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A Non-Newtonian model of Two Phase Hepatic Blood Flow with Special Reference to Liver Abscess

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--ABSTRACT--

In the present paper, we formulate the Hepatic Blood Flow in Liver, Keeping in view the nature of hepatic circulatory system in human body, the viscosity increases in the arterioles due to formation of rolex along axis by red blood cells, as we no the arterioles are remote form heart and proximate to the Liver. P.N. Pandey and V.Upadhyay have considered the blood flow of two phase, one of which is that of red blood cells and other is Plasma. They have also applied the Herschel Bulkley non-Newtonian Model in Bio- fluid mechanical set-up. We have collected a clinical data in case of Liver Abscess for Hematocrit v/s Blood Pressure. The graphical presentation for particular parametric value is much closer to the clinical observation. The over all presentation is in tensorial form an solution technique adapted is analytical as well as numerical. The role of Hematocrit is explicit in the determination of blood pressure in case of hepatic disease liver abscess.

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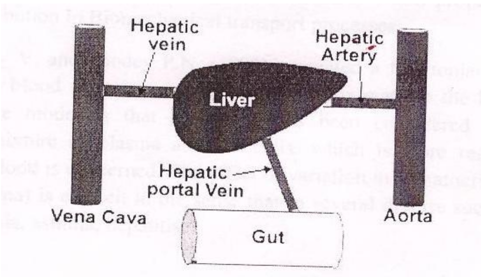
I. INTRODUCTION (DESCRIPTION OF BIO-PHYSICAL PROBLEM)

The liver is large reddish brown solid organ that sits in the upper part of the abdomen, tucked away under the right side of the rib cage. Above it is the heart and the right lung, and it is separated from them by a thin sheet of muscle called the diaphragm. Our liver is probably four to five times the size of our fist.

The liver has two lobes, the right lobes and left lobe, but right lobe is larger then the left. Blood flows into the liver through two large blood vessels that inter it from below- the hepatic artery which brings blood rich in oxygen, pumped out from the heart, and the portal vein which brings blood flowing out of the gut, rich in absorbed food material.

Blood is complex fluid consisting of particulate solids suspended in a non- Newtonian fluid. The particulate solids are red blood cells (RBC), white blood cells(WBC) and platelets. The fluid is plasma, which itself is a complex mixture of proteins and other intergradient in an aqueous base. 50% of the plasma and 40% of the blood cells and in 45% of the blood is RBC and there is few part of the other cells. Which are ignorable, so one phase of the blood is plasma and 2nd phase of the blood in RBC.

Two phase hepatic blood flew is study of measuring the blood pressure if the hemoglobin is known. The percentage of volume covered by blood cells in the whole blood is called hematocrit. This work will focus on two phase hepatic blood flow in liver with special reference two liver abscess.



A lot of work is available, but P.N. Pandey and V. Upadhyay discussed some phenomena in two phase blood flow gave in idea on the two phase hepatic blood flow in liver with a hepatic disease liver abscess. The work of P.N. Pandey and V. Upadhyay is in whole circulatory system but our work will focus on hepatic circulatory system and hepatic circulatory system which is a sub system of whole circulatory system. In this work we apply the Herschel Bulkley non-Newtonian model, here hepatic blood flow means blood flow liver tissue. We present an improvement on the previous work in the field and this is discussed separately below. The ultimate use of this model is to predict normal reference levels of two phase blood flow in liver for individual patients under going to liver abscess.

A liver abscess is a local accumulation of pus in the liver. The abscess causes an increase of pressure with in the liver as well as a killing of healthy surrounding liver tissue. The end results can be that an overwhelming infection can suddenly gain entrance into the blood stream at which point the patient gets extremely sick with blood poisoning. There are three major forms of liver abscess. 1- pyogenic liver abscess accounts for 80% of hepatic abscess cases in the United states. 2- Amoebic liver abscess accounts for 10% of cases.3- Fungal abscess accounts for 10% of cases. Despite the best therapy the mortality rate of liver abscess is still between 10% and 30%.

