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Effect of Depth and Location of Minima of a Double-Well Potential on Vibrational Resonance in a Quintic Oscillator

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Abstract

We analyze the effect of depth and location of minima of a double-well potential on vibrational resonance in a linearly damped quintic oscillator driven by both low-frequency force $f\sin\omega t$ and high-frequency force f $\sin\Omega t$ with $\Omega >> \omega$. The response consists of a slow motion with frequency ω and a fast motion with frequency Ω. We obtain an analytical expression for the response amplitude Q at the low-frequency ω. From the analytical expression Q, we determine the values of ω and g (denoted as ω_{VR} and g_{VR}) at which vibrational resonance occurs. The depth and the location of the minima of the potential well have distinct effect on vibrational resonance. We show that the number of resonances can be altered by varying the depth and the location of the minima of the potential wells. Also we show that the dependence of ω_{VR} and g_{VR} by varying the above two quantities. The theoretical predictions are found to be in good agreement with the numerical result.

Keywords - Vibrational resonance; Quintic oscillator; Double-well potential; Biperiodic force.

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1. Introduction

In the last three decades the influence of noise on the dynamics of nonlinear and the chaotic systems was extensively investigated. A particular interesting example of the effects of the noise within the frame work of signal processing by nonlinear systems is stochastic resonance (SR) ie., the amplification of a weak input signal by the concerted actions of noise and the nonlinearity of the system. Recently, a great deal of interest has been shown in research on nonlinear systems that are subjected to both low- and high-frequency periodic signals and the associated resonance is termed as vibrational resonance (VR) [1, 2]. It is important to mention that two-frequency signals are widely applied in many fields such as brain dynamics [3, 4], laser physics [5], acoustics [6], telecomm-unications [7], physics of the ionosphere [8] etc. The study of occurrence of VR due to a biharmonical external force with two different frequencies ω and Ω with $\omega >> \Omega$ has received much interest. For example, Landa and McClintock [1] have shown the occurrence of resonant behavior with respect to a low-frequency force caused by the high-frequency force in a bistable system. Analytical treatment for this resonance phenomenon is proposed by Gitterman [2]. In a double-well Duffing oscillator, Blekhman and Landa [9] found single and double resonances when the amplitude or frequency of the high-frequency modulation is varied. So far this phenomenon has been studied in a monostable system [10], a multistable system [11], coupled oscillators [12, 13], spatially periodic potential system [14, 15], time-delayed systems [16, 17], noise induced structure [18], the FitzHugh-Nagumo equation [19], asymmetric Duffing oscillator [20], biological nonlinear maps [21] and so on.

In this paper we investigate the effect of depth of the potential wells and the distance between the location of a minimum and a local maximum of the symmetric double-well potential in a linearly damped quintic oscillator on vibrational resonance.

The equation of motion of the linearly damped quintic oscillator driven by two periodic forces is

$$\ddot{x} + d\dot{x} + \mathrm{A}\omega_0^2 x + \mathrm{B}\beta x^3 + \mathrm{C}\gamma x^5 = f\cos\omega t + g\cos\Omega t, \quad (1)$$

where $\Omega >> \omega$ and the potential of the system in the absence of damping and external force is

$$V(x) = \frac{1}{2}\mathrm{A}\omega_0^2 x^2 + \frac{1}{4}\mathrm{B}\beta x^4 + \frac{1}{6}\mathrm{C}\gamma x^6 \quad (2)$$

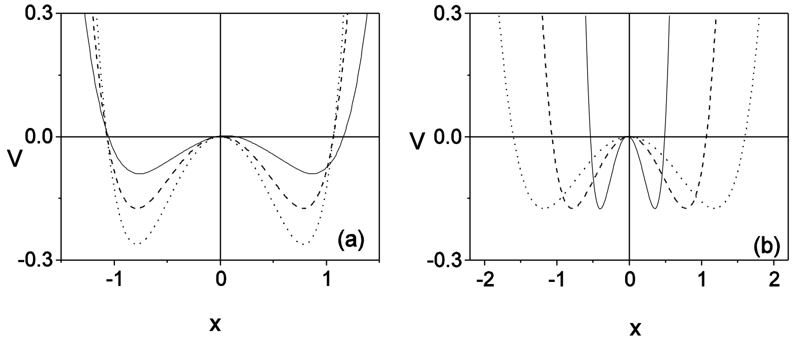

Fig.1: Shape of the double-well potential $V(x)$ for $\omega_0^2 = -1, \beta = 1$ and $\gamma = 1$ (a) $\mathrm{A} = \mathrm{B} = \mathrm{C} = \alpha_1$ and (b) $\mathrm{A} = \frac{1}{\alpha_2^2}$, $\mathrm{B} = \frac{1}{\alpha_2^4}$, $\mathrm{C} = \frac{1}{\alpha_2^6}$. In the sub plots, the values of α_1 (a) and α_2 (b) for continuous line, dashed line and painted circles are $0.5, 1$ and 1.5 respectively.

