ordinary inequality constraints. The work thus presented an algorithm to generate such optimal solution in quadratic optimization problems. There are several other classical local search algorithms and their extensions which can be used in solving quadratic optimization problems.

VII. METHODOLOGY

ALGEBRAIC DEVELOPMENT OF THE ALGORITHM

The general formulation of the problem is to find the optimizer particularly the minimum of the given objective function f(x) subject to K-constraints.

In the common polynomial regression model of degree m of n variant, we consider the problem of determining the optimal of a given function subject to certain constraints:

$$\tilde{x} = \underline{x}, c^1 \underline{x} \leq, =, \geq, \underline{b}_k \} \qquad \ldots \qquad (18)$$

We represent f(x) with $\tilde{x}$ by 1st – order linear equation

$$\underline{y}(x) = x\underline{a} + e \qquad \ldots \qquad (19)$$

To generate matrix (X),

$$y(\underline{x}) = \frac{\partial f}{\partial x_i} = \underline{b} \qquad \ldots \qquad (20)$$

$$\frac{\partial f}{\delta x_i} = \underline{a}_1 + w\underline{a}_2 = b \qquad \ldots \qquad (21)$$

i.e. taking partial derivative of f(X) w.r.t.X

The direction $\underline{d}$ becomes

$$\underline{d} = H\underline{a}_1 + (I - H)W\underline{a}_2 = B\underline{a} \qquad \ldots \qquad (22)$$

and,

$$\underline{d} = \begin{pmatrix} d_0 \\ d_1 \\ : \\ d_n \end{pmatrix} \qquad \ldots \qquad (23)$$

$$\underline{a}_1 = \begin{pmatrix} a_{01} \\ a_{02} \\ \vdots \\ a_{0n} \end{pmatrix} \qquad \ldots \qquad (24)$$

$$a_2 = \begin{pmatrix} a_{Bo} \\ a_{B1} \\ \vdots \\ a_{Bn} \end{pmatrix} \qquad \ldots \qquad (25)$$

$$dd_0 = h_{01}a_0 + (1-h)a_{B0}$$
$$d_1 = h_{11}a_1 + (1-h_{11})a_{B1} \qquad \ldots \qquad (26)$$
$$\vdots$$
$$d_n = h_{n1}a_n + (1-h_{n1})a_{Bn}$$

Making change of parameters to obtain an equivalent model

$$\text{Var}(d) = H\ \text{Var}(a_1)\ H^1 + (I-H)\ W\ \text{var}(a_2)W^1\ (I-H)^1 \pm 2\,\text{cov} \qquad \ldots \qquad (27)$$

Where $H = \begin{pmatrix} h_{0_1} & & & \\ & h_{11} & & \\ & & \vdots & \\ & & & h_{n_1} \end{pmatrix}$

$X = X_1, X_2$

$$\text{var}\begin{pmatrix} \underline{a} \\ \underline{a}_2 \end{pmatrix} = \begin{pmatrix} \text{var}(\underline{a}_1) & \text{cov}(\underline{a}_1 \bar{\underline{a}}_2) \\ \text{cov}(\underline{a}_2 \underline{a}_1) & \text{var}(\underline{a}_2) \end{pmatrix}$$

$$HH^1 + (I-H)\ WW^1\ (I-H)^1 = \underline{\bar{1}}$$

$$(H(I-H)W)\begin{pmatrix} H \\ ((I-H)W)^1 \end{pmatrix}$$

$$BB^1 = \underline{\bar{1}}$$

Note. B is normalized using the Gaussian elimination techniques as cited by Onukogu (1997).

$$\therefore \quad M = B(X^1X)\ B^1 \qquad \ldots \qquad (28)$$

Ƶ is obtained by partitioning the M-matrix and substituting in to the objective function values of X_1 andX_2.

$$\mathbf{M} = \left(\begin{array}{c|ccc} m_{00} & m_{01} & \ldots & m_{0n} \\ m_{10} & m_{11} & \ldots & m_{1n} \\ \vdots & \vdots & \vdots & \vdots \\ m_{no} & m_{n1} & \ldots & m_{nn} \end{array}\right) \qquad \ldots \qquad (29)$$

$$f(m_{0_1} m_{0_2}) = Ƶ_0 \qquad \ldots \qquad (30)$$
$$f(m_{11} m_{12}) = Ƶ_1 \qquad \ldots \qquad (31)$$
$$\vdots$$
$$f(m_{n1}, m_{n2}) = Ƶ_n \qquad \ldots \qquad (32)$$

$$\underline{d} = M^{-1} z \rightarrow \begin{matrix} d_0 \\ d_1 \\ \vdots \\ d_n \end{matrix} \rightarrow \underline{\underline{d}} \qquad \ldots \qquad (33)$$