According to A.C. Guyton the hepatic circulation occurs due to blood pressure gradient, the high pressure of glomerular capillaries about 60 mm Hg cause rapid fluid filtration, whereas much lower hydrostatic pressure in the particular capillaries about 30 mm Hg permits rapid re-absorptio. Hepatic blood flow is not a linear function of the blood pressure gradient.

II. MATHEMATICAL MODELING

Let us the problem of blood flow in hepatic circulatory system which different from the problems in cylindrical tube and select generalized three dimensional orthogonal curvilinear coordinate system. Briefly described as E^3 Called as Euclidean space. According to Mishra, the biophysical laws thus expressed fully hold good in any co-ordinate system which is a compulsion for the truth fullness of the laws. According to I.W. Sherman and V.G. Sherman, blood is mixed fluid. Mainly there are two phase in blood. The first phase is plasma, while the other phase is that of blood cells of enclosed with a semi permeable membrane whose density is greater than that of plasma. These blood cells are uniformly distributed in plasma. Thus blood can be considered as a homogeneous mixture of two phases.

2.1 Equation of continuity for two phase blood flow

According to P. Singh and K.S. Upadhyay, the flow of blood is effected by the presence of blood cells. This effect is directly proportional to the volume occupied by blood cells. Let the volume portion covered by blood cells is unit volume be X, this X is replaced by H/100, Where Hematocrit the volume percentage of blood cells. Then the volume portion covered by plasma will be 1-X. if the mass ratio of blood cells to plasma is r, then clearly

$$r = \frac{X\rho_c}{(1-X)\rho_p} \tag{1}$$

Where ρ_c and ρ_p are densities of blood cells and blood plasma respectively. This mass ratio is not a constant, even then this may be supposed to constant in present context.

The both phase of blood i.e. blood cells and plasma move with the common velocity. Campbell and pitcher have presented a model for this situation. According to this model, we consider the two phase of blood separately. Hence the equation of continuity for two phase according two the principle of conservation of mass defined by J.N. Mishra and R.C Gupta as follows.

$$\frac{\partial(X\rho_c)}{\partial t} + \left(X\rho_c V^i\right)_{,i} = 0 \tag{2}$$

$$\frac{\partial(1-X)\rho_p}{\partial t} + \left(X\rho_c V^i\right)_{,i} = 0 \tag{3}$$

Where v is the common velocity of the two phase hepatic blood cells and plasma. Again $(X\rho_c V^i)_{,i}$ is co-variant derivative of $(X\rho_c V^i)$ with respect to x^i.

If v define the uniform density of hepatic blood ρ_m as follows.

$$\frac{1+r}{\rho_m} = \frac{r}{\rho_c} + \frac{1}{\rho_p} \tag{4}$$

Then equation (2) and (3) can be combined together as follows :

$$\frac{\partial\rho_m}{\partial t} + \left(\rho_m v^i\right)_{,i} = 0 \tag{5}$$

2.2 Equation of motion for two phase hepatic blood-flow

According to H.D. and T.C. Ruche. The hydro dynamical pressure p between the two phases of blood can be supposed to be uniform because the both phases i.e. blood cells and plasma are always equilibrium state in blood. Taking viscosity coefficient of blood cells to be η_c and applying the principle of conservation of momentum, we get the equation of motion for two phase of Hepatic blood cells as follows.

$$\frac{X\rho_c \partial v^i}{\partial t} + \left(X\rho_c V^j\right) v,^i_j = -Xp,_j g^{ij} + X\eta_c \left(g^{jk} v,^i_k\right),_j \tag{6}$$

Similarly, taking the viscosity coefficient of plasma to be η_p , the equation of motion for plasma will be as follows

$$(1-X)\rho_p \frac{\partial v^i}{\partial t} + \left\{(1-X)\rho_p v^i\right\} v^i_j = -(1-X)\rho_{p,j} g^{ij} + (1-X)\rho, jg^{ij} + (1-X)\eta_p \left(g^{jk} V^i_k\right), j \tag{7}$$