For $\omega_0^2 < 1$, β, γ, A, B, $\mathrm{C} > 0$ the potential $V(x)$ is of a symmetric double-well form. When $\omega_0^2 = -1, \beta = 1, \gamma = 1$ and $\mathrm{A} = \mathrm{B} = \mathrm{C} = 1$ there are equilibrium points at (0,0) and $\left(\pm\sqrt{\frac{\sqrt{5}-1}{2}}, 0\right)$. At (0,0), $\lambda = \pm 1$, so this point is a saddle. At $\left(\pm\sqrt{\frac{\sqrt{5}-1}{2}}, 0\right)$, $\lambda \cong \pm 0.935i$, so these points are centres. Recently, Jeyakumari et al [10, 11] analysed the occurrence of VR in the quintic oscillator with single-well, double-well and triple well forms of potential. When $\mathrm{A}=\mathrm{B}=\mathrm{C}=\alpha_1$ potential has a local maximum at $x_0^* = 0$ and two minima at $x_\pm^* = \pm\sqrt{\frac{-\beta \pm \sqrt{\beta^2 - 4\gamma\omega_0^2}}{2\gamma}}$. The depths of the left-and right-wells denoted by D_L and D_R respectively are same and equal to $\alpha_1[6\omega_0^2 p + 3\beta p^2 + 2\gamma p^3]/12$ where $p = \frac{-\beta \pm \sqrt{\beta^2 - 4\gamma\omega_0^2}}{2\gamma}$. By varying the parameter α_1 the depths of the two wells can be varied keeping the values of $x_\pm^*$ unaltered. We call the damped system with $\mathrm{A}=\mathrm{B}=\mathrm{C}=\alpha_1$ as DS1. We call the damped system with $\mathrm{A} = \frac{1}{\alpha_2^2}, \mathrm{B} = \frac{1}{\alpha_2^4}, \mathrm{C} = \frac{1}{\alpha_2^6}$ as DS2 in which case $x_0^* = 0$ where as $x_\pm^* = \pm\alpha_2\sqrt{\frac{-\beta \pm \sqrt{\beta^2 - 4\gamma\omega_0^2}}{2\gamma}}$ and $D_L = D_R = [6\omega_0^2 p + 3\beta p^2 + 2\gamma p^3]/12$ is independent of α_2. Thus, by varying α_2 the depth of the wells of $V(x)$ can be kept constant while the distance between the local maximum and the minima can be changed. Figures (1a) and (1b) illustrate the effect of α_1 and α_2.

In a very recent work, Rajasekar et al [22] analysed the role of depth of the wells and the distance of a minimum and local maximum of the symmetric double-well potential in both underdamped and overdamped Duffing oscillators on vibrational resonance. They obtained the theoretical expression for the response amplitude and the occurrence of resonances is shown by varying the control parameters ω, g, α and Ω. In the present work, we consider the system (1) with α_1 and α_2 arbitrary, obtain an analytical expression for the values of g at which resonance occurs and analyze the effect of depth of the wells and the distance between the location of a minimum and local maximum of the potential $V(x)$ on resonance.

The outline of the paper is as follows, for $\Omega >> \omega$ the solution of the system (1) consists of a slow motion $X(t)$ and a fast motion $\psi(t,\Omega t)$ with frequencies ω and Ω respectively. We obtain the equation of motion for the slow motion and an approximate analytical expression for theresponse amplitude Q of the low-frequency (ω) output oscillation in section II. From the theoretical expression of Q we obtain the theoretical expressions for the values of g and ω at which resonance occurs and analyse the effect of depth of the potential wells in section III and the distance between the location of a minimum and a local maximum of the symmetric double-well potential in section IV. We show that the number of resonances can be changed by varying the above two quantities such as α_1 and α_2. Finally section V contains conclusion.

2. Theoretical Description Of Vibrational Resonance

An approximate solution of Eq. (1) for $\Omega >> \omega$ can be obtained by the method of separation where solution is written as a sum of slow motion $X(t)$ and fast motion $\psi(t,\Omega t)$:

$$x(t) = X(t) + \psi(t,\Omega t) \qquad (3)$$

We assume that ψ is a periodic function with period $2\pi/\Omega$ or 2π-periodic function of fast time $\tau = \Omega t$ and its mean value with respect to the time τ is given by

$$\bar{\psi} = \frac{1}{2\pi}\int_0^{2\pi} \psi \, d\tau = 0. \qquad (4)$$

Substituting the solution Eq. (3) in Eq. (1) and using Eq. (4), we obtain the following equations of motion for X and ψ:

$$\begin{aligned} \ddot{X} + d\dot{X} + (\mathrm{A}\omega_0^2 + 3\mathrm{B}\beta\bar{\psi}^2 + 5\mathrm{C}\gamma\bar{\psi}^4)X \\ + (\mathrm{B}\beta + 10\mathrm{C}\gamma\bar{\psi}^2)X^3 + \mathrm{C}\gamma X^5 \\ + \mathrm{B}\beta\bar{\psi}^3 + \mathrm{C}\gamma\bar{\psi}^5 = f\cos\omega t, \end{aligned} \qquad (5)$$