Onukogu (2003) cited one of the ways of obtaining the step-length ρ for constrained response function whose minimizer is a boundary point.
Let the ith constraint be defined by:

$$\underline{C}_i^1 \underline{X} = b_i; \quad i = 1,2,..., m \qquad \ldots \qquad (34)$$

where C_i and X are vector and b_i is a scalar.
Then:

$$\rho^* = \min_i \left[\frac{\underline{c}_i^1 \overline{\underline{x}} * b_i}{\underline{c}_i^1 \underline{d}^*} \right] \qquad \ldots \qquad (35)$$

The iterate X_j^* is obtained as

$$X_j^* - \rho_i d_j \qquad \ldots \qquad (36)$$

where:

$\overline{\underline{x}}$ is a vector of the starting point

$\underline{d}$ is the direction vector

$\underline{\rho} -$ is the step-length.

This approach is based on super convergent line search techniques to find the optimal value of an optimization problem.

VIII. THE ALGORITHM

The algorithm is defined by the following sequence of steps.

1) At the initial step, obtain $X_{0,} \in \tilde{X}$ … (37)
det $\left(X_0^1 X_0 \right) > 0$, and
move to $\underline{x}_1 = \overline{\underline{x}}_0 - \rho_0 \underline{d}_0; \; \overline{\underline{x}}_0 = \underline{\underline{1}}^1 x_0 / N_0$ … (38)

2) at the jth step move to $\underline{x}_j = \underline{x}_{j-1} - \rho_{j-1} \underline{d}_{j-1}; \underline{d}_{j-1}^1 \underline{d}_{j-1} = 1$ … (39)

$$\rho_j = \min\left(\underline{c}_k^1 \underline{x}_j - b_k \right) \underline{c}_k \underline{d}_j \qquad \ldots \qquad (40)$$

3) Stop if $\left\| f(\underline{x}_j) - f(\underline{x}_{j-1}) \right\| \leq \in > 0$ … (41)

Whenever step (3) does not hold, define

$$X_{j-1} = \begin{pmatrix} X_j \\ \underline{x}_j \end{pmatrix} \Rightarrow X_{j+1}^1 X_{j+1} = X_j^1 X_j + \underline{x}_j \underline{x}_j^1 \qquad \ldots \qquad (42)$$

go back to step 1

$$M_{j+1} > M_j \forall_j \qquad \ldots \qquad (43)$$

IX. TOPPING RULE

The following conditions can be adopted as the stopping rule.
A sequence converges if

1. $P_j > P_{j+1} > P_{j+2} > \dots P_{j+n}$ … (44)
2. $\mathrm{Var}(d_j) > \mathrm{Var}(d_{j+1})$ … (45)
3. $\left(X_j^1 X_j\right) < \left(X_{j+1}^1 X_{j+1}\right)$ … (46)
4. $\left|f\left(X_1^*, X_2^*\right) f\left(X_1, X_2\right)\right| \le \delta > 0$ … (47)
5. $M_{j+1} > M_j$ also $\delta_{j+1} < \sigma_j^2 < \sigma_{j-1}^2$ … (48)

$$M_j < M_{j+1} < M_{j+2} \quad \dots \quad (49)$$

X. REGRESSION MODELING

A good model gives adequate representation of the response surface with the fewest number of parameters. The model of the objective function to be used in this work is called the response function for bivariate quadratic surface.

The model is thus represented as:

$$f(X_1, X_2) = a_0 + a_{10}X_1 + a_{20}X_2 + a_{12}X_1X_2 + a_{11}X_1^2 + a_{22}X_2^2 + e \quad \dots \quad (50)$$

The biasing parameters vectors for the above model is defined as

$$C\underline{B} = (a_{12}, a_{11}, a_{22}) \quad \dots \quad (51)$$

XI. ANALYSIS

NUMERICAL EXAMPLE

A typical quadratic optimization problem was used for illustration of this proposed method.

