State Estimation for Multi-Area Power System

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Abstract
This paper gives a two level estimator for interconnected power system also says the system is stable or not. The information is interchanged among the border buses. In this paper we propose a decentralized algorithm which requires local buses information and also border buses information. The procedure is tested by using IEEE 14 bus system

Keywords – State Estimation, Multi-area State Estimation, WLS Method, Decentralized decomposition

I. INTRODUCTION
In today’s power system, the power transactions takes place over large areas and distances which involves many control areas has increased and are expected to grow in the future. In energy control centers the raw measurements obtained through the SCADA system from the grid are processed by the state estimator[1], which provides the estimate of the operating state of the system. In order to monitor the large scale power transactions taking place over many areas, a wide area state estimator is required. This is a two level estimator in the first level each area runs their state estimation algorithm by using their local area measurements. In the second level the central coordinator[2] collects the data from individual state estimators then coordinate to get two-level state estimation. In this a new method is proposed which uses boundary injection measurements without transferring the topologies of internal transmission lines connected to the boundary buses to central coordinator[2].

II. MULTI-AREA STATE ESTIMATION

State estimation technique was developed initially to reduce the computational burden on the state estimators due to the limited calculation capabilities of the computers. Now-a-days the same concept is used for the security of large area power system and also for reliable operation over several interconnected areas. This type of state estimation performs calculations in two level[3][5]. In first level state estimation is performed by using conventional measurements which is independent of all sub systems and there is no coordination level which coordinates all local estimations[3]. As a result this can reduce the received and processed number of variables compared to facilitate a whole system control monitoring instruments. This was based on conventional weighted least square state estimation algorithm[6]

The state vector of the central coordinator is defined as

\[ X_C = [U^T, X_b^T] \]

Where

\[ U = [u_1, u_2, \ldots, u_N]^T \]
\[ X_b = [X_{1,b}, X_{2,b}, \ldots, X_{N,b}]^T \]

\( u_i \) is the phase angle of \( i^{th} \) area slack bus with respect to the global reference. Here area 1 is chosen as the global reference bus.

The measurement set available for the central coordinator is given as

\[ Z = [Z_b, \hat{X}_{b\text{ \text{int}}}, \hat{X}_{b\text{ \text{ext}}}] \]

Where

\[ \hat{X}_{b\text{ \text{int}}} = [\hat{X}_{1,b\text{ \text{int}}}, \hat{X}_{2,b\text{ \text{int}}}, \ldots, \hat{X}_{N,b\text{ \text{int}}}] \]
\[ \hat{X}_{b\text{ \text{ext}}} = [\hat{X}_{1,b\text{ \text{ext}}}, \hat{X}_{2,b\text{ \text{ext}}}, \ldots, \hat{X}_{N,b\text{ \text{ext}}}] \]
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\( Z_b \) is the set of border measurements which include tie-line power flows, modified injection and voltage measurements at the border buses.

The corresponding measurement model is

\[
Z_c = h_c(x_c) + e_c
\]

Where

- \( h_c \) is the nonlinear function of \( x_c \).
- \( e_c \) is the error vector of measurements.

The objective function of central coordinator state estimation is

\[
J_c = [Z_c - h_c(x_c)]^T R_c^{-1} [Z_c - h_c(x_c)]
\]

### III. WLS METHOD

The conventional WLS method can be applied to the system, once all the loads are defined as pseudo measurements. The approach differs mainly in the selection of states, handling of measurements, and model details. It is pointed out that WLS estimator performs similar to the first approach, in that it scales the loads in a more consistent way so that the total load at measurement point agree with the measurements.

In state estimation, the model used to relate the measurements and the state variables is

\[
Z = h(x,y) + N
\]

We choose bus voltages as state variables. \( N \) is assumed to be a Gaussian distribution with zero mean and variance \( \sigma^2 \).

\( \sigma^2 \) is used to weight each individual measurement. WLS estimation computes the state variable vectors \( x \) and \( y \) which minimize the following function

\[
\min J(x) = \sum w_i (Z_i - h_i(x))^2 = [Z - h(x)]^T W [Z - h(x)]
\]

where

- \( w_i \) and \( h(x) \) represents the weight and the measurement function associated with measurement \( Z_i \). Gain matrix can be obtained by using the following function

\[
G(x) = H^T(x) WH(x)
\]

The algorithm for WLS state estimation problem can be outlined as follows:

a) Start iterations, Set the iteration index \( k = 0 \).
b) Initialize the state vector \( x_k \), typically as a flat start.
c) Calculate the gain matrix, \( G(x_k) \).
d) Calculate the right hand side \( t_k = H^T(x_k) R^{-1} [Z - h(x_k)] \).
e) Decompose \( G(x_k) \) and solve for \( \Delta x_k \).
f) Test for convergence, \( \max |\Delta x_k| < \varepsilon \).
g) If no, update \( x_{k+1} = x_k + \Delta x_k, k = k + 1 \), and go to step c. Else, stop.