$$\begin{aligned} \ddot{\psi} + d\dot{\psi} + \mathrm{A}\omega_0^2\psi + 3\mathrm{B}\beta X^2(\psi - \bar{\psi}) + 3\mathrm{B}\beta X(\psi^2 - \bar{\psi}^2) \\ + \mathrm{B}\beta(\psi^3 - \bar{\psi}^3) + 5\mathrm{C}\gamma X^4(\psi - \bar{\psi}) + 10\mathrm{C}\gamma X^3(\psi^2 - \bar{\psi}^2) \\ + 10\mathrm{C}\gamma X^2(\psi^3 - \bar{\psi}^3) + 5\mathrm{C}\gamma X(\psi^4 - \bar{\psi}^4) \\ + \mathrm{C}\gamma(\psi^5 - \bar{\psi}^5) + 10\mathrm{C}\gamma X^2\bar{\psi}^3 = g\cos\Omega t. \end{aligned} \qquad (6)$$

Because ψ is a fast motion we assume that $\ddot{\psi} >> \dot{\psi}, \ \psi, \ \psi^2, \ \psi^3, \ \psi^4, \ \psi^5$ and neglect all the terms in the left-hand-side of Eq. (6) except the term $\ddot{\psi}$. This approximation called *inertial approximation* leads to the equation $\ddot{\psi} = g\cos\Omega t$ the solution of which is given by

$$\psi = -\frac{g}{\Omega^2}\cos\Omega t. \qquad (7)$$

For the ψ given by Eq. (7) we find

$$\overline{\psi^2} = \frac{g^2}{2\Omega^4}, \ \overline{\psi^3} = 0, \ \overline{\psi^4} = \frac{3g^4}{8\Omega^4}, \ \overline{\psi^5} = 0. \qquad (8)$$

Then Eq. (5) for the slow motion becomes

$$\ddot{X} + d\dot{X} + C_1 X + C_2 X^3 + C\gamma X^5 = f \cos\omega t, \quad (9a)$$

where

$$C_1 = A\omega_0^2 + \frac{3B\beta g^2}{2\Omega^4} + \frac{15C\gamma g^4}{8\Omega^8}, \quad C_2 = B\beta + \frac{5C\gamma g^2}{\Omega^4}. (9b)$$

The effective potential corresponding to the slow motion of the system described by the Eq. (9) is

$$V_{eff}(X) = \frac{1}{2} C_1 X^2 + \frac{1}{4} C_2 X^4 + \frac{1}{6} C\gamma X^6. \qquad (10)$$

The equilibrium points about which slow oscillations take place can be calculated from Eq. (9). The equilibrium points of Eq. (9) are given by

$$X_1^* = 0, \quad X^*_{2,3} = \pm\left[\frac{-C_2 + \sqrt{C_2^2 - 4CC_1\gamma}}{2C\gamma}\right]^{1/2},$$

$$X^*_{4,5} = \pm\left[\frac{-C_2 - \sqrt{C_2^2 - 4CC_1\gamma}}{2C\gamma}\right]^{1/2} \qquad (11)$$

The shape, the number of local maxima and minima and their location of the potential $V(x)$ (Eq. (2)) depend on the parameters ω_0^2, β and γ. For the effective potential (V_{eff}) these depend also on the parameters g and Ω. Consequently, by varying g or Ω new equilibrium states can be created or the number of equilibrium states can be reduced.

We obtain the equation for the deviation of the slow motion X from an equilibrium point X^*. Introducing the change of variable $Y = X - X^*$ in Eq. (9a) we get

$$\ddot{Y} + d\dot{Y} + \eta_1 Y + \eta_2 Y^2 + \eta_3 Y^3 + \eta_4 Y^4 + C\gamma Y^5 = f \cos\omega t, \qquad (12a)$$

where

$$\eta_1 = C_1 + 3C_2 X^{*2} + 5C\gamma X^{*4}, \qquad (12b)$$

$$\eta_2 = 3C_2 X^* + 10C\gamma X^{*3}, \qquad (12c)$$

$$\eta_3 = C_2 + 10C\gamma X^{*2}, \qquad \eta_4 = 5C\gamma X^*. \quad (12d)$$

For $f \ll 1$ and in the limit $t \to \infty$ we assume that $|Y| \ll 1$ and neglect the nonlinear terms in Eq. (12). Then, the solution of linear version of Eq. (12a) in the limit $t \to \infty$ is $A_L \cos(\omega t - \varphi)$ where

$$A_L = \frac{f}{[(\omega_r^2 - \omega^2)^2 + d^2\omega^2]^{1/2}}, \quad \varphi = \tan^{-1}\left(\frac{\omega^2 - \eta_1}{d\omega}\right) \quad (13)$$

and the resonant frequency is $\omega_r = \sqrt{\eta_1}$. When the slow motion takes place around the equilibrium point $X^* = 0$, then $\omega_r = \sqrt{C_1}$.