The problem is to find the vector (X_1, X_2) that minimizes Ƶ.

$$\text{Min } Ƶ = 6X_1 + 3X_2 - 4X_1X_2 - 2X_1^2 - 3X_2^2 + e \quad \dots \quad (52)$$

s.t. : $X_1 + X_2 \ge 1$

$2X_1 + 3X_2 \le 0$

To see the biasing terms clearly, we bracket the biasing effects.

$$\text{Min } Ƶ = 6X_1 + 3X_2(-4X_1X_2 - 2X_1^2 - 3X_2^2) \quad \dots \quad (53)$$

$$CK = \begin{pmatrix} 1 & 1 \\ 2 & 3 \end{pmatrix}, \; bk = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$$

$$\underline{x} = \begin{pmatrix} 0.93 \\ 0.59 \end{pmatrix}$$

$$X = \left(X_1 : X_q\right)$$

We evaluate the possible values of X_1, X_2 that will satisfy each of the constraint equation, using ENCARTAR DVD 2009 MICROSOFT MATHS:

Following the steps of Algorithm above in (7). The initial seven point designs obtained are:

$$X_l = \begin{pmatrix} X_0 & X_1 & X_2 \\ 1 & 1 & 2/3 \\ 1 & 1/2 & 1/2 \\ 1 & 3/2 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 3/2 & 0 \end{pmatrix} \qquad X_q = \begin{pmatrix} X_1X_2 & X_1^{\,2} & X_2^{\,2} \\ 0.67 & 1 & 0.45 \\ 0.25 & 0.25 & 0.25 \\ 1.5 & 2.25 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 0.75 & 0 \end{pmatrix}$$

X_l = The design matrix

X_q = The matrix of the biasing effects

The information matrices for X_l and X_q are

$$X_l^1 X_l = \begin{pmatrix} 7 & 6.5 & 4.2 \\ 6.5 & 7.8 & 3.4 \\ 4.2 & 3.4 & 3.7 \end{pmatrix} \qquad (X_q^1 X_q) = \begin{pmatrix} 3.8 & 5.1 & 2.9 \\ 5.1 & 8.7 & 3.8 \\ 2.9 & 3.8 & 3.3 \end{pmatrix}$$

$(X_l^1 X_l)\det = 13.1 \quad (X_q^1 X_q)\det = 7.1$

$$H = \begin{pmatrix} 0.61 & 0 & 0 \\ 0 & 0.47 & 0 \\ 0 & 0 & 0.51 \end{pmatrix} \qquad \bar{\underline{1}} - H \begin{pmatrix} 0.4 & 0 & 0 \\ 0 & 0.53 & 0 \\ 0 & 0 & 0.5 \end{pmatrix}$$

$$W = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 6 & -2.4 & -3.7 \\ 0 & 0 & 1 & 3 & -3.7 & -3.6 \end{pmatrix}$$

$$B = \begin{pmatrix} 0.24 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.3 & 0 & 1.5 & -0.6 & -0.9 \\ 0 & 0 & 0.3 & 0.8 & -0.9 & -0.89 \end{pmatrix}$$

$$B = \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0.599 & -2.36 & -3.71 \\ 0 & 0 & 0 & 3 & -3.71 & -3.57 \end{array}\right)$$

$B = U^1DU$

where $D = \text{diag}(d_1, d_2, d_3)$

$$B = \begin{pmatrix} 117.8 & 73.72 & 91.24 \\ 73.72 & -62.61 & -68.33 \\ 91.24 & -68.33 & -78.30 \end{pmatrix}$$

Also $BB^1 = \bar{\underline{1}}$

$$M = \begin{pmatrix} 13514.6 & -11505.7 & -12538.3 \\ -11505.7 & 12953.9 & 12819.7 \\ -12538.3 & 12819.7 & 13089.4 \end{pmatrix}$$

$$\cancel{Z} = \begin{pmatrix} -1313543487 \\ -1492995336 \\ -1513976535 \end{pmatrix}$$

$$M^{-1}Z = \begin{pmatrix} -2.4\times10^{19} \\ 4.7\times10^{19} \\ -6.8\times10^{19} \end{pmatrix} \Rightarrow \begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix}$$

Normalized $\underline{\underline{d}}$ becomes

$\underline{\underline{d}}_1 = 0.5618$

$\underline{\underline{d}}_2 = -0.8273$ (See appendix)

The step-length becomes

$$\rho_1 = \frac{(1,1)\begin{pmatrix}0.928\\0.595\end{pmatrix} - 1}{(1,1)\begin{pmatrix}-0.561\\-0.827\end{pmatrix}} = -1.973, \quad \rho_2 = \frac{(2,3)\begin{pmatrix}0.928\\0.595\end{pmatrix} - 4}{(2,3)\begin{pmatrix}-0.561\\-0.827\end{pmatrix}} = 0.263$$

