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Effects of Slots on Deflection and Stresses in Belleville Spring

[1,]H.K.Dubey [2,] Dr. D.V. Bhope [3,] Sneha Tahilyani, [4,] Kushal Singh

[1,]Pg Student Of Dept Of Mechanical Engineering R.C.E.R.T, Chandrapur, India
[2,]Professor, Dept Of Mechanical Engineering R.C.E.R.T, Chandrapur, India
[3,]Pg Student Of Dept Of Mechanical Engineering
[4,]Asst.Professor Department Of Aeronautical Engineering, PCE

Abstract

This paper reports stress and deflection analysis of a Belleville Spring with slots and without slots using finite element method. For a particular dimension of its outer diameter and inner diameter i.e. (OD/ID), diameter between slots and outer diameter (TD/OD) and its Height to thickness i.e. (H/T) have been considered to investigate the Von-misses stresses in the spring along with the deflections. Finite element method is used for analysis. The FE results are compared with existing analytical results.

***Keywords*—** Stress, Deflection, Finite element method, slotted Belleville Spring

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I. Introduction

The slotted conical disk spring is a modification of the regular conical disk spring or Belleville spring in as much as it has regularly arranged slots extending from the inside diameter. A single disk of the slotted type undergoes a larger deflection at a smaller load than a regular disk spring of comparable dimensions, thereby combining some of the advantages of the disk spring and the cantilever type spring in a single unit. It is used, where stacking is undesirable, a relatively large outside diameter is tolerable, and a regressive load-deflection-characteristic is desired, like in clutch applications. The analytical equations for the slotted Belleville spring have been taken from G. SCHRFMMER research paper on “The Slotted Conical Disk Spring”, ASME, 1973,as shown in Figure 1.

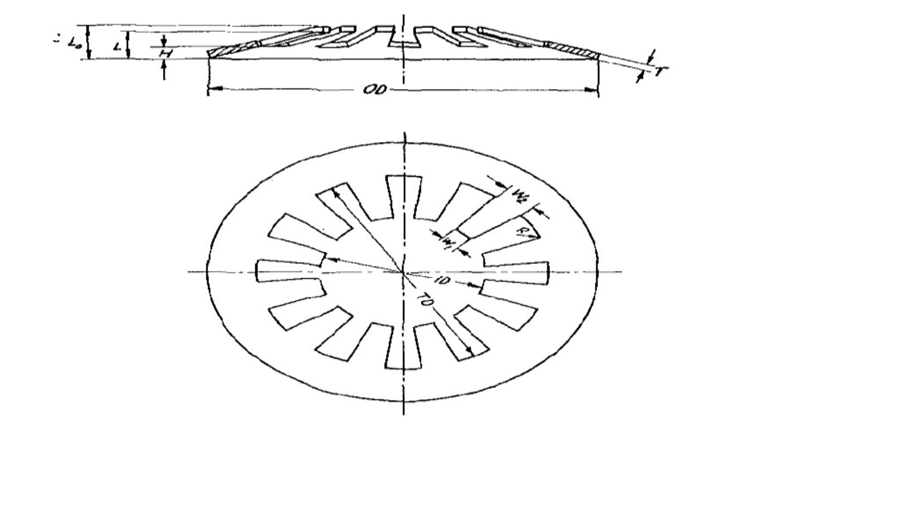


Figure 1: A Slotted Belleville Spring

This work deals with the deflection and the Von-misses stresses induced in Belleville spring due to variation in axial force for a particular dimensions of the Belleville spring with slots and without slots.

II. Literature Review

Many researchers have carried out stress and deflection analysis of a Belleville spring. Monica Carfagni [1] carried out the stress and deflection analysis to prepare a CAD method for the checkout and design of the Belleville springs. The method eliminates the need to resort to conventional trial-and-error techniques. In a matter of seconds, it rapidly and accurately checks out and designs Belleville springs, outputting the load-deflection characteristics in graphic and table formats and can generate a dimensioned drawing. G. Schrfmmer [2] carried out the stress and deflection analysis of a slotted Belleville spring to develop a analytical relationship for deflection and stress of a slotted conical spring.