Now adding equation (6) and (7) and using relation (4) the equation of motion for blood flow with the both phases will be as follows:

$$\rho_m \frac{\partial v^i}{\partial t} + \left(\rho_m v^j\right) v^i,_j = -p,_j g^{ij} \eta_m \left(g^{jk} v^i,_k\right),_j \tag{8}$$

Where $\eta_m = X\eta_c+(1-X)\ \eta_p$ is the viscosity coefficient of blood as a mixture of two phases. As the velocity of blood flow decreases, the viscosity of blood increases. The velocity of blood decreases successively because of the fact that arteriles, veinules and veins. These vessels are relatively a far enough from the heart. Hence the pumping of the heart of the vessels is relatively low. Secondly these vessels relatively narrow down more rapidly. In this situation, the blood cells line up on the exis to build up rouleaux. Hence a yield stress is produced. Though this yield stress is very small, even then viscosity of blood is increased nearly ten times.

$T' = \eta_m e^n + T'_O (T' \geq T'_O)$

and $e=0\ (T' < T'_O)$

T'_O is the yield stress. When strain rate $e=0 (T' < T'_O)$ a core region is formed which flows just like a plug.let the radius of the plug be r_p. The stress acting on the surface of plug will be T'_O. Equating the forces acting on the plug, we get.

$P.\pi\, r_p^2 = T'_O.2\,\pi\, r_p$

$$\Longrightarrow\ r_p = \frac{2\,T'_O}{p} \tag{9}$$

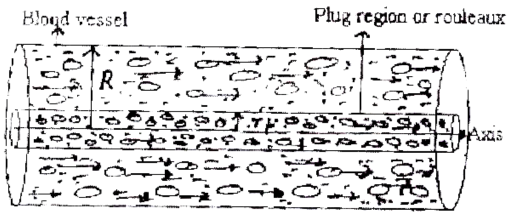


Fig (3.1) : Herschel-Bulkley blood flow

The constituting equation for rest part of blood vessel is

$T' = \eta_m e^n + T_p$

$\Rightarrow\ T' - T_p = \eta_m e^n = T_e$

The Constituting equation for rest part of blood vessel is

$T' = \eta_m e^n + Tp$ or $T' - T_p = \eta_m e^n = T_e$

Where T_e= Effective stress.

Whose generalized form will be as Follows:

$T^{ij} = -p\, g^{ij} + T_e^{ij}$

Where g^{ij} = metric tensor

Equation of Continuity –

$1/\sqrt{g}(\sqrt{g}v^i)_{,i}=0$

Equation of motion

$$\rho_m \frac{\partial v^i}{\partial t} + \rho_m v^j v^i{}_{,j} = -T_{,j}^{ij} \quad (10)$$

Where all the symbols have their usual meaning.

2.3 Solution and discussion-

The blood vessels are cylindrical, but we have derived equations in tensorial form so we transform tensorial form into cylindrical form by using the following cylindrical co-ordinate system such as.

$X^1 = r,\ X^2 = \theta,\ X^3 = Z$

Matrix of metric tensor in cylindrical co-ordinates is as follows

$$[g_{ij}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & r^2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

While matrix of conjugate metric tensor is as follows:

$$[g_{ij}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{r^2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Whereas the Christoffel's symbols of 2nd kind are as follows:

$$\left\{ \begin{matrix} 1 \\ 2\ 2 \end{matrix} \right\} = -r, \left\{ \begin{matrix} 2 \\ 2\ 1 \end{matrix} \right\} = \left\{ \begin{matrix} 2 \\ 1\ 2 \end{matrix} \right\} = \frac{1}{r}, \text{remaining others are zero}$$

Relation between contravariant and physical components of velocity of blood flow will be as follows:

$\sqrt{g_{11}}\ v^1 = v_r \Longrightarrow v_r = v^1$

$\sqrt{g_{22}}\ v^2 = V_\theta \Longrightarrow V_\theta = rv^2$

$\sqrt{g_{33}}\ v^3 = V_z \Longrightarrow V_z = v^3$

The Equation used in Herschel- bulkley Model are transformed into cylindrical form so as to solve them as power law model we get

$$-\frac{dv}{dr} = \left[\frac{\frac{1}{2}Pr - T'_0}{\eta_m} \right]^{\frac{1}{n}}$$

By substituting the value we get

$$-\frac{dv}{dr} = \left[\frac{\frac{1}{2}Pr - \frac{1}{2}pr_p}{\eta_m}\right]^{\frac{1}{n}} \tag{11}$$

$$\frac{dv}{dr} = -\left[\frac{p}{2\eta_m}\right]^{\frac{1}{n}} \left(r - r_p\right)^{\frac{1}{n}} \tag{12}$$

Integrating above equation under the no slip boundary condition: v = 0 and r = R so as to get

$$v = \left[\frac{p}{2\eta_m}\right]^{\frac{1}{n}} \frac{n}{n+1}\left[\left(R - r_p\right)^{\frac{1}{n+1}} - \left(r - r_p\right)^{\frac{1}{n+1}}\right] \tag{13}$$

This is the formula for the velocity of blood flow in hepatic veinules and veins.

2.4 Results : (Bio- Physical interpretation)
Observations: Hematocrit vs. Blood pressure from authorized. Health center of Kanpur by Dr. S.K. katiyar (Gastrolagist)

Patient Name : Mr. Rajendra
Age : 42 year
Diagnosis : Liver abscess
Lab No : AB 07064801

S.No.	Date	Hemoglobin	Hematocrit	Blood Pressure
1	14.11.12	5.7	17.10	100/60
2	15.11.12	5.68	17.04	96/60
3	17.11.12	5.0	15	90/60
4	20.11.12	7.6	22.8	130/80

The Flow Flux of two phased blood flow in arterioles veinules and veins is

$$Q = \int_0^{r_p} 2\pi r v_p dr + \int_{r_p}^{R} 2\pi r v\, dr$$

$$= \int_0^{r_p} 2\pi r \frac{n}{n+1}\left[\frac{p}{2\eta_m}\right]^{\frac{1}{n}} \left(R - r_p\right)^{\frac{1}{n}+1} dr + \int_{rp}^{R} 2\pi r \frac{n}{n+1}\left[\frac{p}{2\eta_m}\right]^{\frac{1}{n}} \left[\left(R - r_p\right)^{\frac{1}{n}+1} - \left(r - r_p\right)^{\frac{1}{n}+1}\right] dr$$

using 12 and 13 we get

$$Q = \frac{\pi\eta}{n+1}\left[\frac{p}{2\eta_m}\right]^{\frac{1}{n}} R^{\frac{1}{n}+3} \left[\frac{r^2{}_p}{R^2}\left(1 - \frac{r_p}{R}\right)^{\frac{1}{n}+1} + \left(1 + \frac{r_p}{R}\right)\left(1 - \frac{r_p}{R}\right)^{\frac{1}{n}+2} - \frac{2\left(1 - \frac{r_p}{R}\right)^{\frac{1}{n}+2}}{\left(\frac{1}{n}+2\right)} + \frac{2\left(1 - \frac{r_p}{R}\right)^{\frac{1}{n}+3}}{\left(\frac{1}{n}+2\right)\left(\frac{1}{n}+3\right)}\right]$$

Now let R = 1 And r_p= 1/3 ve get

$$Q = \frac{\pi n}{(n+1)}\left[\frac{p}{2\eta_m}\cdot\frac{2}{3}\right]^{\frac{1}{n}}\frac{2}{27}\left[\frac{26n^2+33n+9}{6n^2+5n+1}\right]$$

$$\frac{27Q}{2\pi} = \left[\frac{p}{3\eta_m}\right]^{\frac{1}{n}}\left[\frac{26n^3+33n^2+9n}{6n^3+11n^2+6n+1}\right] \qquad (14)$$

Now

Q = 1000ml/min, η_m =.035(pascal-sec.)

