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Application of Assumed Mode Method in Nonlinear Dynamic Analysis of Elastic Robot Arms

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--ABSTRACT---

This paper presents nonlinear dynamic analysis of elastic robot arms using assumed mode method. Elastic manipulators exhibit many advantages over rigid ones and their dynamic analysis has attracted a great deal of interests recently. In the presented work, the assumed mode method is employed and the dynamic equations of flexible manipulators are derived. The total dynamic motion of the system is modeled as a rigid motion and a flexible displacement. Then the link flexibility represented by a truncated finite model series, in terms of spatial mode eigen functions and time-varying mode amplitudes. Hence the eigen function are relevant to boundary conditions of the system, some different boundary conditions are presented. Then, the total displacement of the elastic arm in reference coordinate system is developed and the Lagrange principle is employed to derive the nonlinear dynamic motion of the elastic robot arm. Finally, the nonlinear dynamic equations of a single-link flexible manipulator are presented using the Assumed mode method.

***Key words*:** elastic robot, nonlinear dynamic, assumed mode method, modal eigen function.

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I. INTRODUCTION

Robotic arms are widely used to implement in various activities. As the rigid manipulators are designed and build in a manner to maximize stiffness of the robot in an attempt to avoid unwanted vibration of the end-effector, but the high stiffness is achieved by using heavy material and a bulky design, which results in inefficient power consumption. In despite, the elastic robot arms exhibit many advantages over their rigid ones such as their less weights, more maneuverability, smaller actuators, and for the sake of these advantages, flexible manipulators have been developed in variant scientific fields [1-6]. In fact, dynamic analysis of elastic manipulators is a complex task that plays a crucial role in the design and application of robots in task space. This complexity arises from very lengthy, fluctuating and highly nonlinear and coupled set of dynamic equations due to the flexible nature of robot links. Therefore, a great deal of interests has been received for dynamic modeling of the flexible robots, recently. Martins et al. [7, 8] studied the dynamic modeling of single-link flexible manipulators. Rakhsha and Goldenberg [9] employed an analytical approach to model the dynamic of an elastic robot based on Newton-Euler formulation. Singh [10] used an extended Hamilton's principle to derive the equation of motion of the flexible manipulator. Megahed et al. [11] developed a variation of lumped model to simulate motion of planar flexible link robot. In their procedure, a consistent mass matrix is used in order to provide better approximation than traditional approaches. Meghdari and Fahimi [12] presented an analytical method to decouple the dynamic equation of motion of flexible robots.

In this paper, the nonlinear dynamic analysis of elastic robot arms is presented using assumed mode method. The dynamic motion of the system is modeled as a rigid motion and a flexible displacement. Then the link flexibility represented by a truncated finite model series, in terms of spatial mode eigen functions and time-varying mode amplitudes. Hence the eigen function are relevant to boundary conditions of the system, some different boundary conditions are presented. Then, the total displacement of the elastic arm in reference coordinate system is developed and the Lagrange principle is employed to derive the nonlinear dynamic motion of the elastic robot arm. Finally, the nonlinear dynamic equations of the system are presented.

II. DYNAMIC MODEL OF ELASTIC ROBOTIC ARMS

In this section, the nonlinear dynamic model of the flexible manipulator is derived via Assumed mode method (AMM). Figure 1 shows a schematic of the ith link of an elastic manipulator.