III. Introduction To Problem, Scope & Methodology

Though the geometry of the Belleville spring appears to be simple with conical shape but the stress distribution is quiet complex due to the axial load. It is predicted that the axial load is responsible for axial compressive stress and also for bending stress induced in the Belleville spring. The analytical equations are derived largely on the basis of bending moments to simplify the derivations. In finite element analysis it is possible to model the exact geometry of the spring and to investigate the effect of axial load on the stresses, and deflections of the spring. Hence it is possible to determine the exact values of stresses in Belleville spring which are induced on account of the combination of axial stress and bending stress.Therefore, the present work deals with the determination of stresses and deflection in Belleville spring using FEA. The results obtained from FE analysis are compared with existing analytical equations. This study will lead to justify the validity of existing analytical equations and to estimate the conditions where it may become error prone. The scope & methodology is described as follows:

- In the Present research work an approach has been made to study the effects of cut out's on stresses and deflections in a Belleville Spring for a specific dimension of D, d, h & t, so that the comparison of stresses and deflections coming on the Belleville spring for the same dimensions without slots can be carried out. Therefore in short an approach to study the difference in behavior of the deflection and stresses coming on a Belleville spring with slots and without slots has been done.
- Lastly, the FE results have been compared with the existing analytical equations available for the slotted Belleville Spring for a particular dimension and an effort has been made to show the variations in the stresses and deflections with respect to the variation in force for a Belleville spring with slots and without slots, i.e. the load-deflection and load-stress characteristic is also investigated.

IV. Finite Element Analysis Of A Slotted Belleville Spring

In this work a simple slotted Belleville Spring analysis has been done. For a particular dimension of its various parameters given below the force has been calculated. The specifications of the Slotted Belleville spring considered are as follows.

- OD = 144mm
- TD= 124.8mm
- ID= 72mm
- T= 1.92mm
- Modulus of Elasticity (E) = 206.7×10^3 N/mm²
- Poisson's Ratio (μ) = 0.3
- L_O = 12mm
- L=10.08mm
- W_1 = 8.4mm
- Z= 12

The analysis is done by imposing boundary conditions such that the spring could deflect only along X&Z-direction. The analytical equations for the deflection and stresses for a slotted Belleville spring are given from equation (1) to (7).

$$P=\frac{E}{1-\mu^2}\times\frac{T^3}{OD^2}K_1\times F_1\left[1+\left(\frac{H}{T}-\frac{F_1}{T}\right)\left(\frac{H}{T}-\frac{F_1}{2T}\right)\right]\left[\left(1-\frac{TD}{OD}\right)\div\left(1-\frac{ID}{OD}\right)\right] \quad \text{…. (1)}$$

$$S=\frac{E}{1-\mu^2}\times\frac{T}{OD^2}\times\frac{TD}{OD}K_2\times F_1\left[1+K_3\left(\frac{H}{T}-\frac{F_1}{2T}\right)\right] \quad \text{…. (2)}$$

Where K_1, K_2, K_3 are Constants.

$$K_1 = \frac{2\times 3.14}{3}\frac{(OD/TD)^2 \ln(OD/TD)}{[(OD/TD)-1]^2} \quad \text{…. (3)}$$

$$K_2 = \frac{2(OD/TD)^2}{[(OD/TD)-1]} \quad \text{…. (4)}$$

$$K_3 = 2-2\left[\frac{1}{\ln(OD/TD} - \frac{1}{(OD/TD)-1}\right] \quad \text{…. (5)}$$

$$H = \frac{1-(TD/OD)}{1-(ID/OD)}\times L \quad \text{…. (6)}$$

$$W_2 = (TD/ID)\times W_1 \quad \text{…. (7)}$$

Where,

P= Force

F_1 = Deflection of the spring

OD= Outside surface Diameter

ID= Inside surface diameter

TD= Diameter between two slots

E= Modulus of Elasticity

μ = Poisson's ratio

H= Height of the spring, and

T= thickness of the spring

L_0 = Vertical distance from upper inner surface to the bottom outer surface

L= Vertical distance from Lower inner surface to the bottom outer surface

Z= Number of tongues (i.e. slots provided in the spring)