η_p = 0.0015(pascal-sec.)

and H = 15.0, P= 90

We know that

$\eta_m = \eta_c X+\eta_p (1-X)$ where X = H/100

$0.035 =\eta_c (15.0/100)+0.0015(1-15.0/100)$

$0.035= \eta_c (0.15)+0.0015(1-0.15)$

$\eta_c = 0.2248$

There for,

$$\eta_m = 0.002233 H+0.0015 \qquad (15)$$

From equation (14)

$$\frac{27Q}{2\pi} = \left[\frac{p}{3\eta_m}\right]^{\frac{1}{n}}\left[\frac{26n^3+33n^2+9n}{6n^3+11n^2+6n+1}\right]$$

$$\frac{27X1000}{2x3.14} = \left[\frac{90}{3x0.035}\right]^{\frac{1}{n}}\left[\frac{26n^3+33n^2+9n}{6n^3+11n^2+6n+1}\right]$$

$$4299.36 = (857.1428)^{\frac{1}{n}}\left[\frac{26n^3+33n^2+9n}{6n^3+11n^2+6n+1}\right]$$

Solved by Numerical Method

We get **n =0.91843**

Now using equation (15) in equation (14) we get

$$P = (0.006699H+0.0045)\left[\frac{27Q}{2\pi}X\frac{26n^3+33n^2+9n}{6n^3+11n^2+6n+1}\right]^n$$

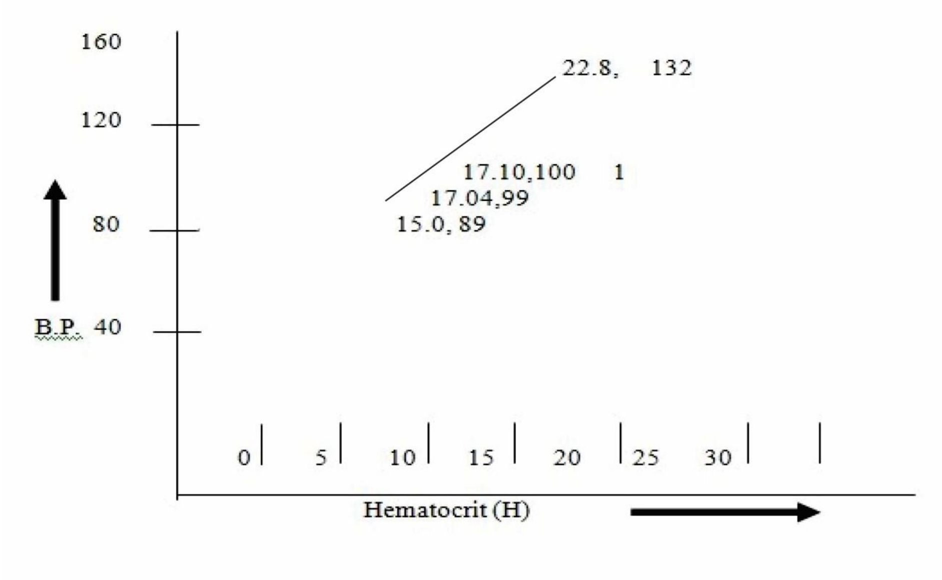
Where **n** = 0.91843

At H = 17.10 P = 99.98≈100

At H = 17.04 P = 98.9≈99

At H = 15 P = 89.17≈89

At H = 22.8 P = 131.92≈132



III. CONCLUSION -

A simple survey of the graph between blood pressure and hematocrit in Liver abscess patient shows that when hematocrit is increased the blood pressure also increased. Hence Hematocrit is proportional to blood pressure.

IV. ACKNOWLEDGEMENT -

I owe my scincere thanks to Dr. S.K. Katiyar. Astha Health Center Kanpur and thanks are due to Dr. Arun Kumar Gupta, Gyan pathology Center Kanpur.

Remark: If this good have been possible to get blood pressure on the particular tissue (Liver) then the relation between blood pressure and hemoglobin has been measured more accurately.

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