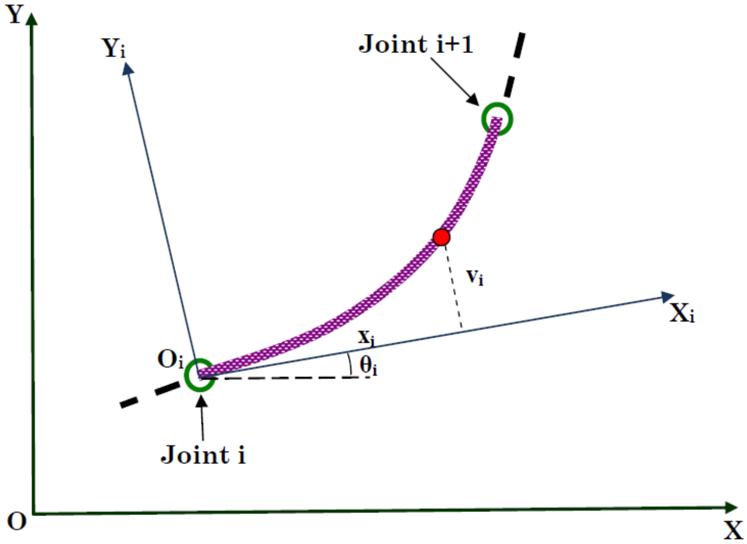

Figure 1. The flexible manipulator

To derive the nonlinear dynamic equation of the system, assume the kinetic and the potential energies of the system can be written as follows:

$$T(\vec{q},\dot{\vec{q}}) = \sum_{i=1}^{n} T_i$$
$$U(\vec{q}) = \sum_{i=1}^{n} U_i \tag{1}$$

Where n is the number of links, T_i and U_i are the kinetic and the potential energies of the ith link of the elastic robot, and $\vec{q}$ and $\dot{\vec{q}}$ are the generalized coordinates and velocities of flexible system, respectively.
By implementation of Lagrange principle, the Lagrangian function is $L = T - U$, and the dynamic equations of flexible manipulator can be written as:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} = Q_j \tag{2}$$

Where Q_j represents the torques applied to joints of links.
As it is shown in Fig. 1, the parameters of the system are indexed as: *OXY* is referred to the global coordinate system, *OiXiYi* is indicated to local coordinate system attached to *ith* link of robot, $\vec{r}_i$ is the displacement of the element in global system, $\vec{r}_{o_i}$ is the displacement of the origin of the local coordinate *OiXiYi* in the global system, T_0^i is the transformation matrix between *OXY* and $O_iX_iY_i$ coordinate systems, θ_i indicates the angular displacement of joint of *ith* arm, l_i is the length of *ith* link, m_i is the uniform mass per meter of *ith* arm, E_i is the elasticity modulus of *ith* link, and I_i is referred to the moment inertia of *ith* link.
According to Eqs. 1-2, for dynamic modeling of elastic links robot, it is sufficient to derive the kinetic and potential energy of *ith* link of the system.
By assuming $\vec{r}_i$ as a displacement vector of a given point of link *i* in the reference coordinate system, the kinetic energy of the link is stated as Eq. 3:

$$T_i = \frac{1}{2}\int_0^{l_i} m_i \left[\frac{\partial \vec{r}_i^{\,T}}{\partial t} . \frac{\partial \vec{r}_i}{\partial t}\right] dx_i \qquad 0 < x_i < l_i \tag{3}$$

Moreover, the total displacement vector $\vec{r}_i$ is a combination of a rigid displacement of the link, and a flexible deformation related to elastic deflection of the arm in local coordinate system $O_iX_iY_i$ attached to *ith* arm. So the displacement of the link is expressed as following:

$$\vec{r}_i = \vec{r}_{o_i} + T_0^i \begin{bmatrix} x_i \\ v_i \end{bmatrix} \quad (4)$$

Where v_i is the lateral deflection of *ith* link in the local coordinate system referred to flexibility of the link, and is presented as a summation of finite model eigen functions multiplied to time-dependent amplitudes:

$$v_i(x_i, t) = \sum_{j=1}^{n_i} \varphi_{ij}(x_i) e_{ij}(t) \quad (5)$$

where $v_i(x_i, t)$ is the bending deflection of the *ith* link at a spatial point x_i, n_i is the number of modes used to describe the deflection of link I, $\varphi_{ij}(x_i)$ is the *jth* assumed mode shape eigen function of the link, and $e_{ij}(t)$ is the *jth* time-varying displacement of the spatial point.