W_1 & W_2= width of the slots on upper and lower side. The representation of the stress and deflection contours for the above said dimensions of the Belleville spring with slots and without slots are shown in figures 2&3 as illustration. Further the load-deflection characteristic and load stress characteristic have been studied by varying the load obtained in the above case. The results are presented in forthcoming section

V. Results

The FE analysis revealed the Von-misses stresses along with the deflections for a particular dimension of a Belleville spring with slots and without slots. The analytical calculations for the above dimensions are in very close approximation with the FE values thereby validating the correctness of the analytical equations. Substituting the given values the constants are found to be: - K_1 =16.8, K_2= 17.3, K_3=1.024 Now substituting these values of constants in Eq (1) & Eq (2) mentioned above to calculate the respective Load (force) and the stress.

$$P=\frac{206.7\times10^{3}}{1-0.3^{2}}\times16.8\times\frac{1.92^{3}}{144^{2}}\times2.68\times\left[1+\left(\frac{2.68}{1.92}-\frac{2.68}{1.92}\right)\left(\frac{2.68}{1.92}-\frac{2.68}{2\times1.92}\right)\right]\left[\left(1-\frac{124.8}{144}\right)\div\left(1-\frac{72}{144}\right)\right]$$

P= 1074 N.

$$S=\frac{206.7\times10^{3}}{1-0.3^{2}}\times\frac{1.92}{144^{2}}\times\frac{72}{144}\times17.3\times2.68\times\left[1+1.024\times\left(\frac{2.68}{1.92}-\frac{2.68}{2\times1.92}\right)\right]$$

S= 1453.229 Mpa.

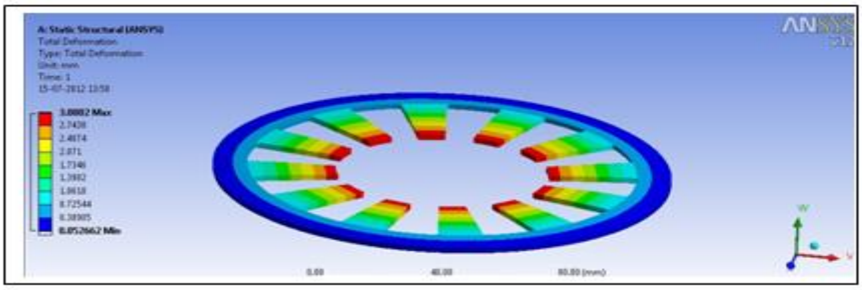


Figure 2: Deflection for slotted Belleville spring

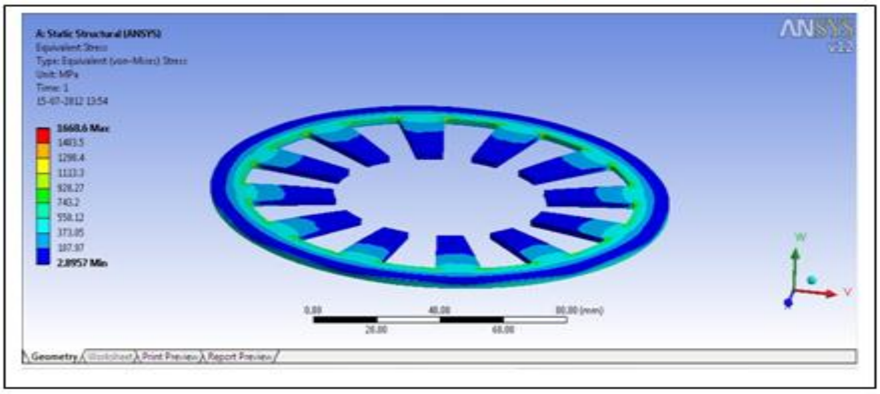
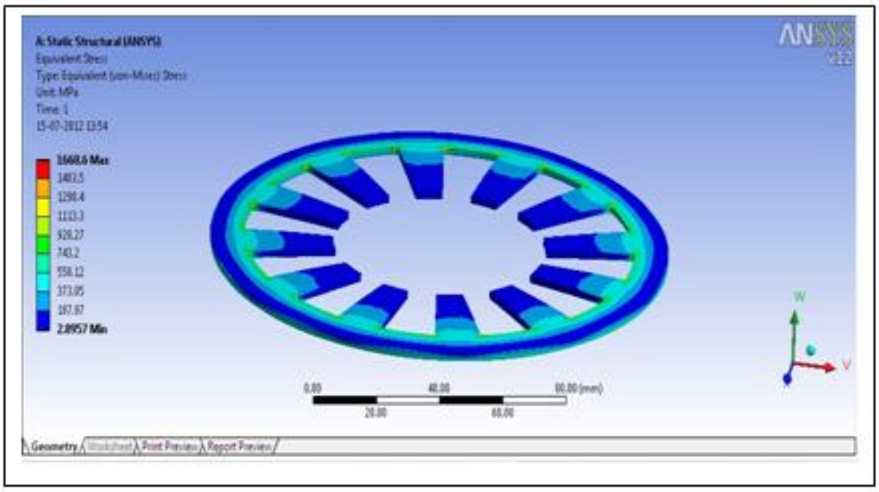