The response amplitude Q is

$$Q = \frac{A_L}{f} = \frac{1}{[(\omega_r^2 - \omega^2)^2 + d^2\omega^2]^{1/2}}. \qquad (14)$$

3. Effect Of Depth Of The Potential Wells On Vibrational Resonance

In this section we analyze the effect of depth of the potential wells on vibrational resonance in the system DS1. From the theoretical expression of Q we can determine the values of a control parameter at which the vibrational resonance occurs. We can rewrite Eq. (14) as $Q = 1/\sqrt{S}$ where

$$S = (\omega_r^2 - \omega^2)^2 + d^2\omega^2 \qquad (15)$$

and ω_r is the natural frequency of the linear version of equation of slow motion (Eq. (9)) in the absence of the external force $f\cos\omega t$. It is called resonant frequency (of the low-frequency oscillation). Moreover, ω_r is independent of f, ω and d and depends on the parameters ω_0^2, β, γ, g, Ω and α_1. When the control parameter g or Ω or α_1 is varied, the occurrence of vibrational resonance is determined by the value of ω_r. Specifically, as the control parameter g or ω or Ω or α_1 varies, the value of ω_r also varies and a resonance occurs if the value of ω_r is such that the function S is a minimum. Thus a local minimum of S represents a resonance. By finding the minima of S, the value of g_{VR} or ω_{VR} or Ω_{VR} at which resonance occurs can be determined. For example

$$\omega_{VR} = \sqrt{\omega_r^2 - \frac{d^2}{2}}, \qquad \omega_r^2 > \frac{d^2}{2}. \qquad (16)$$

For fixed values of the parameters, as ω varies from zero, the response amplitude Q becomes maximum at $\omega = \omega_{VR}$ given by Eq. (16). Resonance does not occur for the parametric choices for which $\omega_r^2 < d^2/2$. When ω is varied from zero, ω_r remains constant because it is independent of ω. In figure (2), ω_{VR} versus g is plotted for three values of d with $\alpha_1 = 0.75$ and 2.0. The values of the other parameters are $\omega_0^2 = -1, \beta = \gamma = 1$ and $\Omega = 10$. ω_{VR} is single valued. Above a certain critical value of d and for certain range of fixed values of g resonance cannot occur when ω is varied. For example, for $d = 0.5$, 1.0, 1.5 and $\alpha_1 = 0.75$ the resonance will not occur if $g \in [64.03,\ 70.49], [56.87,\ 79.81]$ and $[42.54, 91.28]$ respectively. For $\alpha_1 = 2.0$ and $d = 0.5, 1.0, 1.5$ the resonance will not occur if $g \in [65.11, 67.26]$, $[62.96, 72.28]$ and

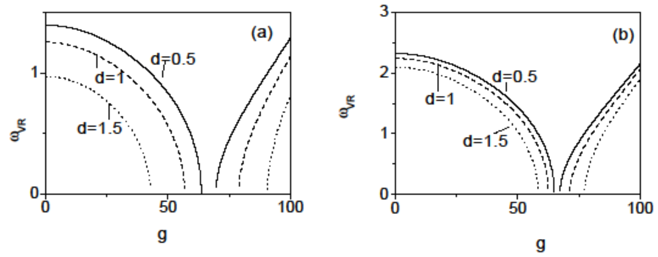

Fig.2: Plot of ω_{VR} versus g for three different values of d with (a) $\alpha_1 = 0.75$ and (b) $\alpha_2 = 2.0$. The value of the other parameters are $\omega_0^2 = -1, \beta = \gamma = 1$ and $\Omega = 10$.

[57.95, 78.02] respectively. Analytical expression for the width of such nonresonance regime is difficult to obtain because $\omega_r^2 = \eta_1$ is a complicated function of g. In fig. (2), we notice that the nonresonance interval of g increases with increase of d but it decreases with increase of α_1. We fix the parameters as $\omega_0^2 = -1, \beta = \gamma = 1, f = 0.05, g = 55$ and $\Omega = 10$. Figures (3a) and (3b) show Q versus ω for $d = 0.5, 1.0, 1.5$ with $\alpha_1 = 0.75$ and 2.0. Continuous curves represent theoretical result obtained from Eq. (14). Painted circles represent numerically calculated Q. We have calculated numerically the sine and cosine components Q_S and Q_C respectively, from the equations

$$Q_S = \frac{2}{nT}\int_0^{nT} x(t)\sin\omega t\,dt, \qquad (17a)$$

$$Q_C = \frac{2}{nT}\int_0^{nT} x(t)\cos\omega t\,dt, \qquad (17b)$$

where $T = 2\pi/\omega$ and n is taken as 500·

Then

$$Q = \frac{\sqrt{Q_S^2 + Q_C^2}}{f} \qquad (17c)$$

numerically computed Q is in good agreement with the theoretical approximation. In fig. (3a**)**, for $\alpha_1 = 0.75$ and $g = 55$ resonsnce occurs for $d = 0.5$ and 1 while for $d = 1.5$ the value of Q decreases continuously when ω is varied. For $\alpha_1 = 0.75$ and $g = 55$ resonance occurs for $d = 0.5$ and the response amplitude Q is found to be maximum at $\omega = 1.02$ and 0.75. For $\alpha_1 = 2.0$ and $g = 55$ resonance occurs for