In the next step, for determining of potential energy U_i of *ith* link, the total potential energy is assumed as a summation of $U_{i,g}$ due to gravity and $U_{i,e}$ due to elasticity of the arm:

$$U_i = U_{i,g} + U_{i,e} \quad (6)$$

The potential energy due to gravity is expressed as:

$$U_{i,g} = \int_0^{l_i} m_i g \begin{bmatrix} 0 & 1 \end{bmatrix} \vec{r}_i \, dx_i \quad (7)$$

And assuming the Euler- Bernoulli model of the flexible link, the potential energy related to elasticity is given as:

$$U_{i,e} = \frac{1}{2} \int_0^{l_i} EI_i \left(\frac{\partial^2 v_i}{\partial x_i^2} \right) dx_i \quad (8)$$

Also, it is must be mentioned that the generalized coordinate vector regarding to the *ith* link of the robot is given as $\vec{q} = \begin{bmatrix} \theta_i & e_{i1} & \dots & e_{in_i} \end{bmatrix}^T$, and proper determining of the generalized coordinates must be performed.

III. MODAL SHAPE EIGEN FUNCTIONS

In the AMM there are numerous ways to choose the boundary conditions. The present study addresses four well-known conditions. Ideally, the optimum set of assumed modes is that closest to natural modes of the system. Hence, there is no stipulation as to which set of assumed modes should be used. Natural modes depend on several factors such as size of hub inertia and size of payload mass. Choosing appropriate conditions is very important and it may cause better consequences in the results [13]. Hence, the ultimate choice requires an assessment based on the actual robot structure and for example, anticipated range of payloads together with its natural modes [14]. Therefore, four normal modes for some familiar mode conditions are described as follows.

3.1. Pinned-pinned boundary condition

As the pinned-pinned boundary condition is presumed, the mode shapes are presented as:

$$\varphi_{ij}(x_i) = A_{ij} \sin(B_{ij} x_i) \quad (9)$$

Where A_{ij} and B_{ij} are determined by the following equation:

$$A_{ij} = \frac{\cosh(B_{ij} l_i)}{\cos(B_{ij} l_i)} \quad (10)$$

$$B_{ij} l_i = 3.14 \quad 6.28 \quad 9.42 \quad 12.56$$

3.2. Clamped-free boundary condition

As the clamped-free boundary condition is presumed, the mode shapes are presented as:

$$\varphi_{ij}(x_i) = \sin(B_{ij} x_i) - \sinh(B_{ij} x_i) + A_{ij} \left(\cos(B_{ij} x_i) - \cosh(B_{ij} x_i) \right) \quad (11)$$

Where A_{ij} and B_{ij} are determined by the following equation:

$$A_{ij} = \frac{\cos(B_{ij} l_i) + \cosh(B_{ij} l_i)}{\sin(B_{ij} l_i) - \sinh(B_{ij} l_i)} \quad (12)$$

$$B_{ij} l_i = 1.87 \quad 4.69 \quad 7.85 \quad 10.99$$

3.3. Clamped-clamped boundary condition

As the clamped-clamped boundary condition is presumed, the mode shapes are presented as:

$$\varphi_{ij}(x_i) = sin(B_{ij}x_i) - sinh(B_{ij}x_i) + A_{ij}\left(cos(B_{ij}x_i) - cosh(B_{ij}x_i)\right) \tag{13}$$

Where A_{ij} and B_{ij} are determined by the following equation:

$$A_{ij} = \frac{cos(B_{ij}l_i) - cosh(B_{ij}l_i)}{sin(B_{ij}l_i) + sinh(B_{ij}l_i)} \tag{14}$$

$$B_{ij}\,l_i = 4.73 \quad 7.85 \quad 10.99 \quad 14.13$$

3.4. Clamped-pinned boundary condition

As the clamped-pinned boundary condition is presumed, the mode shapes are presented as:

$$\varphi_{ij}(x_i) = sin(B_{ij}x_i) - sinh(B_{ij}x_i) + A_{ij}\left(cos(B_{ij}x_i) - cosh(B_{ij}x_i)\right) \tag{15}$$