using ρ_2 to evaluate the iterate

$X_1^* = 0.781$

$X_1^* = 0.813$

$F_1^*(X_1, X_2) = 1.382$

Checking for feasibility

$$(1,1)\begin{pmatrix}0.781\\0.813\end{pmatrix} \geq 1$$

$1.59 \geq 1$

$$(2,3)\begin{pmatrix}0.781\\0.813\end{pmatrix} \leq 4$$

$3.999 \leq 4$

we make move to the second iterate

$$X_1 = \begin{pmatrix} 1 & 1 & 3/2 \\ 1 & 1/2 & 1/2 \\ 1 & 3/2 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 3/2 & 0 \\ 1 & 0.78 & 0.81 \end{pmatrix} \quad X_q = \begin{pmatrix} 0.67 & 1 & 0.45 \\ 0.25 & 0.25 & 0.25 \\ 1.5 & 2.25 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 0.75 & 0 \\ 0.63 & 0.61 & 0.66 \end{pmatrix}$$

The information matrices are

$$(X_1^1 X_1) = \begin{pmatrix} 8 & 7.28 & 4.98 \\ 7.28 & 8.36 & 4.1 \\ 4.98 & 4.1 & 4.4 \end{pmatrix} \quad (X_q^1 X_q) = \begin{pmatrix} 4.2 & 5.5 & 3.3 \\ 5.5 & 9.1 & 4.2 \\ 3.3 & 4.2 & 3.7 \end{pmatrix}$$

$$(X_1^1 X_1)\det = 15.6$$

$$(X_q^1 X_q)\det = 8.3$$

The H-matrix becomes

$$H = \begin{pmatrix} 0.61 & 0 & 0 \\ 0 & 0.47 & 0 \\ 0 & 0 & 0.51 \end{pmatrix}$$

$$\left(\underline{1}H\right) = \begin{pmatrix} 0.396 & 0 & 0 \\ 0 & 0.535 & 0 \\ 0 & 0 & 0.494 \end{pmatrix}$$

$$W = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 6 & -3.3 & -3.1 \\ 0 & 0 & 1 & 3 & -3.1 & -4.9 \end{pmatrix}$$

$$B = \begin{pmatrix} 0.24 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.25 & 0 & 1.5 & -0.8 & -0.8 \\ 0 & 0 & 0.25 & 0.8 & -0.8 & -1.22 \end{pmatrix}$$

$$B = \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 5.99 & -3.25 & -3.12 \\ 0 & 0 & 1 & 3 & -3.12 & -4.88 \end{array}\right)$$

$B = U^1DU$

$$B = \begin{pmatrix} -160.9 & 109.1 & 132.3 \\ 109.1 & 81.9 & -107.3 \\ 132.3 & -107.3 & -147.7 \end{pmatrix}$$

$$M = \begin{pmatrix} 32165.1 & -27823.8 & -39750.6 \\ -27823.8 & -25958.4 & 38568.4 \\ -39750.6 & 38568.4 & 58382.6 \end{pmatrix}$$

$$\underline{Z} = \begin{pmatrix} -107130304445.9 \\ -98146658594 \\ -222071142266.6 \end{pmatrix}$$

$$M^{-1}z = \begin{pmatrix} 8.0\times10^{19} \\ 2.6\times10^{20} \\ -1.2\times10^{20} \end{pmatrix} \Rightarrow \begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix}$$

Normalized $\underline{d}$

$$\underline{d}_1 = 0.9113$$
$$\underline{d}_2 = -0.4117$$

The step-length

$$\rho_1 = \frac{(1,1)\begin{pmatrix}0.781\\0.813\end{pmatrix} - 1}{(1,1)\begin{pmatrix}0.9113\\-0.4117\end{pmatrix}}$$

= 1.188

$$\rho_2 = \frac{(2,3)\begin{pmatrix}0.781\\0.813\end{pmatrix} - 4}{(2,3)\begin{pmatrix}0.9113\\-0.4117\end{pmatrix}}$$

= 3.4×10^{-5}

Using ρ_1 to evaluate the iterate

$$X_1^x = -0.302$$
$$X_2^x = 1.302$$
$$F_2^x(X_1, X_2) = 3.245$$

Checking for feasibility

$$(1,1)\begin{pmatrix}-0.302\\1.302\end{pmatrix} \geq 1$$

$1 \geq 1$

$$(2,3)\begin{pmatrix}-0.302\\1.302\end{pmatrix} \leq 4$$

$3.5 \leq 4$

XII. TEST FOR CONVERGENCE

Considering the stopping rules listed in Section 9, we thus test for convergence as follows:

$$\left|f\left(X_1^x, X_2^x\right) - f\left(X_1, X_2\right)\right| \leq 5 > 0$$

Let $\delta = 0.9$

The value of x_1 and x_2 which satisfies the constraint equations are

$X_2 = 1$, $X_1 = 0.333$

$F(X_1, X_2) = 4.0002$ which is the value of the objective function.