Figure 3: Von-misses Stress for slotted Belleville spring



5.1 COMPARATIVE STUDY OF DEFLECTIONS AND STRESSES FOR THE SAME DIMENSIONS OF THE BELLEVILLE SPRING WITH & WITHOUT SLOTS

The same Belleville spring with similar dimension as mentioned above but without slots is modeled to study the effect of deflection and stresses coming on it for the same force as shown in figure 4 & 5.

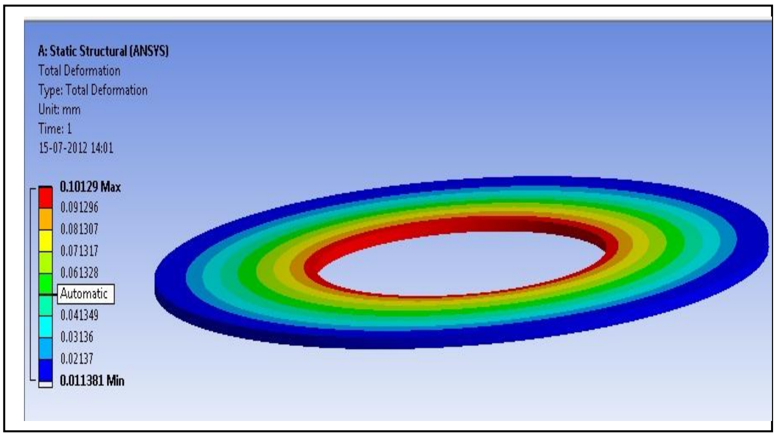
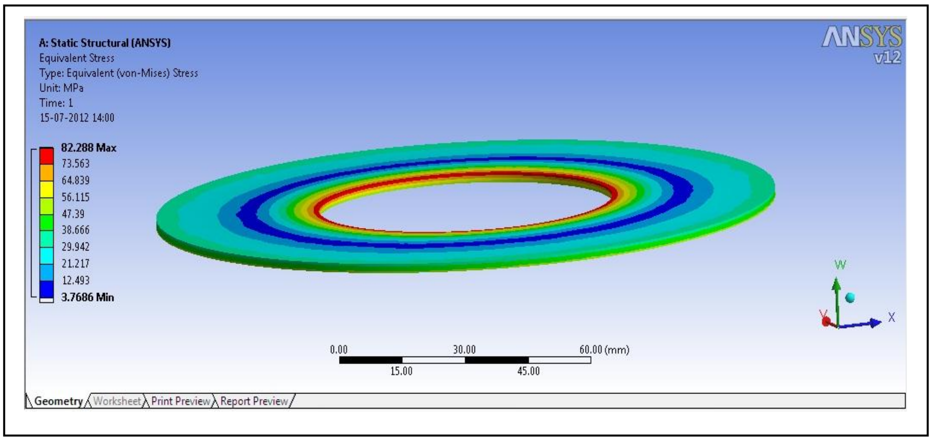


Figure 4: Deflection for Belleville spring of same dimensions without slots



Hence to study the deflection and stress behavior of the spring of same dimensions with slots and without slots, the force is varied and the results are tabulated in the table 6.1. Even 2D- graphs have been plotted to understand the stress and deflection behavior as shown in figure 6.7 & 6.8.