Where A_{ij} and B_{ij} are determined by the following equation:

$$A_{ij} = -\frac{sin(B_{ij}l_i) + sinh(B_{ij}l_i)}{cos(B_{ij}l_i) + cosh(B_{ij}l_i)} \tag{16}$$

$$B_{ij}\,l_i = 3.92 \quad 7.06 \quad 10.21 \quad 13.35$$

IV. DYNAMIC MODEL OF A SINGLE-LINK FLEXIBLE ROBOT

In this section, the nonlinear dynamic modeling of a single-link flexible robotic arm is done. If the boundary condition of the system is presumed as clamped-free condition. Furthermore, only the first modal shape function of the system is considered in modeling of the system. Thus, the lateral deflection of the single link in the local coordinate system attached to the first link is presented as:

$$v_1(x_1,t) = \left[sin(B_{11}x_1) - sinh(B_{11}x_1) + \left(\frac{cos(B_{11}l_1) + cosh(B_{11}l_1)}{sin(B_{11}l_1) - sinh(B_{11}l_1)}\right)\left(cos(B_{11}x_1) - cosh(B_{11}x_1)\right)\right] e_{11} \tag{17}$$

Where l_1 is the length of the arm, x_1 can change between $0 < x_1 < l_1$, and $B_{11}\,l_1$ is equals to 1.87.

Then, the transformation matrix T_0^1 is defined as:

$$T_0^1 = \begin{bmatrix} cos\,\theta_1 & -sin\,\theta_1 \\ sin\,\theta_1 & cos\,\theta_1 \end{bmatrix} \tag{18}$$

And regarding to Eq. 4, the displacement vector of the robot in the reference coordinate system can be written as:

$$\vec{r}_1 = \begin{bmatrix} X_1 \\ Y_1 \end{bmatrix} = \begin{bmatrix} cos\,\theta_1\,x_1 - sin\,\theta_1\,v_1 \\ sin\,\theta_1\,x_1 + cos\,\theta_1\,v_1 \end{bmatrix} \tag{19}$$

The velocity vector of the system is given by:

$$\dot{\vec{r}}_1 = \begin{bmatrix} \dot{X}_1 \\ \dot{Y}_1 \end{bmatrix} = \begin{bmatrix} -sin\,\theta_1\,x_1\,\dot{\theta}_1 - cos\,\theta_1\,v_1\,\dot{\theta}_1 - sin\,\theta_1\,\dot{v}_1 \\ cos\,\theta_1 x_1\,\dot{\theta}_1 - sin\,\theta_1\,v_1\,\dot{\theta}_1 + cos\,\theta_1\,\dot{v}_1 \end{bmatrix} \tag{20}$$

Thus, the kinetic energy of the flexible robotic arm is stated as:

$$T = \frac{1}{2}\int_0^{l_1}\left(\dot{X}_1{}^2 + \dot{Y}_1^2\right)dx_1 \tag{21}$$

Moreover, the potential energy of the system regarding to the gravity is:

$$U_{,g} = \int_0^{l_1} m_1\,g\,Y_1\,dx_1 \tag{21}$$

And the potential energy of the flexible manipulator due to flexible effects can be written as:

$$U_{,e} = \frac{1}{2}\int_0^{l_1} EI_1\left(\frac{\partial^2 v_1}{\partial x_1^2}\right)dx_1 \tag{8}$$

Finally, the generalized coordinate vector is defined as $\vec{q} = [\theta_1 \quad e_{11}]^T$, and the nonlinear dynamic equations of the system can be derived regarding to Eq.1 and Eq. 2.

V. CONCLUSION

In this paper, the nonlinear dynamic analysis of the elastic robot arms have been studied using assumed mode method. The total dynamic displacement of the system has been modeled as a rigid motion and a flexible displacement. Then the link flexibility has been formulates as a summation of spatial modal eigen functions multiplied to time-varying mode amplitudes. Hence the eigen function are relevant to boundary conditions of the system, some different boundary conditions have been considered, and their corresponding eigen functions have been presented. Then, the total displacement of the elastic arm in reference coordinate system has been obtained and the Lagrange principle has been used to derive the nonlinear dynamic motion of the elastic robot arm. Finally, the assumed mode method has been employed to derive the dynamic equations of a single-link manipulator.

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