The measurement Jacobian matrix is modeled as

\[
H = \begin{bmatrix}
\frac{\partial P_{inj}}{\partial \theta} & \frac{\partial P_{inj}}{\partial V} \\
\frac{\partial P_{flow}}{\partial \theta} & \frac{\partial P_{flow}}{\partial V} \\
\frac{\partial Q_{inj}}{\partial \theta} & \frac{\partial Q_{inj}}{\partial V} \\
\frac{\partial Q_{flow}}{\partial \theta} & \frac{\partial Q_{flow}}{\partial V} \\
\frac{\partial I_{mag}}{\partial \theta} & \frac{\partial I_{mag}}{\partial V} \\
0 & \frac{\partial \theta}{\partial V}
\end{bmatrix}
\]
Elements corresponding to real power injection measurements:

\[
\frac{\partial P_i}{\partial \theta_j} = \sum_{j=1}^{N} V_j V_i (-G_{ij} \sin \theta_j + B_{ij} \cos \theta_j) - V_i^2 B_{ii} \frac{\partial P_i}{\partial V_i} = V_i V_j (G_{ij} \sin \theta_j - B_{ij} \cos \theta_j)
\]

\[
\frac{\partial P_i}{\partial V_j} = \sum_{j=1}^{N} V_j (G_{ij} \cos \theta_j + B_{ij} \sin \theta_j) - V_i G_{ii} \frac{\partial P_j}{\partial V_j} = V_i (G_{ij} \sin \theta_j - B_{ij} \cos \theta_j)
\]

Elements corresponding to reactive power injection measurements:

\[
\frac{\partial Q_i}{\partial \theta_j} = \sum_{j=1}^{N} V_j V_i (G_{ij} \cos \theta_j + B_{ij} \sin \theta_j) - V_i^2 G_{ii} \frac{\partial Q_j}{\partial \theta_j} = V_i (G_{ij} \sin \theta_j - B_{ij} \cos \theta_j)
\]

Elements corresponding to real power flow measurements:

\[
\frac{\partial P_i}{\partial \theta_j} = V_i V_j (g_{ij} \sin \theta_j - b_{ij} \cos \theta_j)
\]

\[
\frac{\partial P_i}{\partial V_j} = -V_i V_j (g_{ij} \sin \theta_j - b_{ij} \cos \theta_j)
\]

\[
\frac{\partial P_i}{\partial V_i} = -V_j (g_{ij} \cos \theta_j + b_{ij} \sin \theta_j) + 2(g_{ij} + b_{ij}) V_i
\]

\[
\frac{\partial P_i}{\partial \theta_j} = -V_i (g_{ij} \cos \theta_j + b_{ij} \sin \theta_j)
\]

Elements corresponding to real power flow measurements:

\[
\frac{\partial Q_i}{\partial \theta_j} = -V_i V_j (g_{ij} \cos \theta_j + b_{ij} \sin \theta_j)
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\[
\frac{\partial Q_i}{\partial V_j} = V_i V_j (g_{ij} \cos \theta_j + b_{ij} \sin \theta_j)
\]

\[
\frac{\partial Q_i}{\partial V_i} = -V_j (g_{ij} \sin \theta_j - b_{ij} \cos \theta_j) - 2(b_{ij} + g_{ij}) V_i
\]

\[
\frac{\partial Q_i}{\partial \theta_j} = -V_i (g_{ij} \sin \theta_j - b_{ij} \cos \theta_j)
\]

Elements corresponding to voltage magnitude measurements:

\[
\frac{\partial V_i}{\partial V_j} = 1, \quad \frac{\partial V_i}{\partial \theta_j} = 0, \quad \frac{\partial V_i}{\partial \theta_j} = 0, \quad \frac{\partial V_i}{\partial V_j} = 0
\]

Elements corresponding to current magnitude measurements (ignoring the shunt admittance of the branch):

\[
\frac{\partial I_i}{\partial \theta_j} = \frac{g_{ij}^2 + b_{ij}^2}{I_i} V_j \sin \theta_j
\]
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\[
\frac{\partial I_{ij}}{\partial \theta_j} = -\frac{g_{ij}^2 + b_{ij}^2}{I_{ij}} V_j V_j \sin \theta_j
\]

\[
\frac{\partial I_{ij}}{\partial V_i} = \frac{g_{ij}^2 + b_{ij}^2}{I_{ij}} (V_i - V_j V_j \cos \theta_j)
\]

\[
\frac{\partial I_{ij}}{\partial V_j} = \frac{g_{ij}^2 + b_{ij}^2}{I_{ij}} (V_j - V_i V_j \cos \theta_j)
\]