Table 5.1: Different values of Deflection and stresses obtained by varying the load for same dimension of Belleville spring with and without slots

Sr.No	Deflection with slots in (mm)	Deflection without slots in (mm)	Force in (N)	Stresses with slots (Mpa)	Stresses without slots (Mpa)
1	0.5733	0.0188	200	310	15.324
2	1.1471	0.0377	400	621	30.647
3	1.7208	0.056	600	932.16	45.97
4	2.2941	0.0754	800	1242.8	61.34
5	2.867	0.0943	1000	1553.5	76.618
6	3.01	0.10129	1074	1668.6	82.288

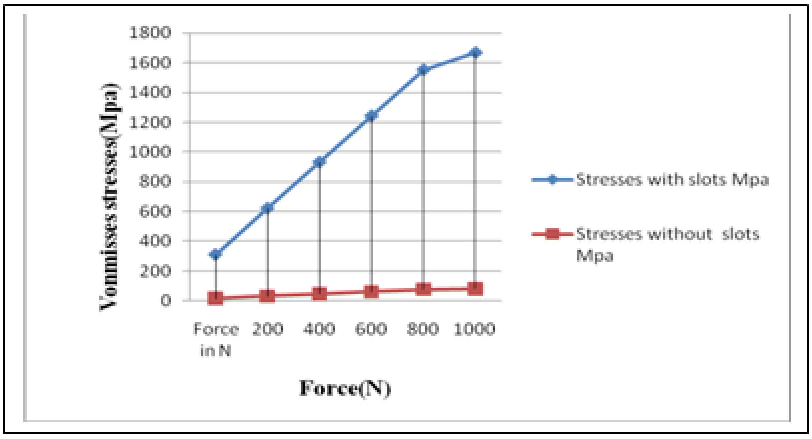


Figure 6: Load –Deflection Characteristic for Belleville spring with and without slots

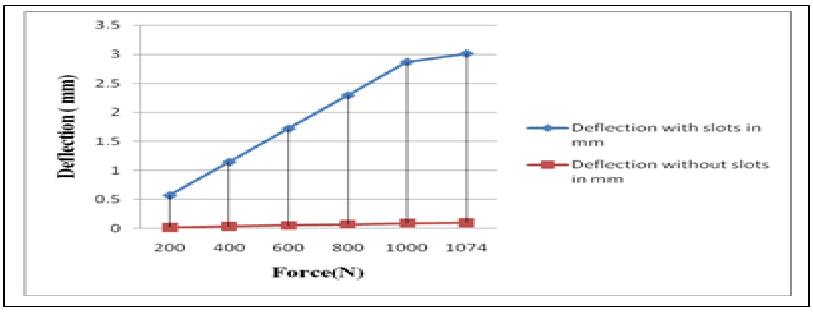


Figure 7: Load – Stress Characteristic for Belleville spring with and without slots

Thus it can be seen that there is drastic difference in deflection and stress for the same load (force) and dimensions of a Belleville spring with & without slots. It is mainly due to the variation in stiffness of the spring due to presence of slots.

VI. DISCUSSION AND CONCLUSION

1. The comparison between the conical springs with & without slots reveals that, deflection drastically increases due to the presence of slots .Thus slots are recommended to decrease the stiffness of spring for similar dimensions. This can be very well observed in figure 6 and table 6.1.

2. The comparison of the stresses reveals that the stresses, in slotted spring are very high as compared to conical spring without slot and this can be observed very clearly from figure 7 and table 6.1, which shows the drastic difference of the Von-misses stress between the two cases i.e. with slots and without slots. This is due to the effect of stress concentration at the corners of the slots. This effect can be significantly reduced by providing suitable fillet at the slot corners, hence slotted springs are recommended for lower load values and greater flexibility.

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