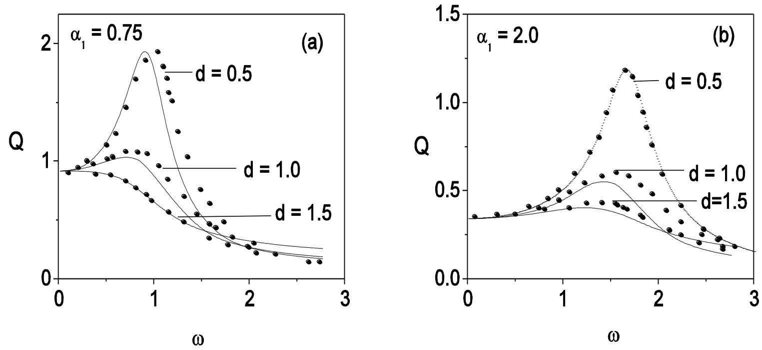

Fig.3: Response amplitude Q versus ω for three values of d with (a) $\alpha_1 = 0.75$ and (b) $\alpha_2 = 2.0$. Continuous curves represent the theoretically calculated Q from Eq. (14) with $\omega_r = \sqrt{C_1}$, while painted circles represent numerically computed Q from Eq. (17). The values of the other parameters are $\omega_0^2 = -1, \beta = \gamma = 1, f = 0.05$, g = 55 and $\Omega = 10$.

$d = 0.5, 1.0, 1.5$. The response amplitude Q is maximum at $\omega = 1.62$, 1.5 and 1.35. Both ω_{VR} and Q at the resonance decreases with increase in d. The above resonance phenomenon is termed as *Vibrational Resonance* as it is due to the presence of the high-frequency external periodic force.

Now we compare the change in the slow motion $X(t)$ described by the Eq. (9) and the actual motion $x(t)$ of the system (1). The effective potential can change into other forms by varying either g or Ω. Figure 4 depicts V_{eff} for three values of g with $\alpha_1 = 0.75$ (fig.4a) and 2.0 (fig.4b). The other parameters are $\omega_0^2 = -1$, $\beta = \gamma = 1$ and $\Omega = 10$. V_{eff} is a double-well potential for $g = 55$ while it becomes a single-well potential for $g = 70$ and $g = 90$. For $\alpha_1 = 0.75, d = 0.5$ and $g = 55$ the value of $\omega_{VR} = 0.765$ and for $\alpha_1 = 2.0, d = 0.5$ and $g = 55$, ω_{VR} is 1.25. The system (1) has two co-existing orbits and the associated

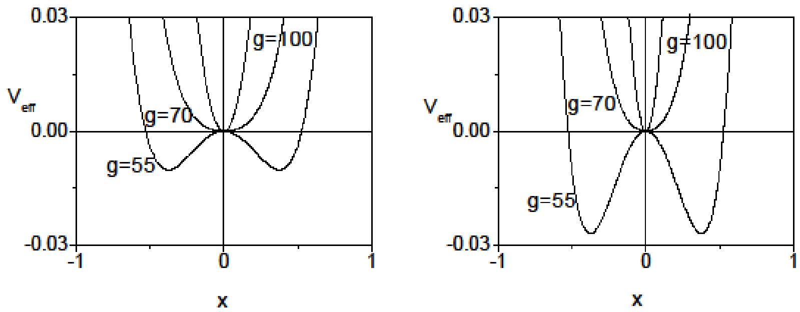

Fig.4: Shape of effective potential V_{eff} for $\omega_0^2 = -1, \beta = \gamma = 1$ and $\Omega = 10$ with (a) $\alpha_1 = 0.75$ and (b) $\alpha_1 = 2$ for three values of g.

slow motion takes place around the two equilibrium points $X^*_{2,3}$. This is shown in fig. (5a) for $\omega = 0.5, 1.0$ and 1.25. The corresponding actual motions of the system (Eq.(1)) are shown in figs. (5(b)-5(d)). For a wide range

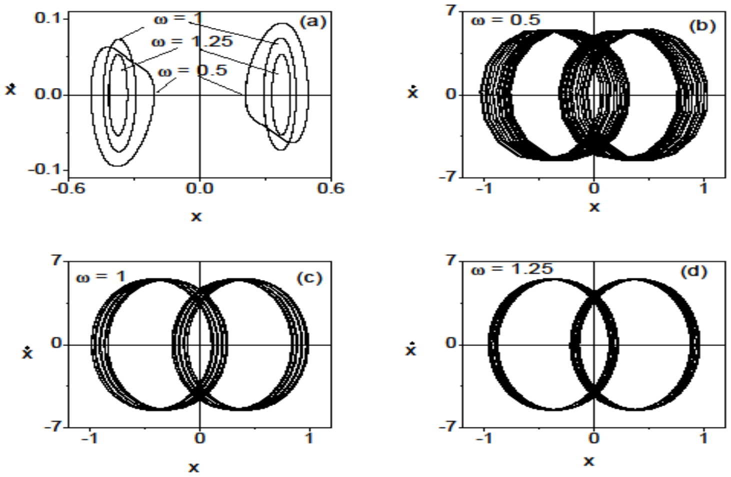

Fig.5: Phase portraits of (a) slow motion and (b-d) actual motion of the system (1) for few values of ω with $g = 55$ and d = 0.5.