The covariance matrix for each set of m measurements, each one corresponding to one row, as shown below:

\[
R = \begin{bmatrix}
R_{11} & 0 & 0 & 0 \\
0 & R_{22} & 0 & 0 \\
. & . & R_{33} & . \\
0 & 0 & . & 0 \\
0 & 0 & . & R_{mm}
\end{bmatrix}
\]

The gain matrix can be re-written as follows

\[
G = \sum_{i=1}^{m} H_i^T R_i^{-1} H_i
\]

IV. SYSTEM DECOMPOSITION

The overall optimization problem is divided into number of sub problems based on the areas, each area consists of its own agent, which solves its own optimization problem individually. To obtain an overall optimal solution the sub problems need to be coordinated among each other as shown in figure.

This method provides a correct estimation of the state of the system that is connected with other systems with interchange of only small amounts of border bus data.

V. IEEE 14 BUS SYSTEM

IEEE 14 bus system is partitioned into two areas for the simulation purpose. Area1 consists of buses 1 to 8 and area2 consists of buses 9 to 14. There are 7 boundary buses in total out of which 3 belong to area1 and 4 belong to area2.

The total number of states in the first level is given as \((2n-1)\).

Where \(n=\text{number of buses}\).

The total number of states in the second level is given as \((1+2n)\).

Where \(n=\text{number of border buses}\).

In this the areas considered to be non-overlapping, i.e., area1 will not restrict its model to buses A1-1 and A1-2, but will also include buses A2-2 and AN-2 from area 2 and N respectively. The buses in each area can be categorized as internal buses, internal boundary buses and external boundary buses.
VI RESULTS

a) Output for Area 1:

<table>
<thead>
<tr>
<th>Bus</th>
<th>Vi (pu)</th>
<th>Angle (Degree)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.06</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>1.045</td>
<td>-5.2499</td>
</tr>
<tr>
<td>3</td>
<td>1.010</td>
<td>-13.3834</td>
</tr>
<tr>
<td>4</td>
<td>1.019</td>
<td>-10.4722</td>
</tr>
<tr>
<td>5</td>
<td>1.020</td>
<td>-9.0086</td>
</tr>
<tr>
<td>6</td>
<td>1.070</td>
<td>-13.0397</td>
</tr>
<tr>
<td>7</td>
<td>1.062</td>
<td>-12.7903</td>
</tr>
<tr>
<td>8</td>
<td>1.097</td>
<td>-13.3919</td>
</tr>
</tbody>
</table>

b) Output for Area 2

<table>
<thead>
<tr>
<th>Bus</th>
<th>Vi (pu)</th>
<th>Angle (Degree)</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>1.0560</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>1.0516</td>
<td>-0.1707</td>
</tr>
<tr>
<td>11</td>
<td>1.0571</td>
<td>0.1475</td>
</tr>
<tr>
<td>12</td>
<td>1.056</td>
<td>-0.1045</td>
</tr>
<tr>
<td>13</td>
<td>1.0524</td>
<td>-0.2716</td>
</tr>
<tr>
<td>14</td>
<td>1.0359</td>
<td>-1.1121</td>
</tr>
</tbody>
</table>

c) Output for border Buses

<table>
<thead>
<tr>
<th>Bus</th>
<th>Vi (pu)</th>
<th>Angle (Degree)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>1.019</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>1.0701</td>
<td>-2.9135</td>
</tr>
<tr>
<td>7</td>
<td>1.0621</td>
<td>-1.7489</td>
</tr>
<tr>
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<td>1.0559</td>
<td>-2.38</td>
</tr>
<tr>
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<td>1.0571</td>
<td>-2.8182</td>
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<tr>
<td>12</td>
<td>1.055</td>
<td>-2.9534</td>
</tr>
<tr>
<td>13</td>
<td>1.0498</td>
<td>-2.8028</td>
</tr>
</tbody>
</table>

VII. CONCLUSION

The two level state estimator requires the individual areas to share their state estimator results only to the central coordinator. To attain the global solution through the proposed decentralized procedure, the information exchange reduces to state variables corresponding to border buses. The state of the two individual areas and the boundary buses area are estimated by using MATLAB. By considering the measurements like voltage magnitude, and phase angles, real and reactive power flows and injections.

REFERENCES

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