And $F(X_1^x, X_2^x) = 3.245$

$$\therefore \quad \left|F\left(X_1^x, X_2^x\right) - F\left(X_1, X_2\right)\right|$$

$= 0.755 < 0 < \delta$

Also

$M_j < M_{j+1}$ and $\delta_{j+1} < \delta_j^2$

It can also be seen that

$$(X_j^1 X_j) < \left(X_{j+1}^1 X_{j+1}\right)$$

XIII. SUMMARY/CONCLUSION AND RECOMMENDATIONS

It has been seen that in just one move the proposed algorithm has reached an optimal.

To achieve this, the optimal starting design of size (7) was selected algebraically to satisfy the constraint equations, which are feasible. The selected points are feasible points.
The B matrix was normalized following the Gaussian elimination techniques as cited by Onukogu (1997).
The determinant of the information matrix of both the linear and quadratic design matrices is greater than one, and increases as the design points increases. The arithmetic mean $(\underline{\bar{X}})$ was used to evaluate the W-matrix at the first iterate and was replaced by the new iterate (X^x) in the second iterate. The direction vector $\underline{d}$ was normalized. The vector $\underline{\bar{X}}^x$, $\underline{d}^x$ and ρ^x were computed accordingly. In just one move the optimal values of x_1, x_2 which were confirmed feasible were obtained.The algorithm converges to a global point. This is because the initial points were picked from the whole feasible region making all the point feasible.

CONCLUSION

However, it can be seen that this method can be adopted as one of the method of solving quadratic optimization problems.

RECOMMENDATION

This method should be applicable to more than two variables and constraints equation.
Other methods of choosing the optimal starting point design should be verified and adopted by future researchers. This method should also be verified using non-linear constraints by further researchers.

REFERENCES

[1.] Box, G.E. and Draper, N.R. (1959): A basis for the selection of a response surface design. Jour. Amer. Statist. Assoc.; 38, 172 – 211.
[2.] Cook, R.D. and Nachtsheim, C. (1982): Model-Robuts, Linear-Optimal design, Technometrics.
[3.] Cook, R.D. and Wong, W.K. (1994): On the equivalence of Constrained and Compound Optimal designs. J. Amer. Statist. Assoc.; 24, 136 – 181.
[4.] Dette, H. (1995): Optimal designs for identifying the degree of a polynomial regression. Ann. Statist. 13,18 – 49.
[5.] Fang, G. and Li, J. (2005): An Optimization model with one linear objective function and finitely many Constraints of fuzzy relation inequalities. SIAM Journal on Optimization, 16(1), 180 – 200.
[6.] Fletcher, R. (1981): Practical Methods of Optimization Vols, 1 and 2 Wiley and sons.
[7.] Fletcher, R. and Mc, Cann A. (1969): Acceleration techniques for non-linear Programming optimization: Academic press, London.
[8.] Keifer, J.C. (1974): General Equivalence Theory of Optimum designs. Ann. Satist.; 178(2):33 – 51.
[9.] Hager, W.W. and Zhang, H. (2009): A new Conjugate Gradient Method with Guaranteed Descent and an efficient line search SIAM Journal on Optimization. 16, 1:170 – 192.
[10.] Inyama, S.C. (2007): Operations Research an Introduction Supreme Publisher, Owerri.
[11.] Jan, B. and Valadimi, T. (2005): Optimization insights and Applications; Princeton University Press, Princeton, N.J.
[12.] Nocedal, J. and Wringht, S.J. (2006): Numerical Optimization (2nd Ed.) Berlin, New York spririger-verlay.
[13.] Onukogu, I.B. (1997): Foundation of Optimal Exploration of Response Surfaces, Ephrata Press, Nsukka.
[14.] Onukoku, I.B. and Chigbu, P.E. (2002): Super Convergent line series in optimal Design of Experiment and Mathematical programming; A.P. Express Publishing.
[15.] Phil-Wolf (1972): On the convergence of gradient methods under constraints. SIAM Rev. &, 42 – 54.
[16.] Shi, Z.J. (2004): Convergence of line search methods for unconstrained and computation. 157, 2, 51 – 72.
[17.] Yuan, Y. and Sun, W. (1999): Theory and Methods of Optimization. Science press of China. Beijing.
[18.] Yuan, G. and Lu. X. (2009): A Modified conjugates Gradient Method. Annals of Operations Research 166,(1)73 – 90.
[19.] Yuan, G. and Wei Z. (2009): New line search method for unconstrained Optimization Journal of Korean Statistics Society. 38,1:29 – 39.