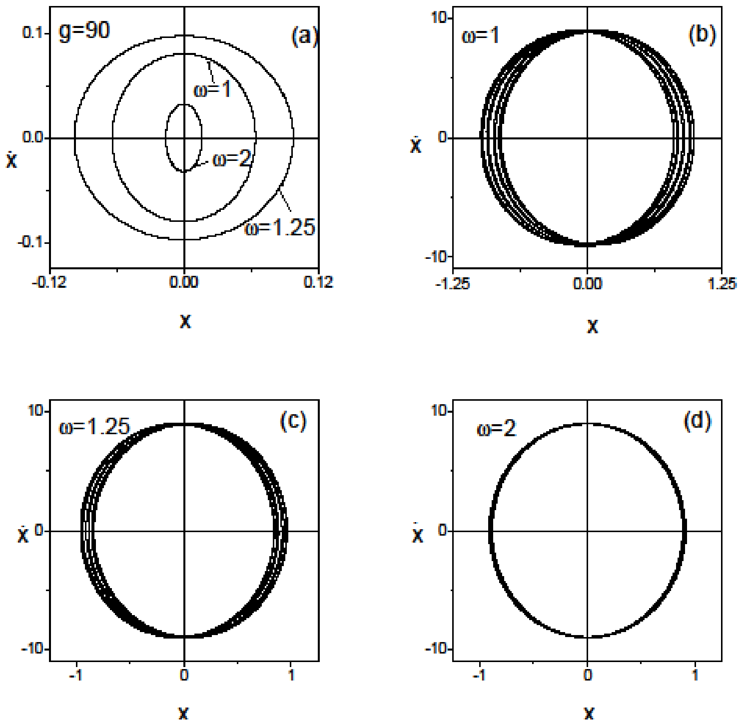

Fig.6: Phase portraits of (a) slow motion and (b-d) actual motion of the system (1) for few values of ω with g = 90 and d = 0.5.

of values of ω including the values of ω_{VR}, $x(t)$ is not a cross-well motion, that is not crossing both the equilibria $(x^*, y^*(=\dot{x}^*))(\pm 0.7521, 0)$. When $g = 90$, V_{eff} is a single-well potential and $\omega_{VR} = 0.9925$ for $\alpha_1 = 0.75$ and for $\alpha_1 = 2.0$, ω_{VR} is 1.72. The slow oscillation takes place around $X_1^* = 0$ [fig.(6a)] and $x(t)$ encloses with the minimum $(x^* = \pm 0.7521)$ and the local maxima $(x = 0)$ of the potential for all valuesof ω [figs. (6b)-6(d)]. Next, we determine g_{VR} which are the roots of $S_g = \frac{dS}{dg} = 4(\omega_r^2 - \omega^2)\omega_r \omega_{rg} = 0$ with $S_{gg}\big|_{g=g_{VR}} > 0$ where $\omega_{VR} = \frac{d\omega_r}{dg}$. The variation in ω_r with g for four values of α_1 is shown in fig.(7). For each fixed value of α_1 as g increases from zero, the resonant frequency ω_r decreases upto $g = g_0 = 65.69$. The value of $g(= g_0)$ at which V_{eff} undergoes bifurcation from a double-well to a single-well is independent of α_1. For $g > g_0$, V_{eff} becomes a single-well potential and ω_r increases with increase in g. Resonance will take place whenever $\omega_r = \omega$. Further, for $g > g_0$, V_{eff} is a single-well potential and $\omega_r^2 = C_1$. In this case an analytical expression for g_{VR} can be easily obtained from $\omega_r = \sqrt{C_1} = \omega$ and is given by

$$g_{VR} = \Omega^2 \left[\frac{-B\beta + \sqrt{B\beta^2 + 10C\gamma \left| (A\omega_0^2 - \omega^2) \right| / 3}}{5C\gamma / 2} \right]^{1/2}, \quad \omega^2 > A\omega_0^2 \qquad (18)$$

For $g < g_0$ the resonance frequency is $\sqrt{\eta_1}$. The value of $g = g_0$ at which V_{eff} undergoes bifurcation from a double-well to a single-well is independent of α_1. The analytical determination of the roots of $S_g = 0$ and g_{VR} is difficult because C_1 and C_2 and $X^*_{2,3}$ are function of g and $\omega_r^2 = \eta_1$ is a complicated function of g.

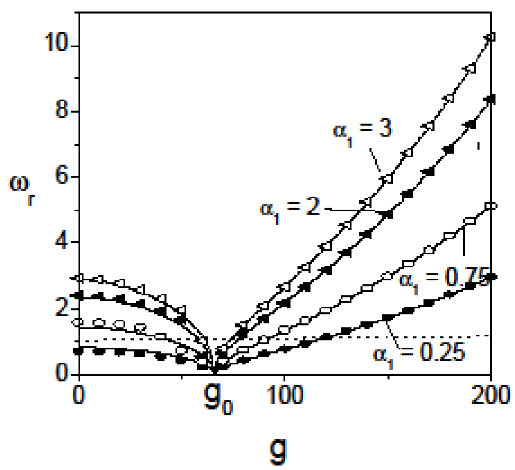

Fig.7: Plot of g versus ω_r for few values of $\alpha_1 = 0.25, 0.75, 2$ and 3. The values of the other parameters are $\omega_0^2 = -1, \beta = 1, \gamma = 1, f = 0.05$ and $\Omega = 10$.

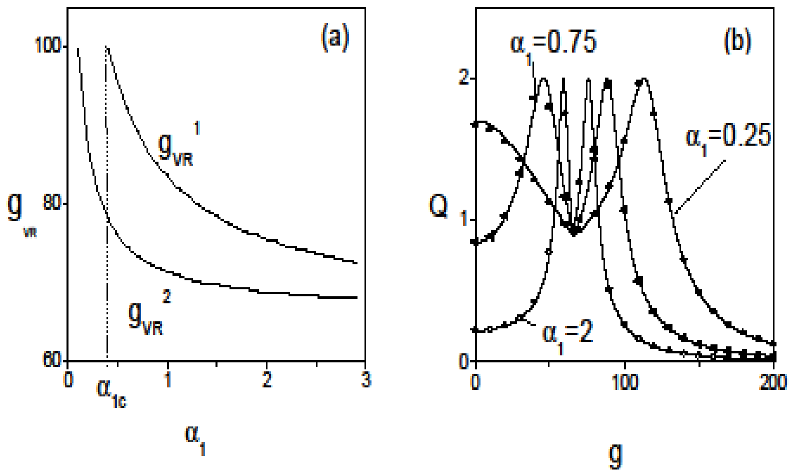

Fig.8: (a) Plot of theoretical g_{VR} versus α_1 for the system (1) and (b) theoretical and numerical response amplitude Q versus g for a few values of $\alpha_1 = 0.25, 0.75$ and 2 with $d = 0.5$. Continuous curves are theoretical result and painted circles are numerical values of Q.

Therefore we determine the roots of $S_g = 0$ and g_{VR} numerically. We analyze the cases $(\omega_r^2 - \omega^2) = 0$ and $\omega_{rg} = 0$. In fig. 8(a), g_{VR} computed numerically for a range of α_1 is plotted. For $\alpha_1 > \alpha_{1c}$ there are two resonances - one at value of $g < g_0 (= 65.69)$ and another at a value of $g > g_0$. In fig. (7) for $\alpha_1 = 0.25 < \alpha_{1c} = 0.4059$ the ω_r curve intersects the $\omega = 1$ dashed lineat only one value of $g > g_0$ and so we get only one resonance for $\alpha_1 < \alpha_c$. Figure (8b) shows Q versus g for $\alpha_1 = 0.25, 0.75$ and 2 .Continuous curve represents theoretical results obtained from Eq. (18). Painted circles represents numerically calculated Q

from Eq. (17). For large values of α_1, the two g_{VR} values are close to $g_0 = 65.69$. As α_1 decreases the values of g_{VR} move away from g_0. For $\alpha_1 = 0.25$ we notice only one resonance. In the double resonance cases the two resonances are almost at equidistance from g_0 and the values of Q at these resonances are the same. However, the response curve is not symmetrical about g_0.

4. Effect Of Location Of Minima Of The Potential On Vibrational Resonance

For DS2 as α_2 increases the location of the two minima of $V(x)$ move away from the origin in opposite direction, ie., the distance between a minimum and local maximum $x_0^* = 0$ of the potential increases with increase in α_2. When the slow oscillation occurs about $x^* \neq 0$ then $\omega_r^2 = \eta_1$ and analytical determination of g_{VR} is difficult because η_1 is a complicated function of g. In this case numerically we can determine the value of g_{VR}. Figure (9) shows the plot of g_{VR} versus α_2 For $\alpha_2 < \alpha_{2c}$ there are two resonances while for $\alpha_2 > \alpha_{2c}$ only resonance. The above prediction is confirmed by numerical simulation. Figure (9b) shows the variation of ω_r with g for three values of α_2. The bifurcation point g_0 increases linearly with α_2. That is, at $\alpha_2 = 0.75, 1.2$ and 1.6, the values of g_0 are 48.54, 78.66 and 105.44 respectively. Sample response curves for three fixed values of α_2 are shown in fig. (9c). The difference in the effect of the distance of $x_\pm^*$ from origin over the depth of the potential wells can be seen by comparing the figs. (9a) and (8a). In DS1 two resonances occur above certain critical depth α_{1c} of the wells. In contrast to this in DS2 two resonances occur only for $\alpha_2 < \alpha_{2c}$. In fig. (9a) we infer that as α_2 increases from a small value (ie., as $x_\pm^*$ moves away from origin) g_{VR}^1 increases and reaches a maximum value 96.5 at $\alpha_2 = 1.15$. Then with further increase in α_2,

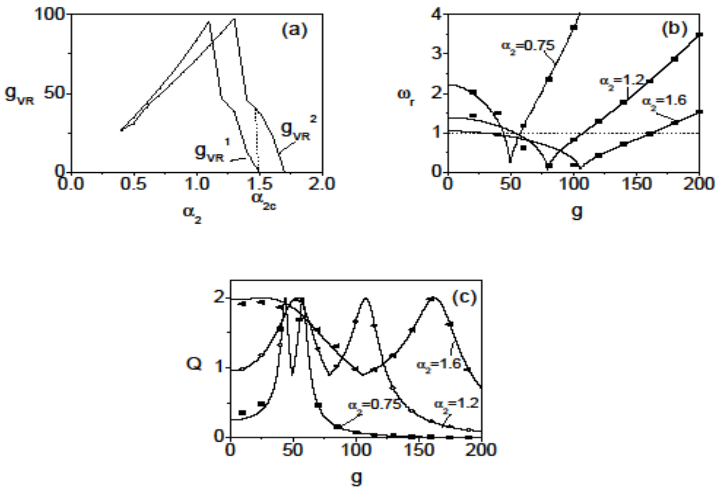

Fig.9: (a) Plot of theoretical g_{VR} versus α_2 for the system (1). (b) Theoretical and numerical ω_r versus g for a few values of $\alpha_2 = 0.75, 1.2$ and 1.6 with $\omega_0^2 = -1,\ \beta = 1,\ \gamma = 1,\ f = 0.05$ and $\Omega = 10$. The horizontal dashed line represents $\omega_r = \omega = 1$ (c) Theoretical and numerical response amplitude Q versus g for a few values of $\alpha_2 = 0.75, 1.2$ and 1.6 with $d = 0.5$. Continuous curves are theoretical result and painted circles are numerical values of Q.

it decreases and $\to 0$ as $\alpha_2 \to \alpha_{2c}$. Similarly g_{VR}^2 increases and reach a maximum value 98.5 at $\alpha_2 = 1.35$.Then with further increase in α_2, it decreases and $\to 0$ at $\alpha_2 = 1.7015$. We note that g_{VR}^1 and g_{VR}^2 of DS1 continuously decreases with increase of α_1. But in DS2, α_2 increases from a small value, g_{VR}^1 and g_{VR}^2 increase and reaches a maximum value. Then with further increase in α_2, it decreases and $\to 0$.

In double resonance case the separation between the two resonances increases with increase in α_2. The converse effect is noticed in DS1. Next we consider the dependence of ω_{VR} on g. ω_{VR} is given by Eq. (16). Figure (10) shows plot of ω_{VR} versus g for three different values of d with $\alpha_2 = 0.75$ and 1.2. For $d = 0.5,\ 1,\ 1.5$ and $\alpha_2 = 0.75$ the nonresonance intervals occur at $g \in [48.98, 51.14]$, $g \in [46.84, 54.72]$ and $g \in [43.25, 59.02]$ and for $\alpha_2 = 1.2$, the nonresonance intervals occur at $g \in [75.92, 84.12]$, $g \in [66.82, 95.95]$ and $g \in [49.50, 111.43]$. That is the nonresonance intervals of g increases with increase in α_1. But in DS1, nonresonance intervals of g decreases with increase in α_1. Figures (2) and (10) can be compared.

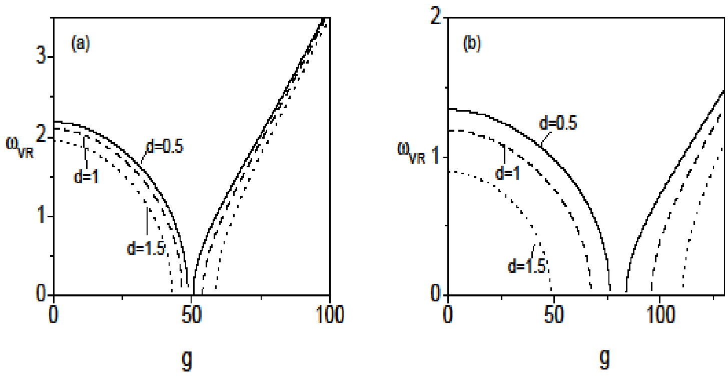

Fig.10: Plot of ω_{VR} versus g for three different of d with (a) $\alpha_2 = 0.75$ and (b) $\alpha_2 = 1.2$. The values of other parameters are $\omega_0^2 = -1, \beta = 1, \gamma = 1$ and $\Omega = 10$.

5. Conclusion

We have analysed the effect of depth of the wells and the distance of a minimum and the local maximum of the symmetric double-well potential in a linearly damped quintic oscillator on vibrational resonance. The effective potential of the system allowed us to obtain an approximate theoretical expression for the response amplitude Q at the low-frequency ω. From the analytical expression of Q, we determined the values of ω and g at which vibrational resonance occurs. In the system (1) there is always one resonance at a value of g, $g > g_0$, while another resonance occurs below g_0 for a range of values of ω. The two quantities α_1 and α_2 have distinct effects. The dependence of g_{VR} and ω_{VR} on these quantities are explicitly determined. The number of resonance and the value of g_{VR} and ω_{VR} can be controlled by varying the parameters α_1 and α_2. Q_{max} is independent of α_1 and α_2 in the system (1). g_{VR} of DS1 is independent of α_1 while in DS2 it depends on α_2.

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